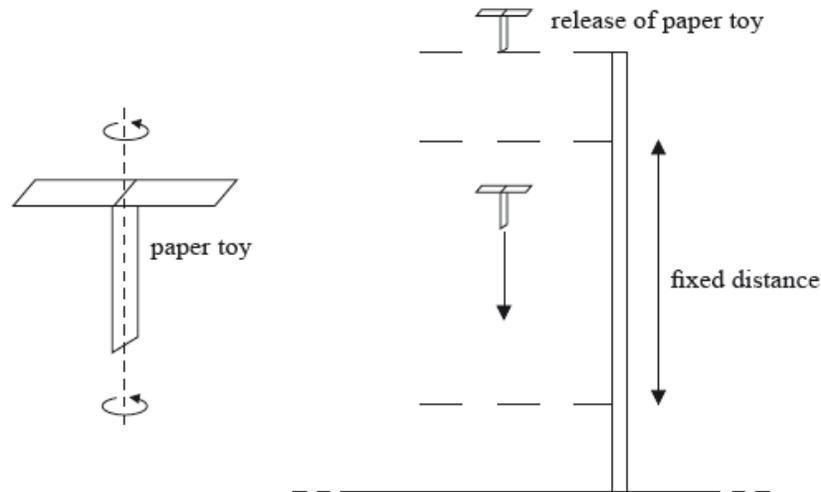


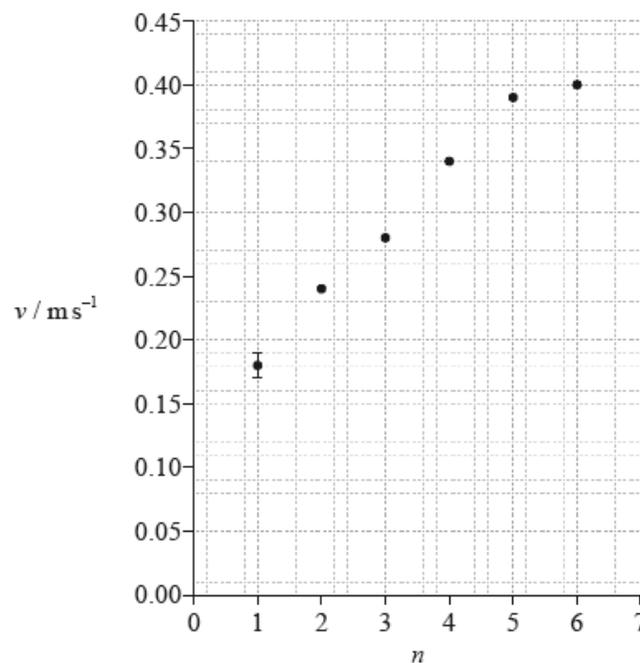
## SL Paper 2

A student performs an experiment with a paper toy that rotates as it falls slowly through the air. After release, the paper toy quickly attains a constant vertical speed as measured over a fixed vertical distance.



The aim of the experiment was to find how the terminal speed of the paper toy varies with its weight. The weight of the paper toy was changed by using different numbers of paper sheets in its construction.

The graph shows a plot of the terminal speed  $v$  of the paper toy (calculated from the raw data) and the number of paper sheets  $n$  used to construct the toy. The uncertainty in  $v$  for  $n = 1$  is shown by the error bar.



The fixed distance is 0.75 m and has an absolute uncertainty of 0.01 m. The percentage uncertainty in the time taken to fall through the fixed distance is 5%.

a.i. Calculate the absolute uncertainty in the terminal speed of the paper toy for  $n = 6$ .

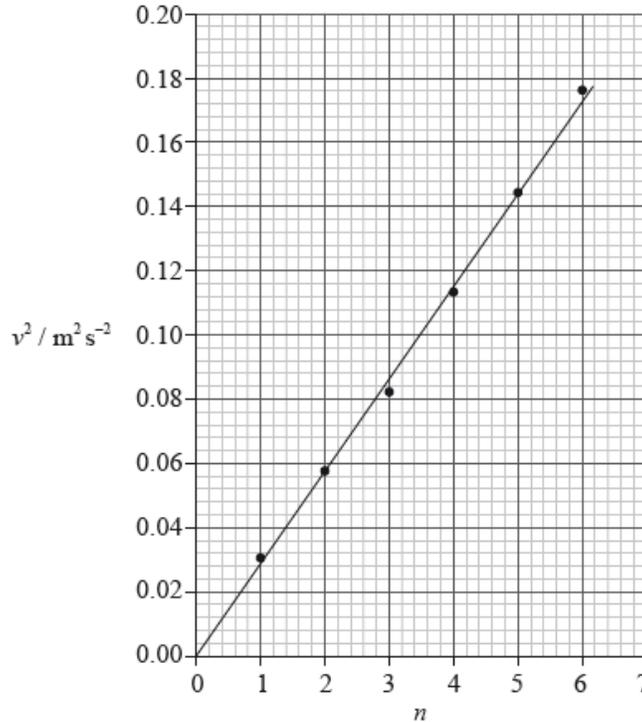
[3]

a.ii. On the graph, draw an error bar on the point corresponding to  $n = 6$ .

b. On the graph, draw a line of best-fit for the data points. [1]

c. The student hypothesizes that  $v$  is proportional to  $n$ . Use the data points for  $n = 2$  and  $n = 4$  from the graph opposite to show that this hypothesis is incorrect. [3]

d. Another student hypothesized that  $v$  might be proportional to  $n$ . To verify this hypothesis he plotted a graph of  $v^2$  against  $\sqrt{n}$  as shown below. [3]



Explain how the graph verifies the hypothesis that  $v$  is proportional to  $\sqrt{n}$ .

## Markscheme

a.i. percentage uncertainty in distance  $\left(\frac{0.01}{0.75} = \right) 1.3\%$ ;

percentage uncertainty in  $v = (5 + 1.3 =) 6.3\%$ ;

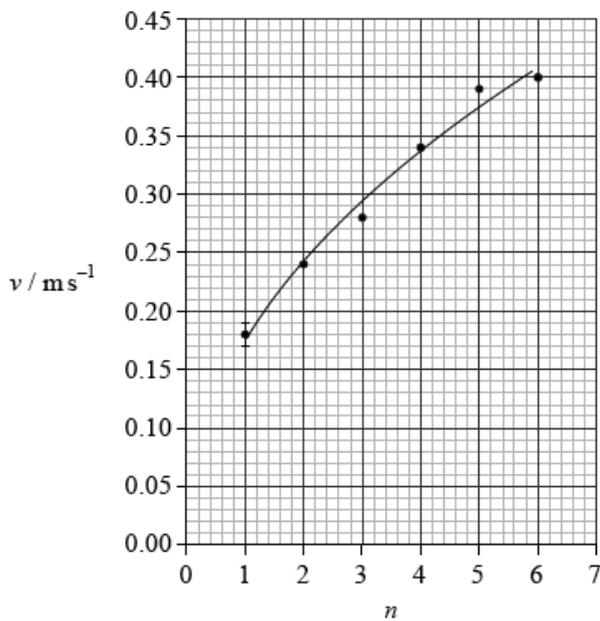
absolute uncertainty in  $n = 6$  point is  $(0.40 \times 0.063 =) 0.025 \text{ m s}^{-1}$ ;

a.ii. overall length of error bar drawn correct to within half a small square;

*Consistent with (a)(i).*

b. any reasonable smooth curve/straight-line passing through error bars

*Do not award where abrupt change of gradient occurs at  $n = 5$ .*



c. tests for  $\frac{v}{n}$  or  $\frac{n}{v}$ ;

$$\frac{v}{n} = 0.12 \text{ for } n = 2 \text{ and } \frac{v}{n} = 0.085 \text{ for } n = 4; \text{ (both needed)}$$

hypothesis incorrect because two values should be equal;

Accept for **[3]** read-off of both 0.24 and 0.34 together with the comment that 0.34 is not double 0.24.

Award **[2 max]** if no use of data but candidate has drawn curve with no straight portion and comments that line is not straight and does not go through origin.

Award **[1 max]** if no use of data but candidate has drawn a straight line and comments that line is straight but not through origin.

d. (if  $v \propto \sqrt{n}$ )  $v^2 \propto n$ ;

graph of  $v^2$  against  $n$  is a straight-line;

that goes through the origin;

## Examiners report

a.i. This was well done with almost all candidates understanding the combination of errors.

a.ii. Only the weakest candidates could not take the uncertainty value calculated in (a)(i) and transfer this correctly to the graph.

b. This was not well done. Too many candidates forced their lines through the printed origin, drew lines that lay outside the two error bars at each end of the range, or gave unrealistic abrupt changes of gradient.

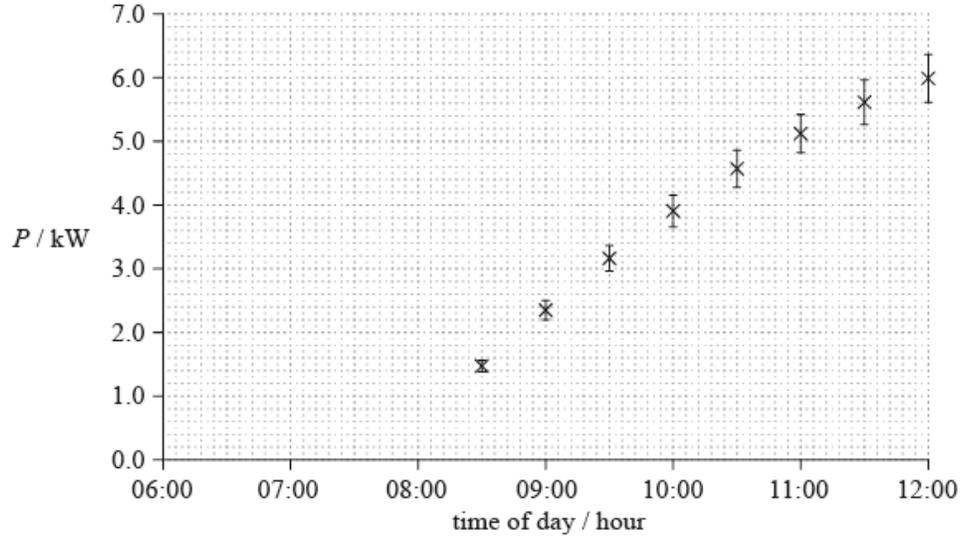
c. Many understood the need to compare ratios in some way and identify that the ratios were not the same for  $n = 2$  or 4. However, some candidates appeared unable to make any attempt at this question.

d. This is an example of a case where a clear and logical presentation is required so that examiners can understand what is in the candidate's mind.

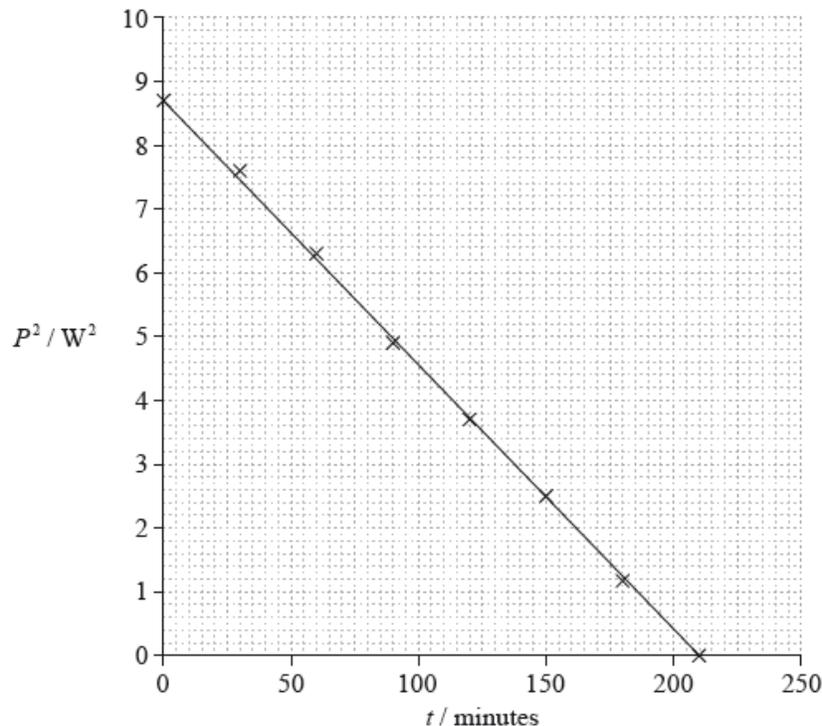
Data analysis question.

An array of photovoltaic cells is used to provide electrical energy for a house. When the array produces more power than the house consumes, the excess power is fed back into the mains electrical supply for use by other consumers.

The graph shows how the power  $P$  produced by the array varies with the time of day. The error bars show the uncertainty in the power supplied. The uncertainty in the time is too small to be shown.



- Using the graph, estimate the time of day at which the array begins to generate energy. [2]
- The average power consumed in the house between 08:00 and 12:00 is 2.0 kW. Determine the energy supplied by the array to the mains electrical supply between 08:00 and 12:00. [3]
- The power  $P$  produced by the array is calculated from the generated emf  $V$  and the fixed resistance  $R$  of the array using the equation  $\frac{V^2}{R}$ . The uncertainty in the value of  $R$  is 2%. Calculate the percentage uncertainty in  $V$  at 12:00. [3]
- Later that day a second set of data was collected starting at  $t = 0$ . The variation of  $P^2$  with time  $t$  since the start of this second data collection is shown in the graph. [3]



Using the graph, determine the relationship between  $P^2$  and  $t$ .

# Markscheme

- a. smooth curve drawn through all error bars and curve extrapolated appropriately to x-axis;

their own intercept correctly read to within  $\pm 6$  minutes (1 small square) / 7:50 hour to 8:00 hour if no line drawn;

- b. calculates total energy used by house = 8 (kWh) **or** 28.8 (MJ);

estimate of total area =  $14 \pm 1$  (kWh) **or**  $50.4 \pm 3.6$  (MJ);

$6 \pm 1$  (kWh) **or**  $21.6 \pm 3.6$  (MJ); (allow ECF from first two marking points)

**or**

clear attempt to estimate any area of graph;

correct calculation of area above 2 kW line on graph;

$6 \pm 1$  (kWh) **or**  $21.6 \pm 3.6$  (MJ); (allow ECF from first two marking points)

- c. read-off error bar at 12:00 hour as 0.4;

calculate uncertainty in  $P = \left( \frac{100 \times 0.4}{6.0} \right) = 6.6\%$  ;

$$\frac{\Delta V}{V} = \frac{\left[ \frac{\Delta P}{P} + \frac{\Delta R}{R} \right]}{2} = 2.3\%;$$

- d. intercept  $8.7 \pm 0.1$ ;

gradient equals  $\left( \frac{8.7}{210} \right) = (-)0.041$ ; (allow ECF from first marking point)

$P^2 = 8.7 - 0.041t$ ; } (negative sign essential) (allow ECF from first and second marking points)

Do not accept "inverse" relationship or "linear".

Award **[3]** for a bald correct answer.

Award **[2 max]** if gradient is left as a fraction.

## Examiners report

- a. Most candidates understood the requirements of the question. They were able to draw an acceptably smooth curve extrapolated to the time axis.

There were fewer poor quality lines than in previous examinations. Nevertheless, a substantial minority ended the curve at a time of 08:30 and then quoted this time as that at which the solar panels began to generate energy. Some drew a straight line (that could not possibly touch all the error bars) and extrapolated this line to a time of about 07:30. Some credit was available for this.

- b. This was poorly done from two points of view. It was clear that there was widespread misunderstanding of the relationship between energy and power units. Many candidates could get no further forward than calculating the energy used by the house in the four-hour time period. Most were unable to recognise that the energy supplied to the grid was related to the area under the graph.

- c. Candidates were usually able to read the error bar as having a total length of 0.8 units and could use this to calculate the uncertainty in  $P$ . Some were able to work through to the correct answer but many made a sign error in the calculation.

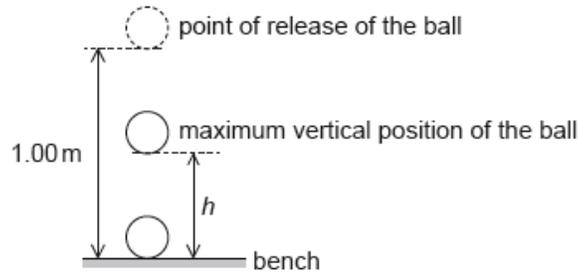
- d. A number of past data-analysis questions have asked for a simple statement of the proportionality or otherwise of a provided graph. The use of the command verb "determine should have indicated to student that this question required more. The full relationship was required for full marks. For

example, a determination of the gradient and the intercept (or solution from data points) to yield the equation relating  $P$  and  $t$  for the graph.

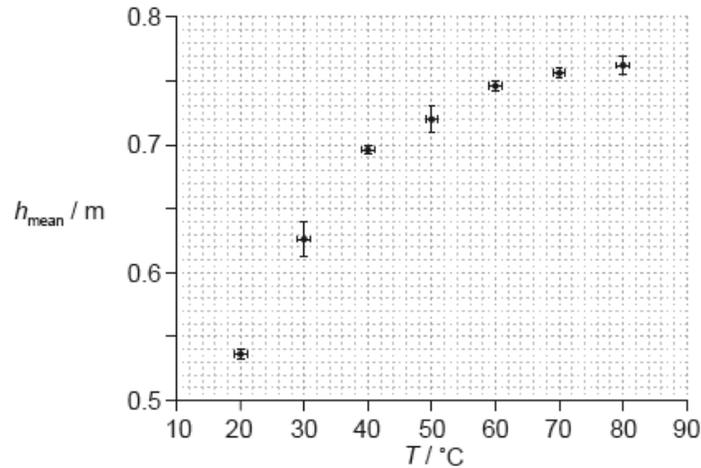
Data analysis question.

An experiment is undertaken to investigate the relationship between the temperature of a ball and the height of its first bounce.

A ball is placed in a beaker of water until the ball and the water are at the same temperature. The ball is released from a height of 1.00 m above a bench. The maximum vertical height  $h$  from the bottom of the ball above the bench is measured for the first bounce. This procedure is repeated twice and an average  $h_{\text{mean}}$  is calculated from the three measurements.



The procedure is repeated for a range of temperatures. The graph shows the variation of  $h_{\text{mean}}$  with temperature  $T$ .



- Draw the line of best-fit for the data. [1]
- State why the line of best-fit suggests that  $h_{\text{mean}}$  is not proportional to  $T$ . [1]
- State the uncertainty in each value of  $T$ . [1]
  - The temperature is measured using a liquid in glass thermometer. State what physical characteristic of the thermometer suggests that the change in the liquid's length is proportional to the change in temperature. [1]
- Another hypothesis is that  $h_{\text{mean}} = KT^3$  where  $K$  is a constant. Using the graph on page 2, calculate the absolute uncertainty in  $K$  corresponding to  $T = 50^\circ\text{C}$ . [4]

## Markscheme

- smooth curve line through all error bars;

*Do not allow kinked or thick lines or double/multiple lines.*

*Ignore any line beyond the range of plotted points.*

b. line (of best-fit) not straight/curved/changing gradient;

ratio of  $h$  to  $T \times 10^{-4}$  is not constant;

Allow “does not pass through origin” **only** if a straight line drawn in (a).

Otherwise treat as neutral.

c.i.  $(\pm)1^\circ \text{ C/K/deg}$ ; (do not allow 2 or more sig figs in the answer)

c.ii. equal graduations / constant cross-section/capillary diameter / (volume of) liquid expands linearly/proportionally to  $T$  / OWTTE;

Accept synonym for “capillary”, eg: “tube”.

d.  $\frac{\Delta h}{h} = \frac{0.01}{0.72}$  **or** 0.014 **or** 1.4% **and**  $\frac{\Delta T}{T} = \frac{1}{50}$  **or** 0.02 **or** 2%; (allow ECF from (c)(i))

$\frac{\Delta K}{K} = 3 \times \frac{1}{50} + \frac{0.01}{0.72}$  **or**  $7.4 \times 10^{-2}$  **or** 7.4%;

$K = 5.8/5.76/6 \times 10^{-6}$ ;

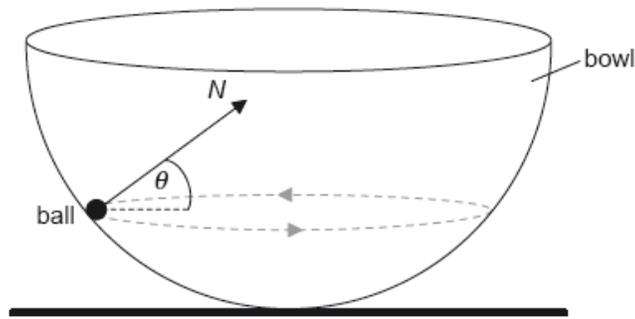
$\Delta K = 4 \times 10^{-7} \text{ m K}^{-3}$  **or**  $\text{m}^\circ\text{C}^{-3}$ ; (1 sig fig **and** correct unit required)

## Examiners report

- a. Many candidates were able to draw acceptably smooth curves but sometimes these failed to stay within the region of the error bar “box”. Only a handful attempted to draw a straight line through the points. On the whole, the technical drawing of the lines was better than in previous years but there are still too many thick, doubled or kinked lines.
- b. Many candidates stated that the line did not go through the origin. Although this answer was counted as neutral it showed that candidates were repeating by rote rather than applying their knowledge to the graph in question.
- c.i. This was usually correct. The main error was to quote the answer to 2 or more significant figures.
- c.ii. This was poorly done. The question asks about a physical characteristic of the thermometer and proportionality. Answers often just repeated the question in other words.
- d. There were many correct and well explained evaluations of the uncertainty in  $K$ . However many candidates failed to link the magnitude of the percentage uncertainty with a sensible significant figure for the final answer. Only 1 significant figure was accepted by examiners following the large final percentage error in the answer. A unit for the answer was also required and this too was frequently omitted.

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A small ball of mass  $m$  is moving in a horizontal circle on the inside surface of a frictionless hemispherical bowl.



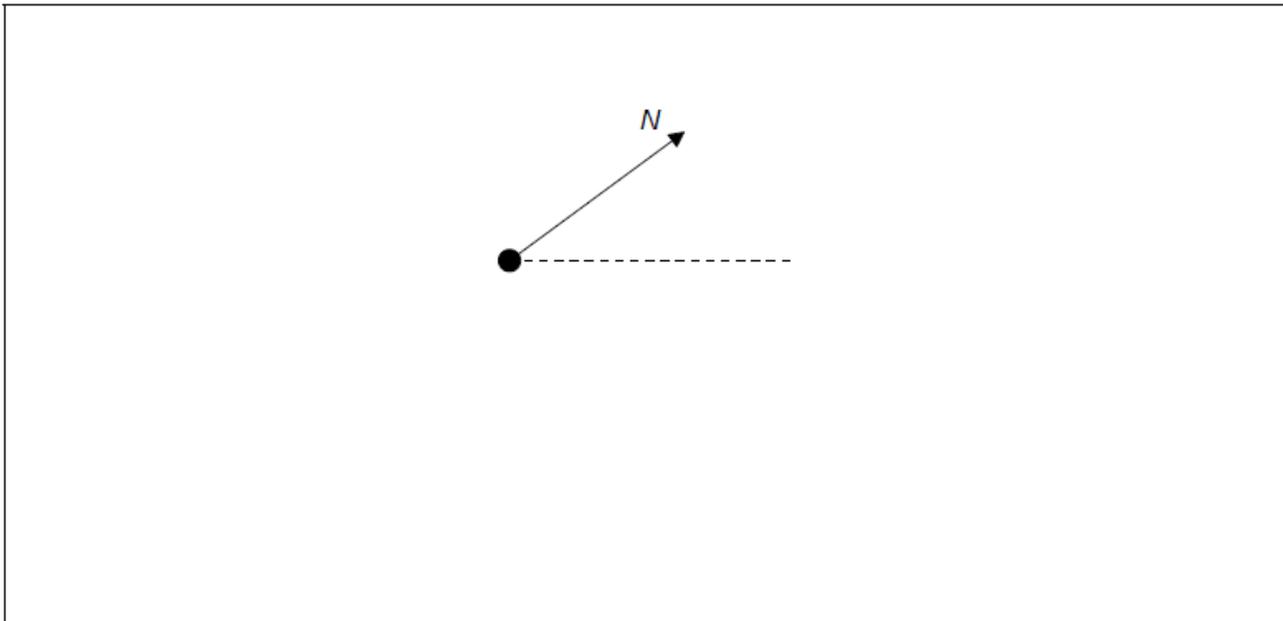
The normal reaction force  $N$  makes an angle  $\theta$  to the horizontal.

a.i. State the direction of the resultant force on the ball.

[1]

a.ii. On the diagram, construct an arrow of the correct length to represent the weight of the ball.

[2]



a.iii. Show that the magnitude of the net force  $F$  on the ball is given by the following equation.

[3]

$$F = \frac{mg}{\tan \theta}$$

b. The radius of the bowl is 8.0 m and  $\theta = 22^\circ$ . Determine the speed of the ball.

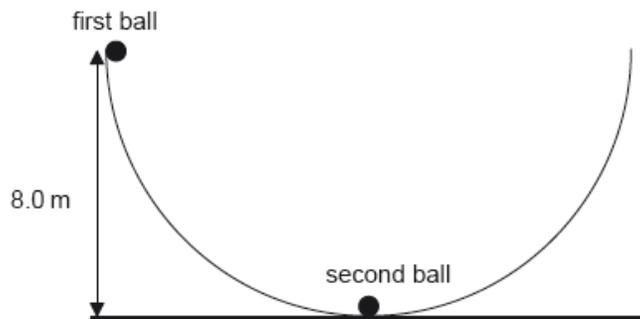
[4]

c. Outline whether this ball can move on a horizontal circular path of radius equal to the radius of the bowl.

[2]

d. A second identical ball is placed at the bottom of the bowl and the first ball is displaced so that its height from the horizontal is equal to 8.0 m.

[3]



The first ball is released and eventually strikes the second ball. The two balls remain in contact. Determine, in terms of any symbols used, the acceleration of the two balls.

# Markscheme

a.i. towards the centre «of the circle» / horizontally to the right

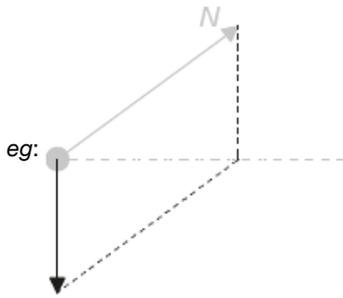
*Do not accept towards the centre of the bowl*

**[1 mark]**

a.ii. downward vertical arrow of any length

arrow of correct length

*Judge the length of the vertical arrow by eye. The construction lines are not required. A label is not required*



**[2 marks]**

a.iii **ALTERNATIVE 1**

$$F = N \cos \theta$$

$$mg = N \sin \theta$$

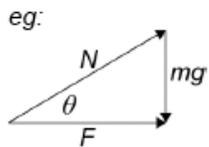
dividing/substituting to get result

**ALTERNATIVE 2**

right angle triangle drawn with  $F$ ,  $N$  and  $W/mg$  labelled

angle correctly labelled and arrows on forces in correct directions

correct use of trigonometry leading to the required relationship



$$\tan \theta = \frac{O}{A} = \frac{mg}{F}$$

**[3 marks]**

b.  $\frac{mg}{\tan \theta} = m \frac{v^2}{r}$

$$r = R \cos \theta$$

$$v = \sqrt{\frac{gR \cos^2 \theta}{\sin \theta}} / \sqrt{\frac{gR \cos \theta}{\tan \theta}} / \sqrt{\frac{9.81 \times 8.0 \cos 22}{\tan 22}}$$

Award **[4]** for a bald correct answer

Award **[3]** for an answer of 13.9/14 «ms<sup>-1</sup>». MP2 omitted

**[4 marks]**

c. there is no force to balance the weight/N is horizontal

so no / it is not possible

Must see correct justification to award MP2

**[2 marks]**

d. speed before collision  $v = \sqrt{2gR} \Rightarrow 12.5 \text{ «ms}^{-1}\text{»}$

«from conservation of momentum» common speed after collision is  $\frac{1}{2}$  initial speed « $v_c = \frac{12.5}{2} = 6.25 \text{ ms}^{-1}$ »

$$h = \frac{v_c^2}{2g} = \frac{6.25^2}{2 \times 9.81} \approx 2.0 \text{ «m»}$$

Allow 12.5 from incorrect use of kinematics equations

Award **[3]** for a bald correct answer

Award **[0]** for  $mg(8) = 2mgh$  leading to  $h = 4 \text{ m}$  if done in one step.

Allow ECF from MP1

Allow ECF from MP2

**[3 marks]**

## Examiners report

a.i. [N/A]

a.ii. [N/A]

a.iii. [N/A]

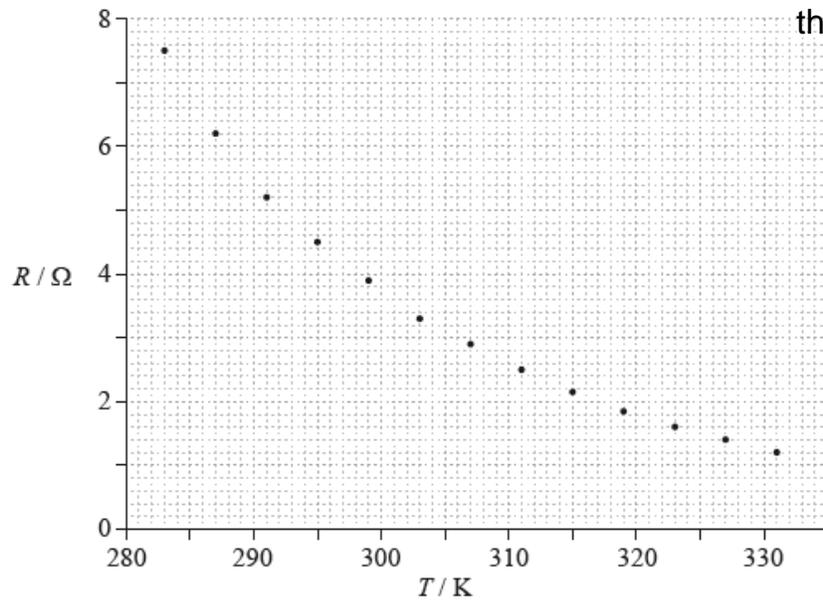
b. [N/A]

c. [N/A]

d. [N/A]

Data analysis question.

A student sets up a circuit to study the variation of resistance  $R$  of a negative temperature coefficient (NTC) thermistor with temperature  $T$ . The data are shown plotted on the graph.



The electric current through the thermistor for  $T = 283 \text{ K}$  is  $0.78 \text{ mA}$ . The uncertainty in the electric current is  $0.01 \text{ mA}$ .

- a. Draw the best-fit line for the data points. [1]
- b.i. Calculate the gradient of the graph when  $T = 291 \text{ K}$ . [3]
- b.ii. State the unit for your answer to (b)(i). [1]
- c. The uncertainty in the resistance value is 5%. The uncertainty in the temperature is negligible. On the graph, draw error bars for the data point at  $T = 283 \text{ K}$  and for the data point at  $T = 319 \text{ K}$ . [2]
- d.i. Calculate the power dissipated by the thermistor at  $T = 283 \text{ K}$ . [1]
- d.ii. Determine the uncertainty in the power dissipated by the thermistor at  $T = 283 \text{ K}$ . [3]

## Markscheme

- a. smooth curve that passes within  $\pm 0.5$  squares of all data points;
- b.i. a tangent drawn at  $[291, 5.2]$  and selection of two extreme points on the tangent that use  $\Delta R > 3.5 \Omega$ ; (*judge by eye*)  
 gradient magnitude determined as  $0.20 \pm 0.02$ ;  
 negative value given;
- b.ii.  $\Omega \text{ K}^{-1}$ ;
- c. correct error bar for  $283 \text{ K}$  (total length of bar 3–5 squares, centred on point);  
 correct error bar for  $319 \text{ K}$  (total length of bar 0.5–2 squares, centred on point);
- d.i. substituting  $I^2 R = \left( [0.78 \times 10^{-3}]^2 \times 7.5 \right) = 4.5 \times 10^{-6} \text{ W}$  **or**  $4.6 \times 10^{-6} \text{ W}$ ;
- d.ii. fractional uncertainty in  $I^2 = 2 \times \frac{0.01}{0.78}$  (=  $0.026$  **or**  $2.6\%$ );

uncertainty in power (=  $[0.026 + 0.05] \times 4.6 \times 10^{-6}$ ) =  $0.34 \times 10^{-6} \text{ W}$  to  $0.35 \times 10^{-6} \text{ W}$ ;

answer rounded to 1 significant figure;

**or**

uncertainty in  $I^2 = 2 \times 1.3\%/0.026$ ;

total uncertainty in  $P = 7.6\%/0.076$ ;

answer rounded to 1 significant figure;

## Examiners report

a. [N/A]

b.i. Very poorly executed. Few SL candidates knew how to draw an acceptable tangent.

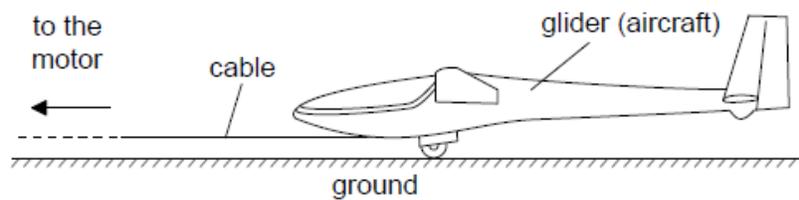
b.ii. [N/A]

c. [N/A]

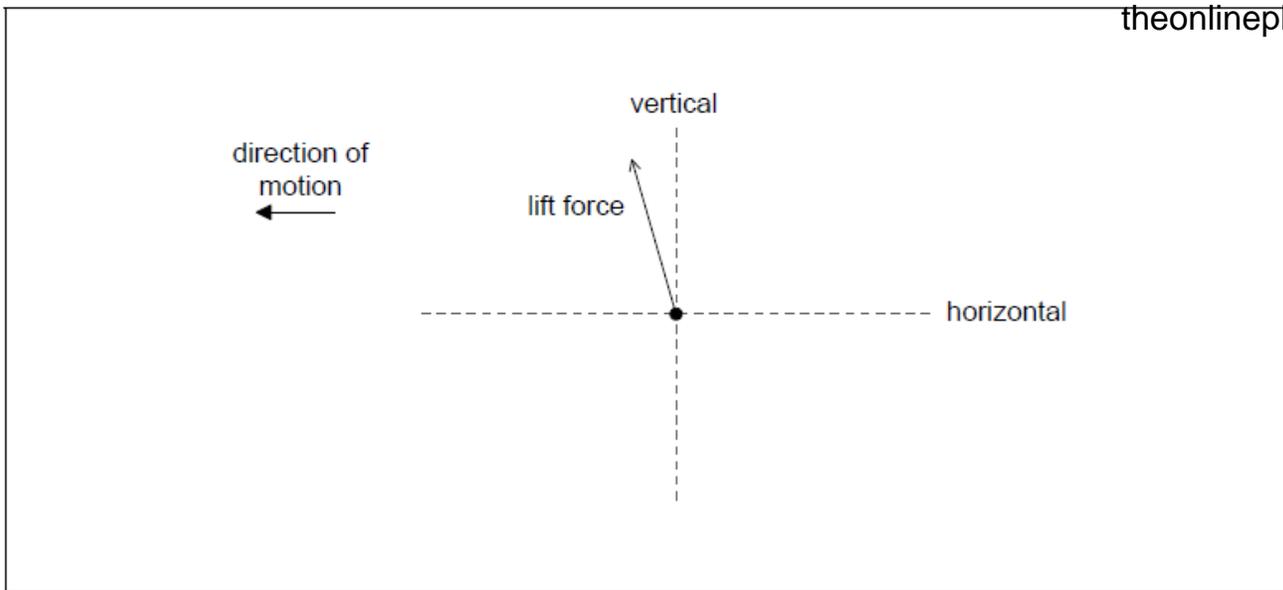
d.i. [N/A]

d.ii. [N/A]

A glider is an aircraft with no engine. To be launched, a glider is uniformly accelerated from rest by a cable pulled by a motor that exerts a horizontal force on the glider throughout the launch.



- a. The glider reaches its launch speed of  $27.0 \text{ m s}^{-1}$  after accelerating for  $11.0 \text{ s}$ . Assume that the glider moves horizontally until it leaves the ground. Calculate the total distance travelled by the glider before it leaves the ground. [2]
- b. The glider and pilot have a total mass of  $492 \text{ kg}$ . During the acceleration the glider is subject to an average resistive force of  $160 \text{ N}$ . Determine the average tension in the cable as the glider accelerates. [3]
- c. The cable is pulled by an electric motor. The motor has an overall efficiency of  $23 \%$ . Determine the average power input to the motor. [3]
- d. The cable is wound onto a cylinder of diameter  $1.2 \text{ m}$ . Calculate the angular velocity of the cylinder at the instant when the glider has a speed of  $27 \text{ m s}^{-1}$ . Include an appropriate unit for your answer. [2]
- e. After takeoff the cable is released and the unpowered glider moves horizontally at constant speed. The wings of the glider provide a lift force. The diagram shows the lift force acting on the glider and the direction of motion of the glider. [2]



Draw the forces acting on the glider to complete the free-body diagram. The dotted lines show the horizontal and vertical directions.

- f. Explain, using appropriate laws of motion, how the forces acting on the glider maintain it in level flight. [2]
- g. At a particular instant in the flight the glider is losing 1.00 m of vertical height for every 6.00 m that it goes forward horizontally. At this instant, the horizontal speed of the glider is  $12.5 \text{ m s}^{-1}$ . Calculate the **velocity** of the glider. Give your answer to an appropriate number of significant figures. [3]

## Markscheme

- a. correct use of kinematic equation/equations

148.5 **or** 149 **or** 150 «m»

*Substitution(s) must be correct.*

- b.  $a = \frac{27}{11}$  **or** 2.45 «m s<sup>-2</sup>»

$$F - 160 = 492 \times 2.45$$

1370 «N»

*Could be seen in part (a).*

*Award [0] for solution that uses  $a = 9.81 \text{ m s}^{-2}$*

- c. **ALTERNATIVE 1**

«work done to launch glider» =  $1370 \times 149$  «= 204 kJ»

$$\text{«work done by motor»} = \frac{204 \times 100}{23}$$

$$\text{«power input to motor»} = \frac{204 \times 100}{23} \times \frac{1}{11} = 80 \text{ **or** } 80.4 \text{ **or** } 81 \text{ kW}»$$

### **ALTERNATIVE 2**

use of average speed  $13.5 \text{ m s}^{-1}$

«useful power output» = force x average speed «=  $1370 \times 13.5$ »

power input =  $\llcorner 1370 \times 13.5 \times \frac{100}{23} \Rightarrow 80 \text{ or } 80.4 \text{ or } 81 \text{ k}\llcorner W \llcorner$

**ALTERNATIVE 3**

work required from motor = KE + work done against friction  $\llcorner = 0.5 \times 492 \times 27^2 + (160 \times 148.5) \llcorner = 204 \llcorner \text{kJ} \llcorner$

$\llcorner \text{energy input} \llcorner = \frac{\text{work required from motor} \times 100}{23}$

power input =  $\frac{883000}{11} = 80.3 \text{ k}\llcorner W \llcorner$

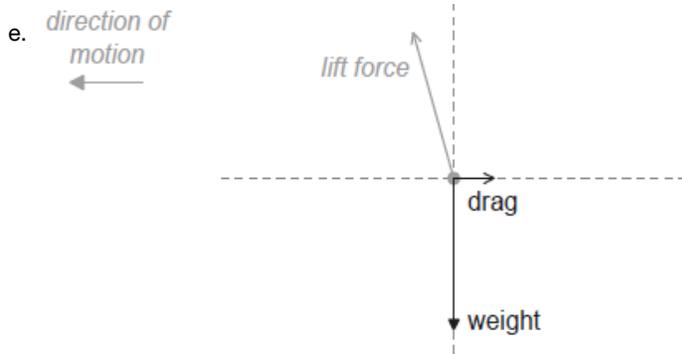
Award **[2 max]** for an answer of  $160 \text{ k}\llcorner W \llcorner$ .

d.  $\omega = \llcorner \frac{v}{r} \Rightarrow \frac{27}{0.6} = 45$

rad  $s^{-1}$

Do not accept Hz.

Award **[1 max]** if unit is missing.



drag correctly labelled and in correct direction

weight correctly labelled and in correct direction **AND** no other incorrect force shown

Award **[1 max]** if forces do not touch the dot, but are otherwise OK.

f. name Newton's first law

vertical/all forces are in equilibrium/balanced/add to zero

**OR**

vertical component of lift mentioned

as equal to weight

g. any speed and any direction quoted together as the answer

quotes their answer(s) to 3 significant figures

speed =  $12.7 \text{ m s}^{-1}$  **or** direction =  $9.46^\circ$  **or**  $0.165 \text{ rad}$   $\llcorner \text{below the horizontal} \llcorner$  **or** gradient of  $-\frac{1}{6}$

## Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

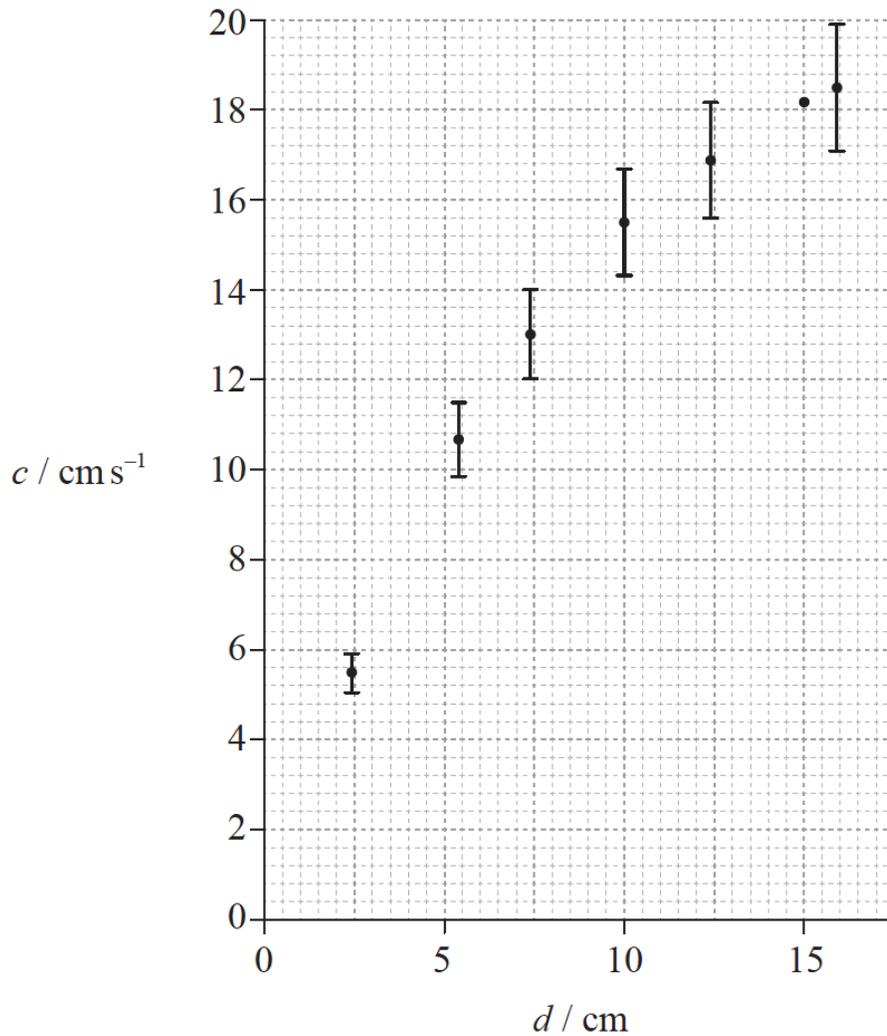
d. [N/A]

e. [N/A]

[N/A]

Data analysis question.

Caroline carried out an experiment to measure the variation with water depth  $d$  of the wave speed  $c$  of a surface water wave. Her data are shown plotted below.



The uncertainty in the water depth  $d$  is too small to be shown. Uncertainties in the measurement of the wave speed  $c$  are shown as error bars on the graph except for the data point corresponding to  $d=15$  cm.

- a. Caroline calculated the wave speed by measuring the time  $t$  for the wave to travel 150 cm. The uncertainty in this distance is 2 cm. For the reading at a water depth of 15 cm, the time  $t$  is 8.3 s with an uncertainty 0.5 s. [4]
- Show that the absolute uncertainty in the wave speed at this time is  $1.3 \text{ cm s}^{-1}$ .
  - On the graph opposite, draw the error bar for the data point corresponding to  $d=15$  cm.
- b. Caroline hypothesized that the wave speed  $c$  is directly proportional to the water depth  $d$ . [3]
- On the graph opposite, draw a line of best-fit for the data.
  - Suggest if the data support this hypothesis.

- c. Another student proposes that  $c$  is proportional to  $d^{0.5}$ .

State a suitable graph that can be plotted to test this proposal.

- d. There is a systematic error in Caroline's determination of the depth. [2]

- (i) State what is meant by a systematic error.  
 (ii) State how the graph in (c) would indicate that there is a systematic error.

## Markscheme

- a. (i)

fractional uncertainty in distance =  $\frac{2}{150}$  and

fractional uncertainty in time =  $\frac{0.5}{8.3}$ ; { (allow use of percentage uncertainty)

fractional uncertainty in speed =  $\frac{2}{150} + \frac{0.5}{8.3}$  (= 0.074 **or** 7.4%);

absolute uncertainty =  $18 \times 0.074$ ;

= 1.3 ( $\text{cm s}^{-1}$ )

**or**

maximum =  $\frac{152}{7.8}$ ;

minimum =  $\frac{148}{8.8}$ ;

shows subtraction of maximum and minimum and division by 2;

(ii) error bars drawn as  $\pm 1.3$ ;

- b. (i) smooth curve within limits of all error bars;

(ii) a straight line cannot be drawn;  
 that goes through all the error bars / that goes through the origin;

- c.  $c$  versus  $\sqrt{d}$  /  $d^{0.5}$  **or**  $c^2$  versus  $d$  **or**  $\lg c$  versus  $\lg d$  **or**  $\ln c$  versus  $\ln d$ ;

Allow as symbols or written in words.

- d. (i) error that is identical for each reading / error caused by zero error in instrument / OWTTE;

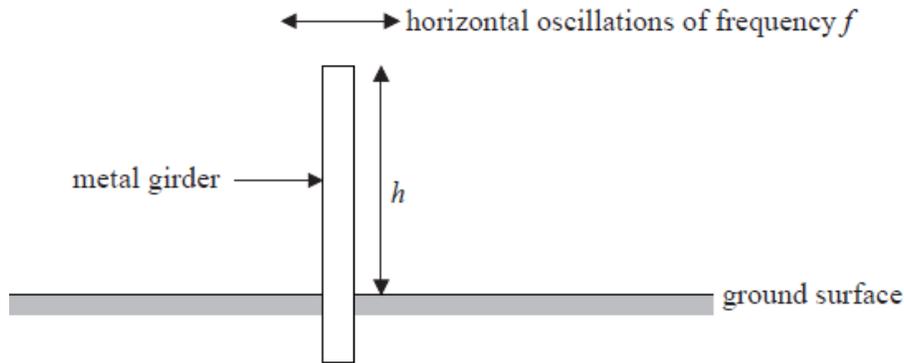
(ii) graph will not go through origin / intercept non-zero;  
 graph will not be straight line/linear;

## Examiners report

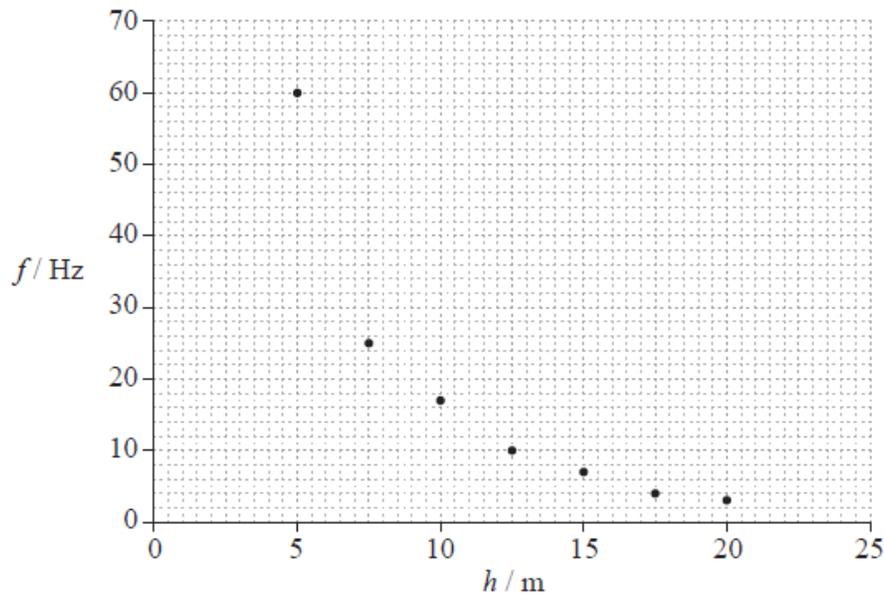
- a. [N/A]  
 b. [N/A]  
 c. [N/A]  
 d. [N/A]

Data analysis question.

Metal girders are often used in buildings that have been constructed to withstand earthquakes. To aid the design of these buildings, experiments are undertaken to measure how the natural frequency  $f$  of horizontal oscillations of metal girders varies with their dimensions. In an experiment,  $f$  was measured for vertically supported girders of the same cross-sectional area but with different heights  $h$ .



The graph shows the plotted data for this experiment. Uncertainties in the data are not shown.



a. Draw a best-fit line for the data. [1]

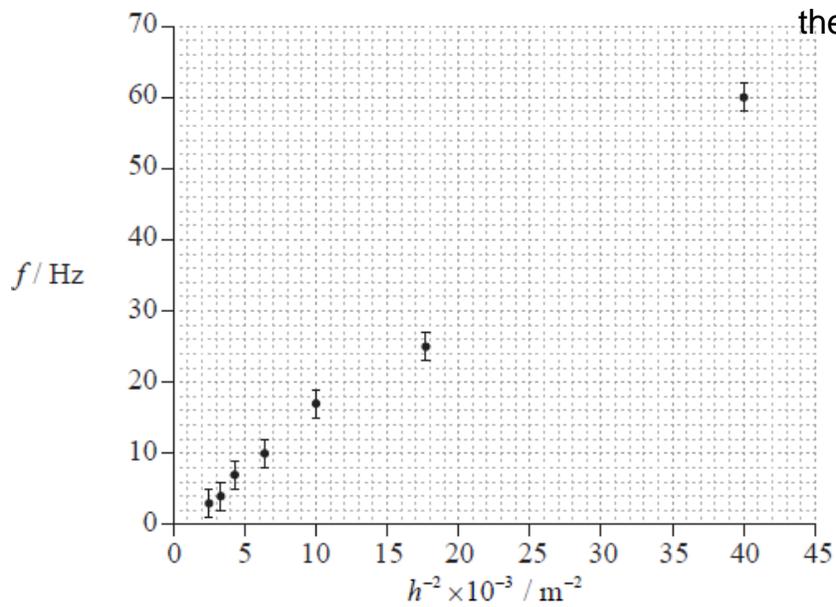
b. It is hypothesized that the frequency  $f$  is inversely proportional to the height  $h$ . [4]

By choosing **two** well separated points on the best-fit line that you have drawn in (a), show that this hypothesis is incorrect.

c. Another suggestion is that the relationship between  $f$  and  $h$  is of the form shown below, where  $k$  is a constant. [5]

$$f = \frac{k}{h^2}$$

The graph shows a plot of  $f$  against  $h^{-2}$ .



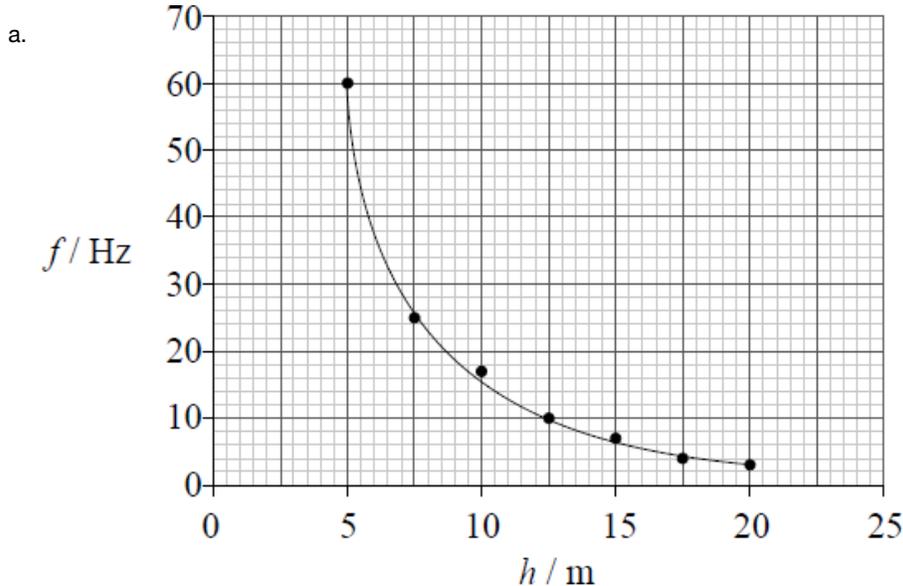
The uncertainties in  $h^{-2}$  are too small to be shown.

(i) Draw a best-fit line for the data that supports the relationship  $f = \frac{k}{h^2}$ .

(ii) Determine, using the graph, the constant  $k$ .

d. State **one** reason why the results of the experiment could not be used to predict the natural frequency of oscillation for girders of height 50 m. [1]

## Markscheme



smooth curve as above; (*judge by eye*)

*Do not allow point-to-point curve.*

*Do not allow curve to "curl round" at low or high  $h$ .*

*Single "non-hairy" line only is acceptable.*

b. choice of points separated by ( $\Delta h \geq 7.5$ ) e.g. [6.0, 37] [15, 6.0];

recognize  $fh = \text{constant}$  for an inverse relation;

calculates  $fh$  correctly for both points;

state that two calculated numbers are not equal (therefore not inverse);

Award **[3 max]** if data points are not on line.

Award **[3 max]** if data points are too close together ( $\Delta h \geq 7.5$ ).

Award **[2 max]** if both of above.

- c. (i) a straight-line that goes through all the error bars; and drawn through the origin; (*allow  $\pm 1/2$  square*)
- (ii) read-off of suitable point(s) on line separated by at least half of drawn line;  
(*allow implicit use of origin*)  
calculation of gradient to give  $1.5(\pm 0.2) \times 10^3$ ;  
 $\text{s}^{-1}\text{m}^2$  **or**  $\text{Hzm}^2$ ;
- d. the relation might not hold/extrapolate for larger values of  $h$  / outside range of experiment / values would be close to origin and with large  
(percentage experimental) error / girders of this height could buckle under their own weight / *OWTTE*;

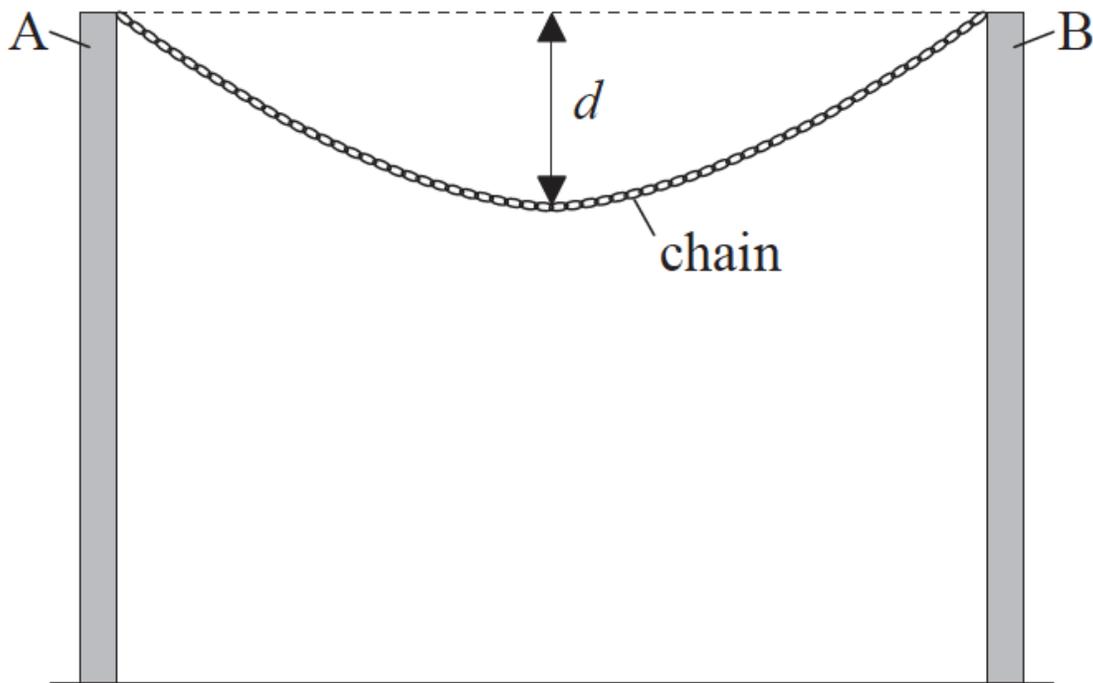
## Examiners report

- a. Candidates were required to draw a smooth curve through a series of points. Few could do this adequately and it was rare to see a good construction. Lines were usually point-to-point, kinked in some way, or “hairy” (meaning that the candidate had a number of separate attempts to draw the line). This is evidently not a skill that all candidates possess.
- There is still a widespread misconception that when the question asks for the candidate to “draw a best-fit line” this implies that the line is straight. This error is seen in work representing all the IB languages and is a simple point that traps candidates year after year.
- b. The question required a test of inverse proportionality. Examiners were expecting candidates to show that  $fh$  was *not* a constant for two well separated points. Only about 75% the cohort could manage this. Many tried to show that  $f/h$  was constant and gained little credit other than for choosing two well separated data points.
- c. (i) This part required a straight line *going through all the error bars*. Here candidates made good attempts. A common error was to fail to draw the line through the origin.
- (ii) It should have been a simple matter to determine the gradient of this graph with its intercept at the origin. Many candidates missed the  $10^{-3}$  in the axis scaling and went on to omit the unit from their answer. A widespread failure to add units to a gradient calculation has been a feature of several recent paper 2 examinations.
- d. Candidates understood the dangers of extrapolation but could not express them well.

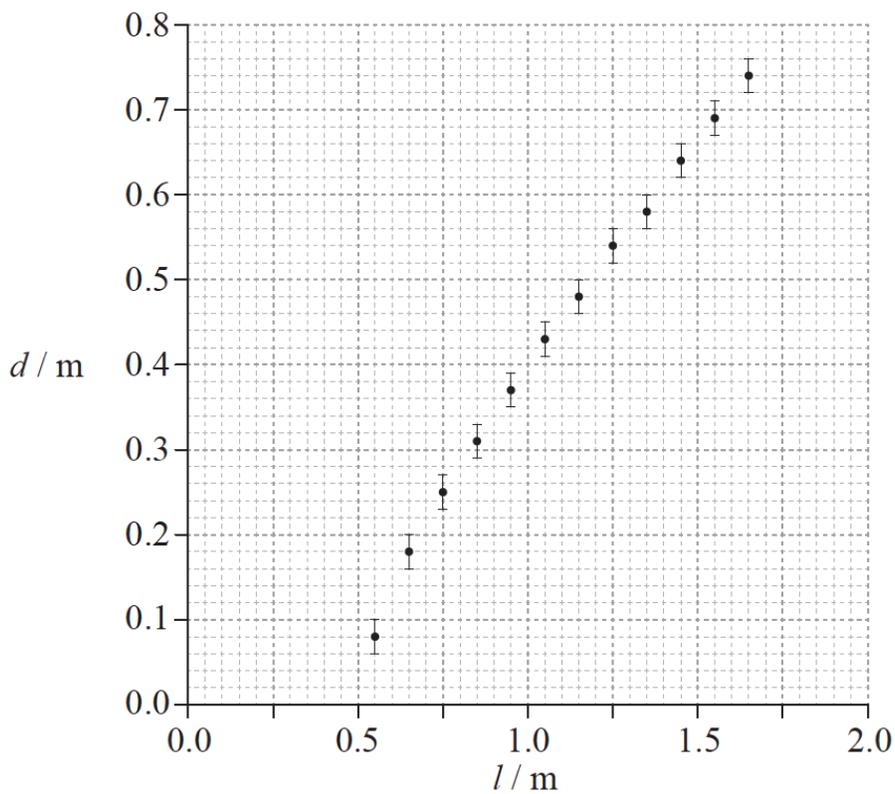
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Data analysis question.

A chain is suspended between two vertical supports A and B. The chain is made of a number of identical metal links.



The length  $l$  of the chain can be increased by adding extra links. An experiment was undertaken to investigate how the sag  $d$  of the midpoint of the chain, measured from the horizontal between A and B, varies with  $l$ . The data obtained are shown plotted below. The uncertainties in  $l$  are too small to be shown.



a. Draw a best-fit line for the data points on the graph opposite. [1]

b. With reference to your answer to (a), [4]

(i) explain why the relationship between  $d$  and  $l$  is not linear.

(ii) estimate the horizontal distance between the supports A and B.

- c. Before the experiment was carried out, it was hypothesized that  $d$  depends on  $\sqrt{l}$ . Determine, using your answer to (a), whether this hypothesis [4]  
is valid.

## Markscheme

- a. smooth curve that goes through all error bars;

*Do not allow thick or hairy or doubled lines, or lines where the curvature changes abruptly.*

*Do not allow lines that touch horizontal ends of error bars but miss the verticals.*

- b. (i) (no)

reference to going through all the error bars;

the line is a curve/not straight / straight line would not pass through all the points / equal increments in  $l$  give rise to unequal increments in  $d$ ;

(ii) mentions or shows clear extrapolation to  $l$  axis; { *(allow from curve or straight line)*

read-off to within a square ( $0.50 \pm 0.05\text{m}$ );

*Award [1max] if no extrapolation seen on graph.*

*Answer must match read-off to 2+sig fig.*

- c. two data points on line correctly read and more than 0.5 apart on  $l$ -axis;

$$d^2 = kl \text{ or } d = k\sqrt{l};$$

two or more correct calculations of  $k$  from readings;

comment that two or more values are not equal (even with error bar consideration) therefore hypothesis is not valid;

*Award [3 max] if  $l$ -axis values differ by less than 0.5.*

## Examiners report

a. [N/A]

b. [N/A]

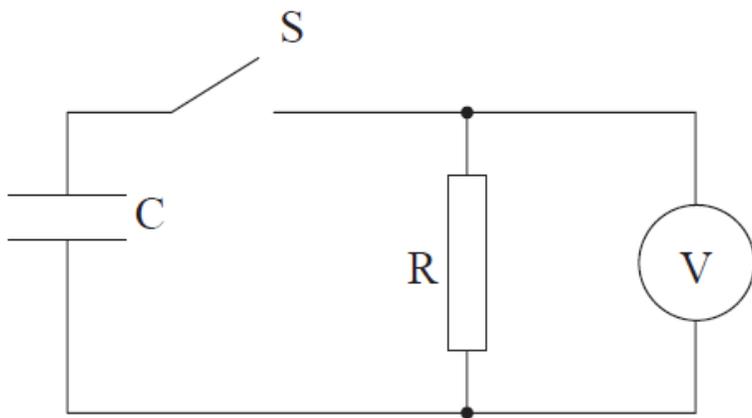
c. [N/A]

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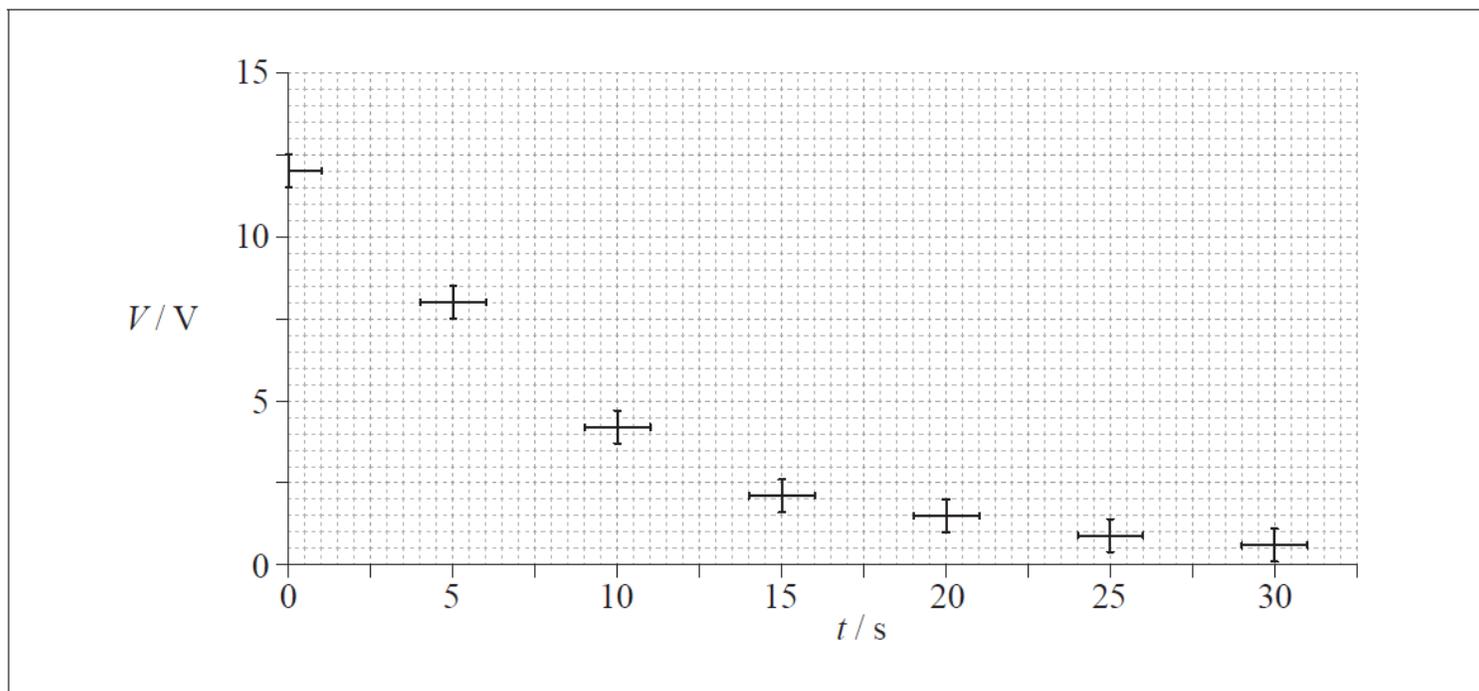
Data analysis question.

A capacitor is a device that can be used to store electric charge.

- a. An experiment was undertaken to investigate one of the circuit properties of a capacitor. A capacitor  $C$  was connected via a switch  $S$  to a [6]  
resistance  $R$  and a voltmeter  $V$ .



The initial potential difference across C was 12V. The switch S was closed and the potential difference  $V$  across R was measured at various times  $t$ . The data collected, along with error bars, are shown plotted below.



(i) On the graph opposite, draw a best-fit line for the data starting from  $t = 0$ .

(ii) It was hypothesized that the decay of the potential difference across the capacitor is exponential. Determine, using the graph, whether this hypothesis is true or not.

b. The time constant  $\tau$  of the circuit is defined as the time it would take for the capacitor to discharge were it to keep discharging at its initial rate. [3]

Use the graph in (a) to calculate the

(i) initial rate of decay of potential difference  $V$ .

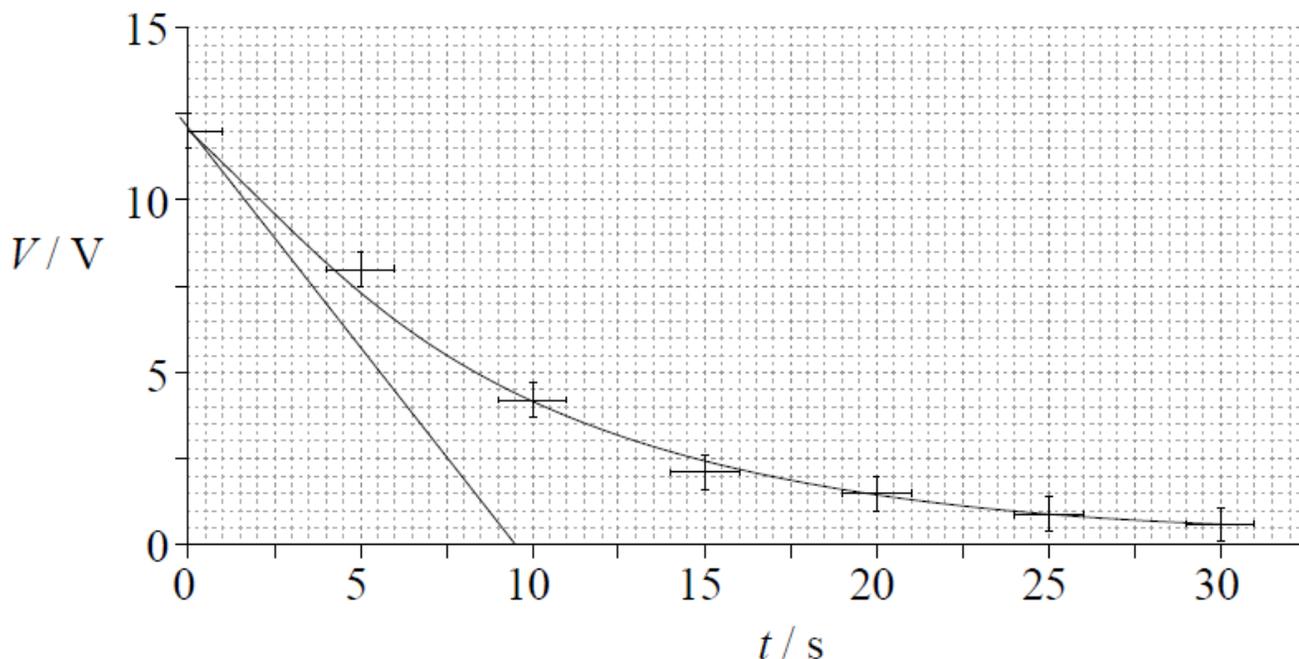
(ii) time constant  $\tau$ .

c. The time constant  $\tau = RC$  where  $R$  is the resistance and  $C$  is a property called capacitance. The effective resistance in the circuit is  $10 \text{ M}\Omega$ . [1]

Calculate the capacitance  $C$ .

## Markscheme

a.



(i) smooth curve; that passes through all error bars;

*Award [1 max] if an obvious straight line is drawn through first three points.*

*Award [1 max] if line touches time axis.*

*Do not penalize if line starts beyond zero.*

*Do not allow upward curve at high t in first marking point.*

*Do not allow double or kinked lines.*

(ii) correctly identifies three points/intervals from own graph;

correctly processes these three using exponential/half-life/constant ratio/relationship;

to conclude that decay is exponential;

within uncertainty;

*Award [0] for a bald statement.*

*Award [1] for a straight line or portion of straight line leading to conclusion that decay is **not** exponential.*

*Award [1] if uncertainties are not considered and conclusion that decay is not exponential.*

b. (i) evaluates a gradient over a minimum of 5 s to give an initial rate for example,  $\left(\frac{12}{9.5} = \right) 1.3 \text{ (Vs}^{-1}\text{)}$  for graph above; *(allow ECF from the graph);*

$\text{Vs}^{-1}$  (unit paper mark)

*Clear evidence of calculation of gradient must be seen.*

*Accept use of (0, 12) (5, 8) to give  $0.8 \text{ (Vs}^{-1}\text{)}$ .*

*Allow answer left as fraction (eg  $\frac{4}{5}$ ).*

*Accept negative gradient.*

(ii) obtains evidenced answer for t intercept;

*Accept **one** of the following methods:*

*Drawing tangent to initial part of graph (yields  $9.5 \pm 3\text{s}$ ).*

*Extending the first two/three points to the time axis (yields 11 – 19).*

*Using answer to (b)(i) to calculate intercept.*

c.  $C = \left( \frac{(b)(ii)}{10 \times 10^6} \right) 1.0 \times 10^{-6} (\Omega^{-1} \text{s/F});$

Expect to see  $10^6$  in denominator. Award [0] for absence of  $10^6$  unless unit is in terms of  $M\Omega$ .

## Examiners report

a. (i) Few candidates scored full marks. Too often examiners saw poor quality draughtsmanship and ruler-straight lines through the first three points.

Most candidates were able to ensure that their lines stayed within the bounds of the error bars. Candidates are encouraged to read through the whole question before attempting to answer – had this been done then they might have gained additional clues from what followed. It should be noted that the skill being tested here was the ability of the candidates to ignore the points and draw a smooth curve through the uncertainty bars.

(ii) Good tests of exponential change were beyond many. Examiners expect to see a systematic test carried through accurately. A suitable test might include identification of halflife behaviour, constant ratio behaviour, or fitting to an exponential function. Each of these approaches could have scored full marks. Often there were vague and meaningless statements about the asymptotic behaviour of the graph.

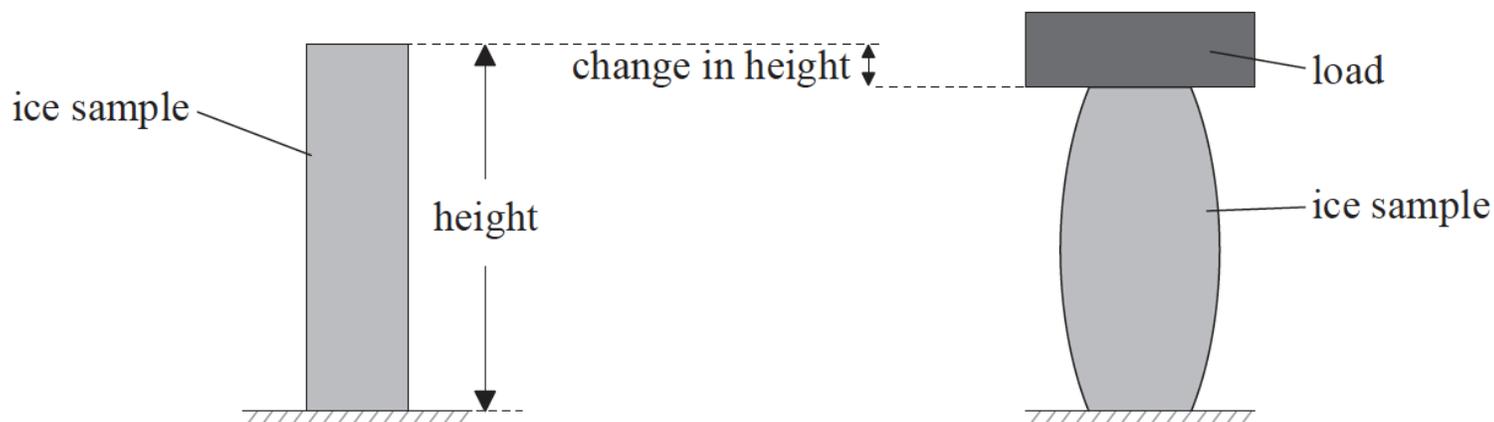
b. (i) This was adequately done by about half of the candidates although there were few confident tangents seen by examiners. Errors were to omit the unit and to try to work out a gradient over the full 30s.

(ii) Examiners expected to see an evidenced solution. Candidates who wrote down the answer without explanation gained little credit.

c. The answer here had to use the answer to (b)(ii) and most candidates were able to do this satisfactorily. A substantial number failed to take account of the prefix to the unit in the resistance and were a factor of  $10^6$  out in their answer.

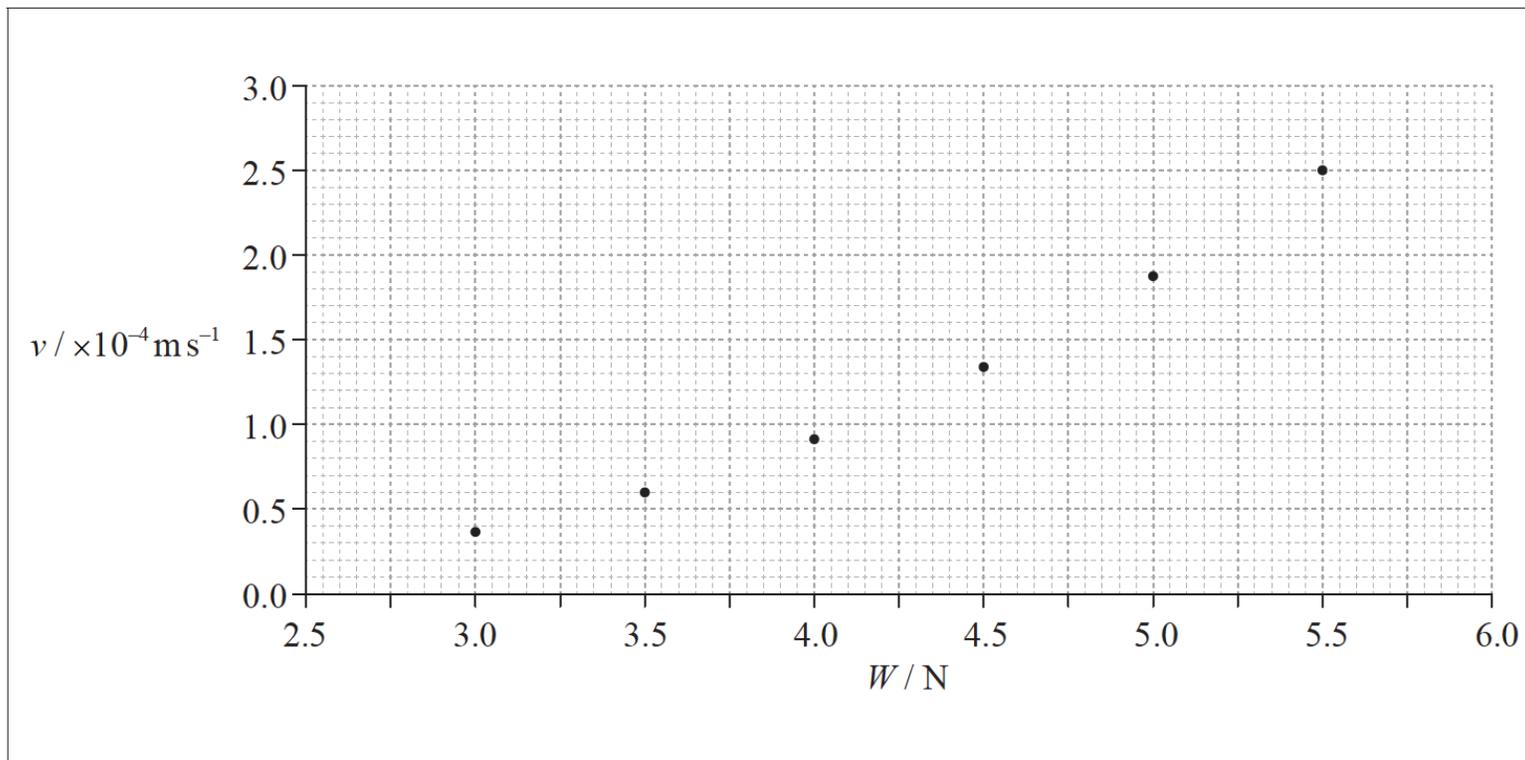
Data analysis question.

The movement of glaciers can be modelled by applying a load to a sample of ice.



After the load has been applied, it is observed to move downwards at a constant speed  $v$  as the ice deforms. The constant speed  $v$  is measured for different loads. The graph shows the variation of  $v$  with load  $W$  for a number of identical samples of ice.

The data points are plotted below.



The uncertainty in  $v$  is  $\pm 20 \mu\text{m s}^{-1}$  and the uncertainty in  $W$  is negligible.

a. (i) On the graph opposite, draw error bars on the first and last points to show the uncertainty in  $v$ . [2]

(ii) On the graph opposite, draw the line of best-fit for the data points.

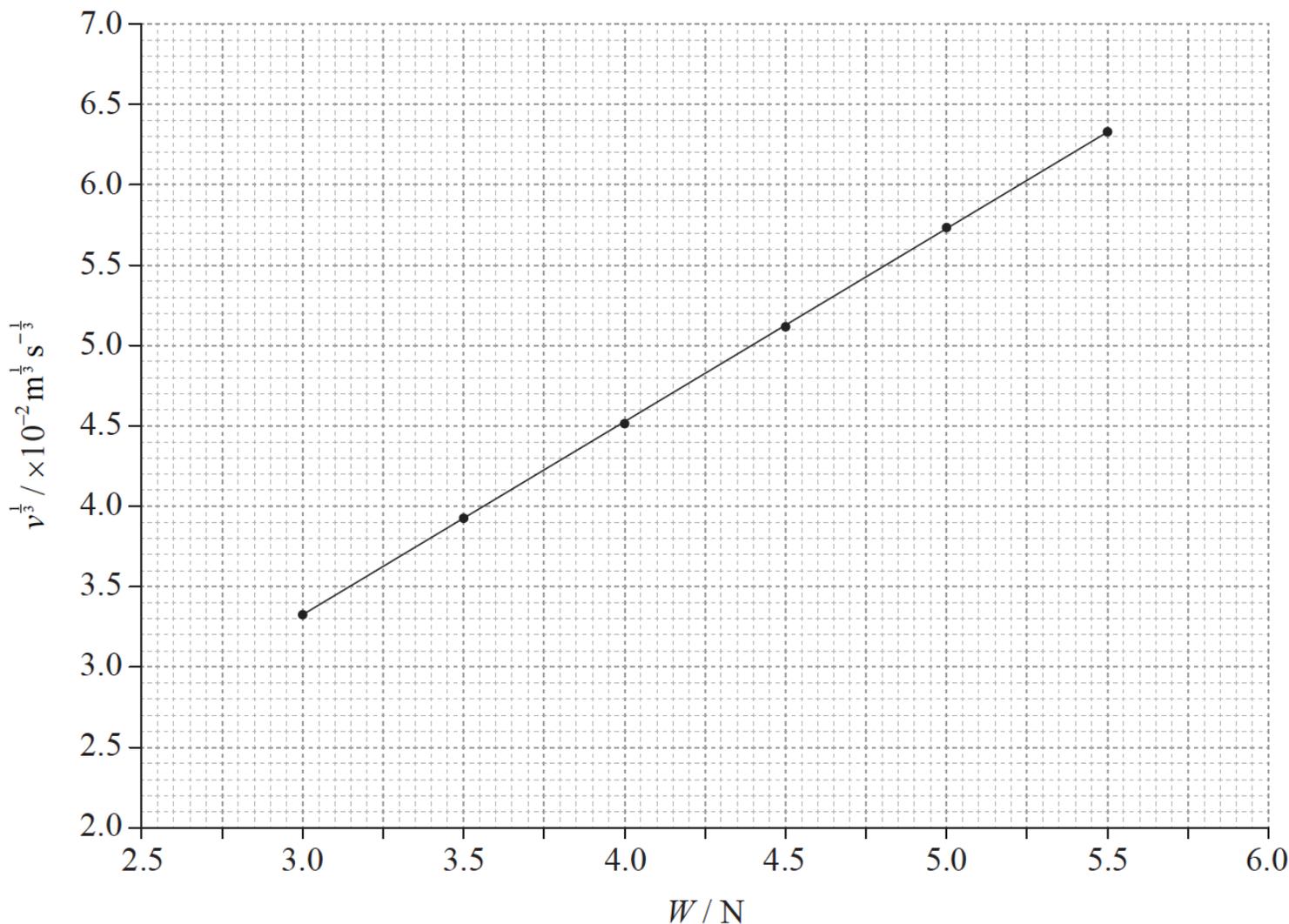
b. Explain whether the data support the hypothesis that  $v$  is directly proportional to  $W$ . [1]

c. Theory suggests that the relation between  $v$  and  $W$  is [3]

$$v = kW^3$$

where  $k$  is a constant.

To test this hypothesis a graph of  $v^{\frac{1}{3}}$  against  $W$  is plotted.



At  $W=5.5 \text{ N}$  the speed is  $250 \pm 20 \mu\text{m s}^{-1}$ .

Calculate the uncertainty in  $v^{1/3}$  for a load of  $5.5 \text{ N}$ .

d. (i) Using the graph in (c), determine  $k$  without its uncertainty.

[5]

(ii) State an appropriate unit for your answer to (d)(i).

## Markscheme

a. (i) both error bars correct (overall length 4 squares)  $\pm \frac{1}{2}$  square;

(ii) smooth curve going through error bars and within half square of other points;

b. not proportional because not straight/trend cannot go through origin;

c. fractional error in  $v = \frac{20}{250} (= 0.080)$ ;

fractional error in  $v^{1/3} = \frac{0.080}{3} (= 0.027)$ ; (allow ECF from first marking point)

uncertainty in  $v^{1/3} = (0.063 \times 0.027 =) 0.00169$ ; (allow  $0.00168-0.00170$ )

Allow expression of answer as  $0.630 \pm 0.002$  if calculation above seen.

Award [3] for a bald correct answer.

or

recognizes uncertainty in  $v^{\frac{1}{3}} = \frac{\sqrt[3]{270} - \sqrt[3]{230}}{2}$  **or**  $\sqrt[3]{250} - \sqrt[3]{230}$  **or**  $\sqrt[3]{270} - \sqrt[3]{250}$ ;

= 0.168 ;

conversion to  $0.00168\text{ms}^{-1}$ ;

d. (i) large triangle > half line used;

read-offs and substitution correct; (allow power of ten error here)

$k^{\frac{1}{3}} = 0.012 \pm 0.001$ ; (allow ECF)

$k = 1.73 \times 10^{-6} \text{ m N}^{-3} \text{ s}^{-1}$ ; (allow correct power of ten only)

Award [0] for use of a single data point.

(ii)  $\text{m N}^{-3} \text{ s}^{-1}$  **or**  $\text{kg}^{-3} \text{ m}^{-2} \text{ s}^5$ ;

## Examiners report

a. [N/A]

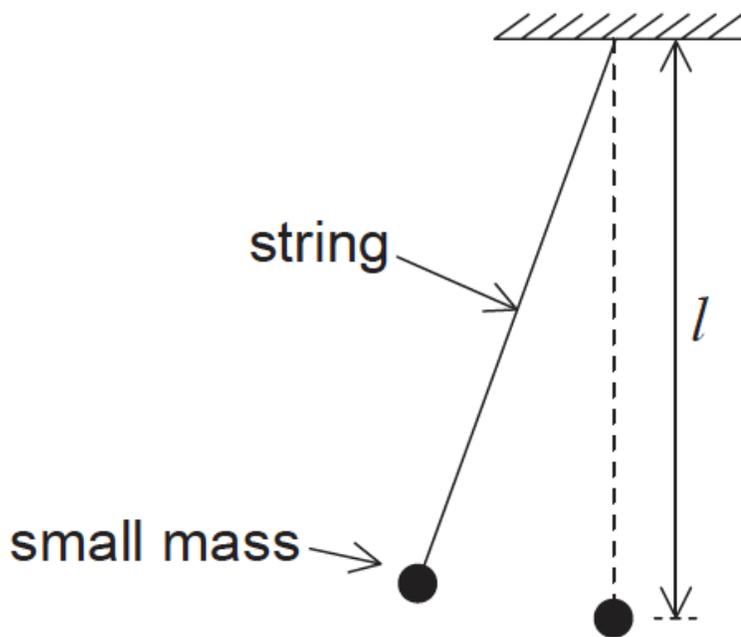
b. [N/A]

c. [N/A]

d. [N/A]

Data analysis question.

A simple pendulum of length  $l$  consists of a small mass attached to the end of a light string.



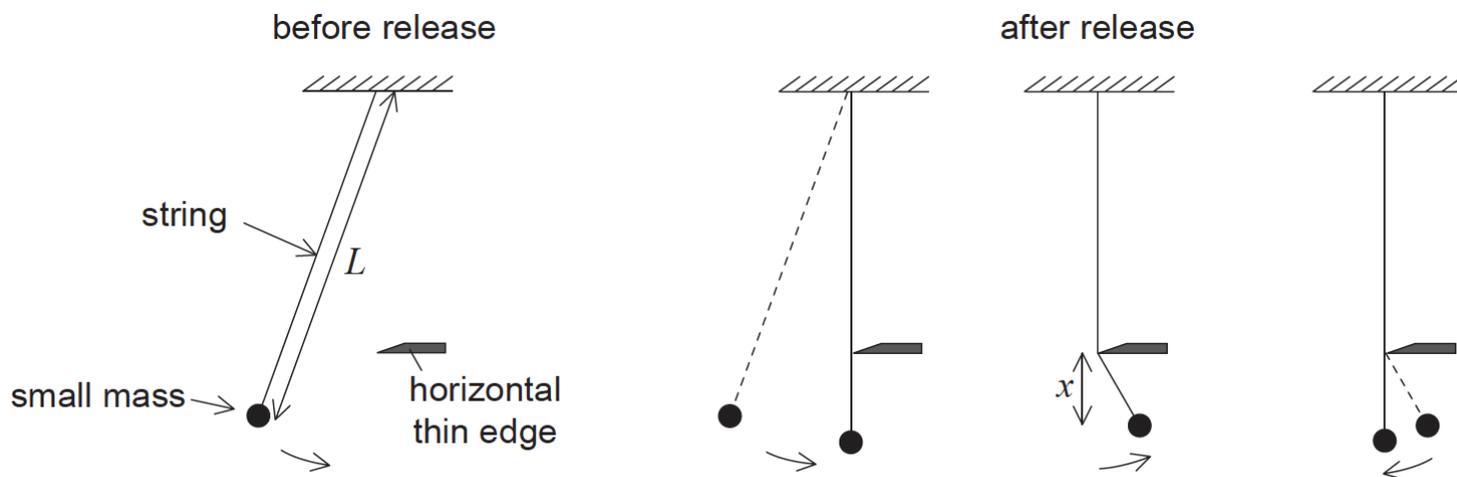
The time  $T$  taken for the mass to swing through one cycle is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where  $g$  is the acceleration due to gravity.

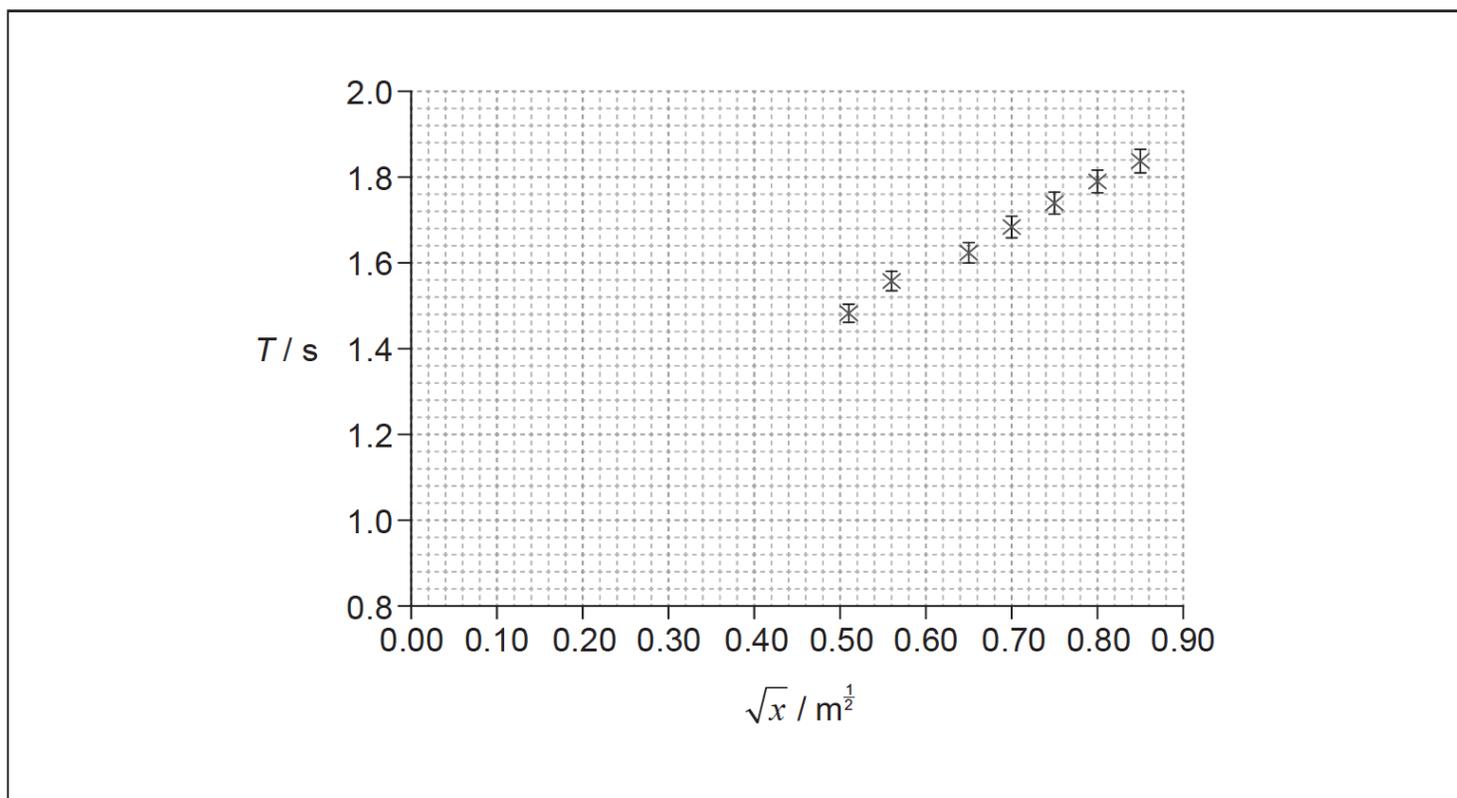
a. A student measures  $T$  for one length  $l$  to determine the value of  $g$ . Time  $T = 1.9s \pm 0.1s$  and length  $l = 0.880m \pm 0.001m$ . Calculate the fractional uncertainty in  $g$ . [2]

b. The student modifies the simple pendulum of length  $L$  so that, after release, it swings for a quarter of a cycle before the string strikes a horizontal thin edge. For the next half cycle, the pendulum swings with a shorter length  $x$ . The string then leaves the horizontal thin edge to swing with its original length  $L$ . [9]



The length  $L$  of the string is kept constant during the experiment. The vertical position of the horizontal thin edge is varied to change  $x$ .

The graph shows the variation of the time period with  $\sqrt{x}$  for data obtained by the student together with error bars for the data points. The error in  $\sqrt{x}$  is too small to be shown.



(i) Deduce that the time period for one complete oscillation of the pendulum is given by

$$T = \frac{\pi}{\sqrt{g}}(\sqrt{L} + \sqrt{x})$$

(ii) On the graph, draw the best-fit line for the data.

(iii) Determine the gradient of the graph.

(iv) State the value of the intercept on the  $T$ -axis.

(v) The equation of a straight line is  $y = mx + c$ . Determine, using your answers to (b)(iii) and (b)(iv), the intercept on the  $\sqrt{x}$ -axis.

(vi) Calculate  $L$ .

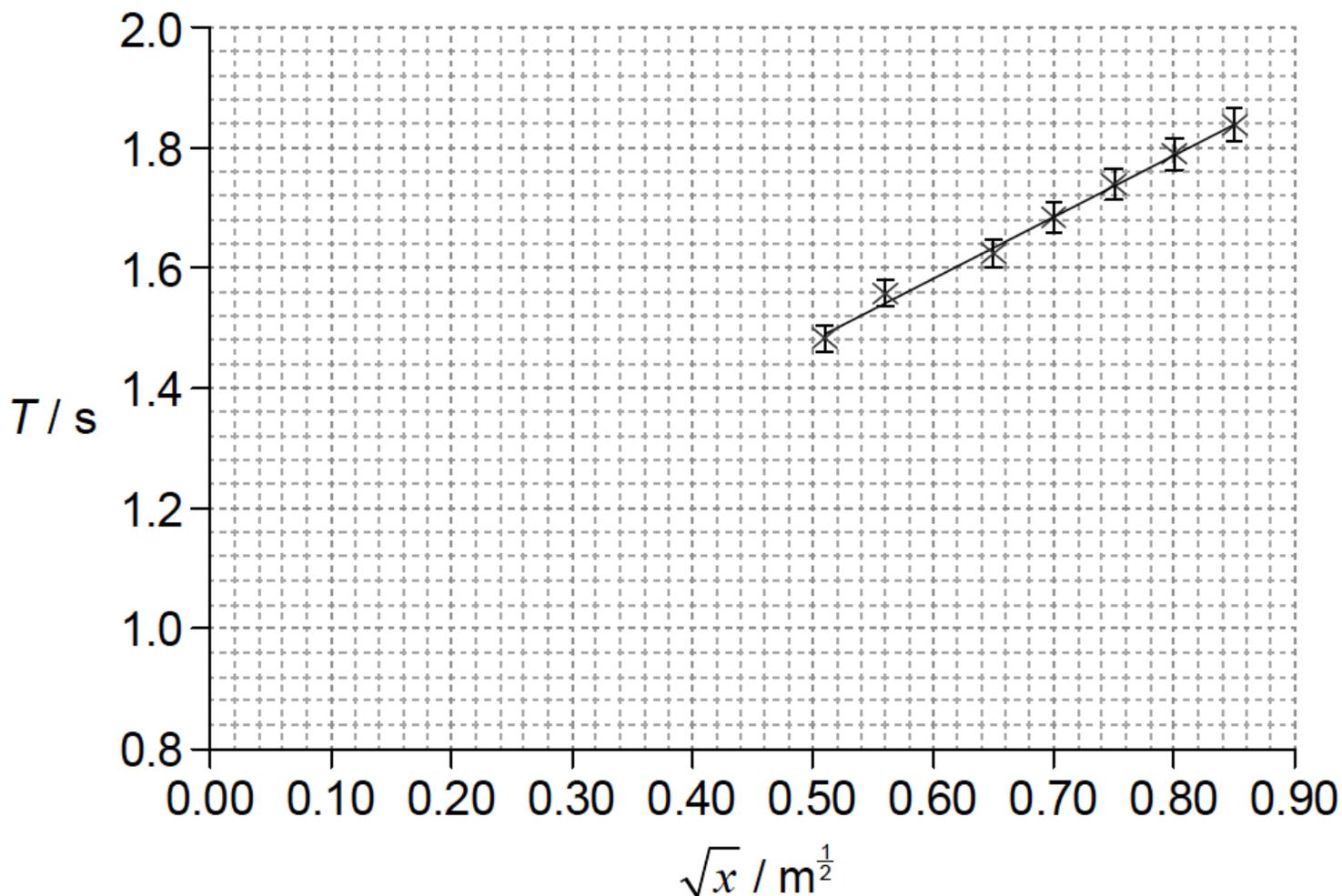
## Markscheme

- a. fractional uncertainty in  $l = \frac{1}{880}$  **or** 0.00114  
and fractional uncertainty in  $T = \frac{1}{19}$  **or** 0.0526; } (both needed)  
 } (accept percentage, or fraction here  
 } – allow candidate to quote  $\frac{2}{19}$   
 } directly if added correctly later)
- fractional uncertainty in  $g = (2 \times 0.0526 + 0.00114 =) 0.106$ ; } (accept percentage,  
 } do not accept fraction)

- b. (i) half of cycle takes  $\pi\sqrt{\frac{L}{g}}$  other half takes  $\pi\sqrt{\frac{x}{g}}$  and combine to give result;

$$\left(\frac{\pi}{g}(\sqrt{L} + \sqrt{x})\right)$$

(ii)



straight line of any length through all error bars;  
Do not accept kinked, fuzzy, doubled lines.

(iii) more than half line of their line used for gradient determination;  
read-offs correct;  
correct working leading to their gradient; (*best straight line gives 1.03*)  
*At least two significant figures are required in answer.*

(iv) their intercept  $\pm$  half a square; (*best straight line gives 0.96 s*)

(v) makes correct substitution for  $T=0$ ;

correct answer from own data including negative sign; (unit not required  $-0.93\text{m}\frac{1}{2}$ )

*Allow substitution into equation for straight line, but data point used **must** lie on candidate line.*

**N.B.**  $x$  in  $y = mx + c$  is  $\sqrt{x}$  on the axis – give BOD if not clear but answer correct.

## Examiners report

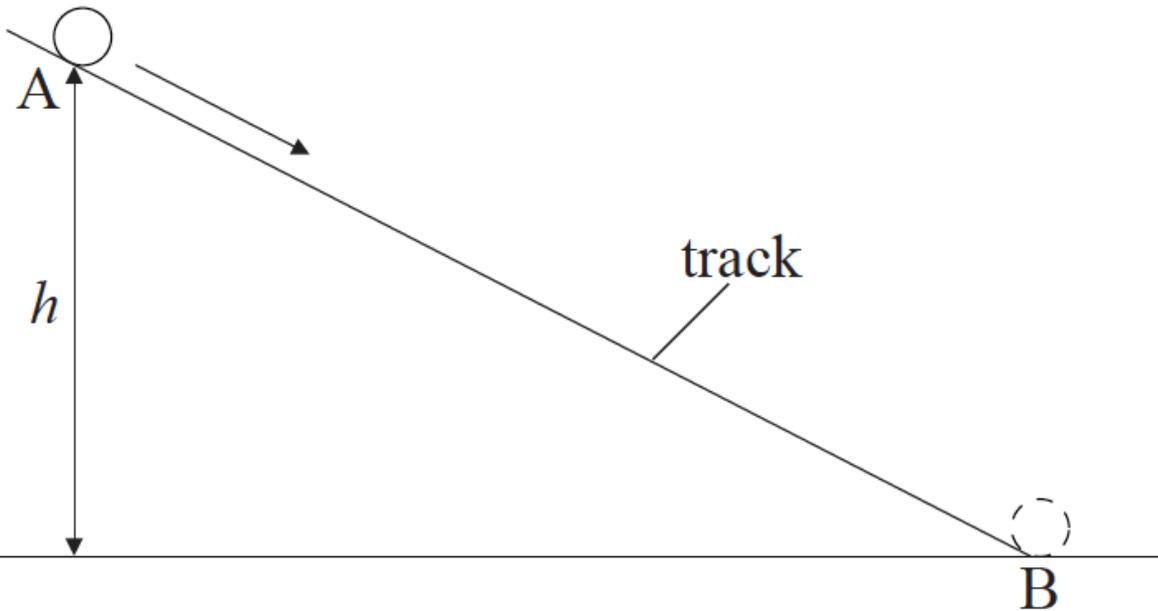
a. [N/A]

b. [N/A]

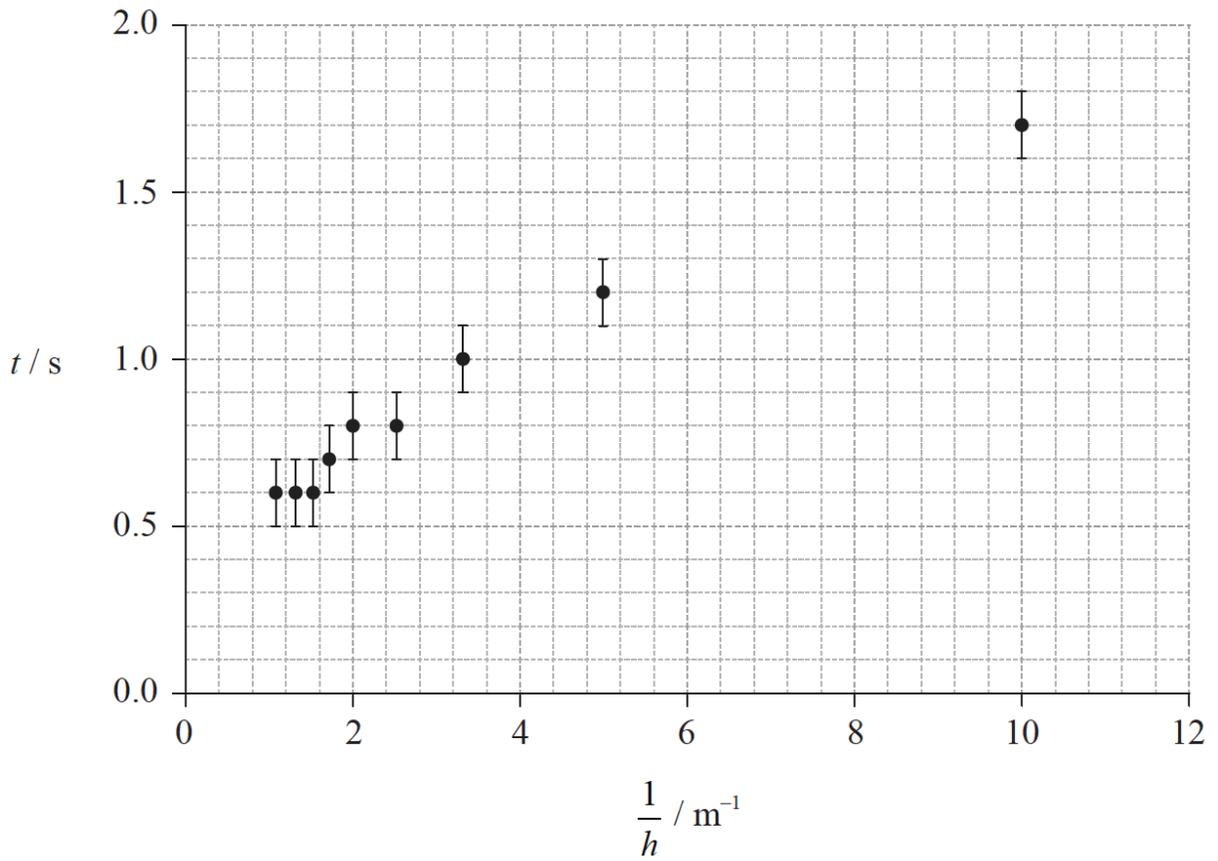
Data analysis question.

A small sphere rolls down a track of constant length AB. The sphere is released from rest at A.

The time  $t$  that the sphere takes to roll from A to B is measured for different values of height  $h$ .



A student suggests that  $t$  is proportional to  $\frac{1}{h}$ . To test this hypothesis a graph of  $t$  against  $\frac{1}{h}$  is plotted as shown on the axes below. The uncertainty in  $t$  is shown and the uncertainty in  $\frac{1}{h}$  is negligible.



a. (i) Draw the straight line that best fits the data. [2]

(ii) State why the data do not support the hypothesis.

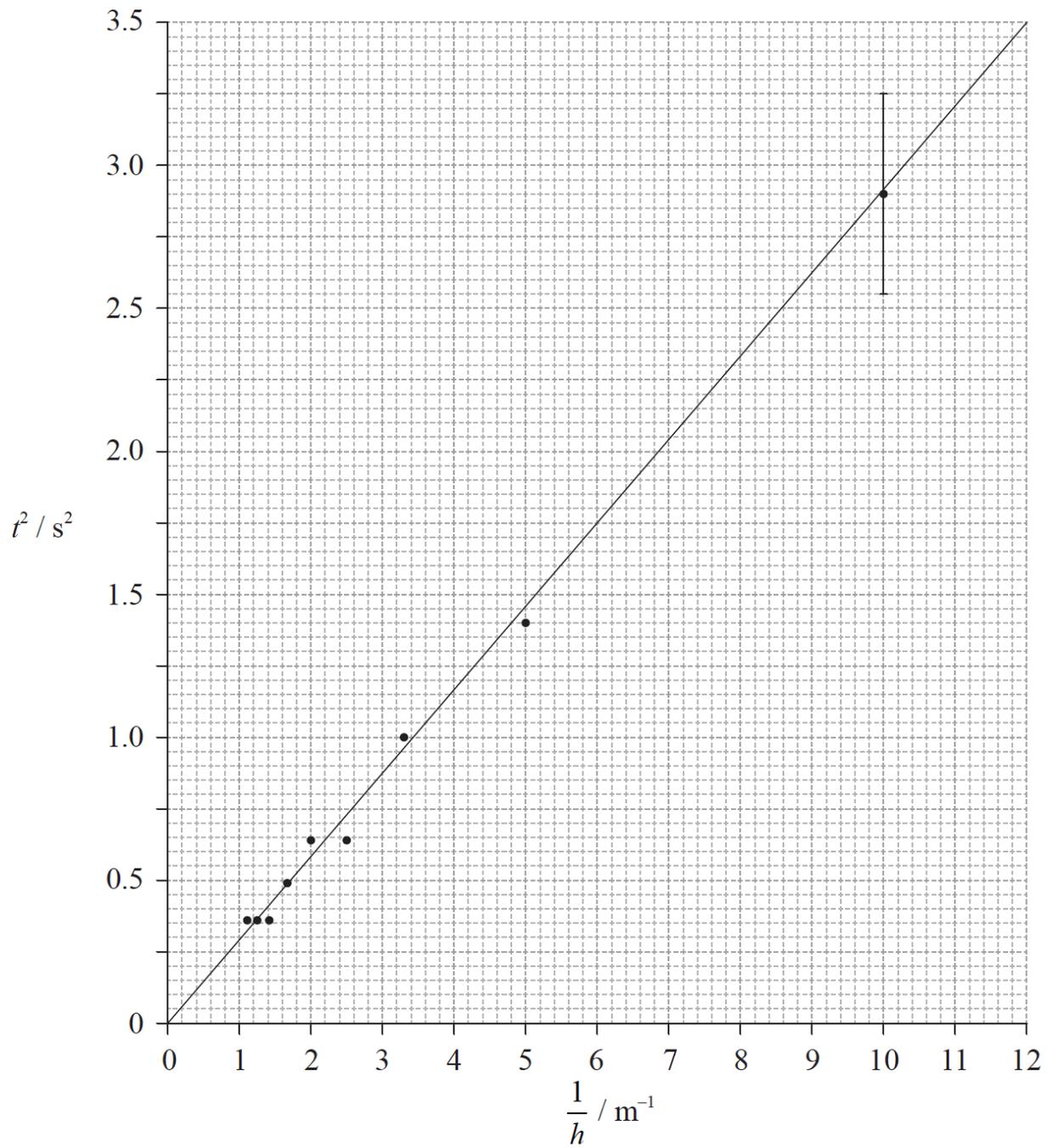
b. Another student suggests that the relationship between  $t$  and  $h$  is of the form [9]

$$t = k\sqrt{\frac{1}{h}}$$

where  $k$  is a constant.

To test whether or not the data support this relationship, a graph of  $t^2$  against  $\frac{1}{h}$  is plotted as shown below.

The best-fit line takes into account the uncertainties for all data points.



The uncertainty in  $t^2$  for the data point where  $\frac{1}{h} = 10.0\text{m}^{-1}$  is shown as an error bar on the graph.

(i) State the value of the uncertainty in  $t^2$  for  $\frac{1}{h} = 10.0\text{m}^{-1}$ .

(ii) Calculate the uncertainty in  $t^2$  when  $t = 0.8 \pm 0.1\text{s}$ . Give your answer to an appropriate number of significant digits.

(iii) Use the graph to determine the value of  $k$ . Do not calculate its uncertainty.

(iv) State the unit of  $k$ .

## Markscheme

a. (i) any straight line that goes through all error bars;

(ii) line does not go through origin / (0,0) / zero;

b. (i)  $\pm 0.35\text{s}^2$ ; (accept answers in range 0.3 to 0.4)

$$(ii) \frac{\Delta(t^2)}{t^2} = 2 \frac{\Delta t}{t};$$

$$\Delta(t^2) = 0.8^2 \times 2 \times \frac{0.1}{0.8};$$

$$\Delta(t^2) = 0.16 \approx 0.2s^2;$$

answer given to one significant figure;

or

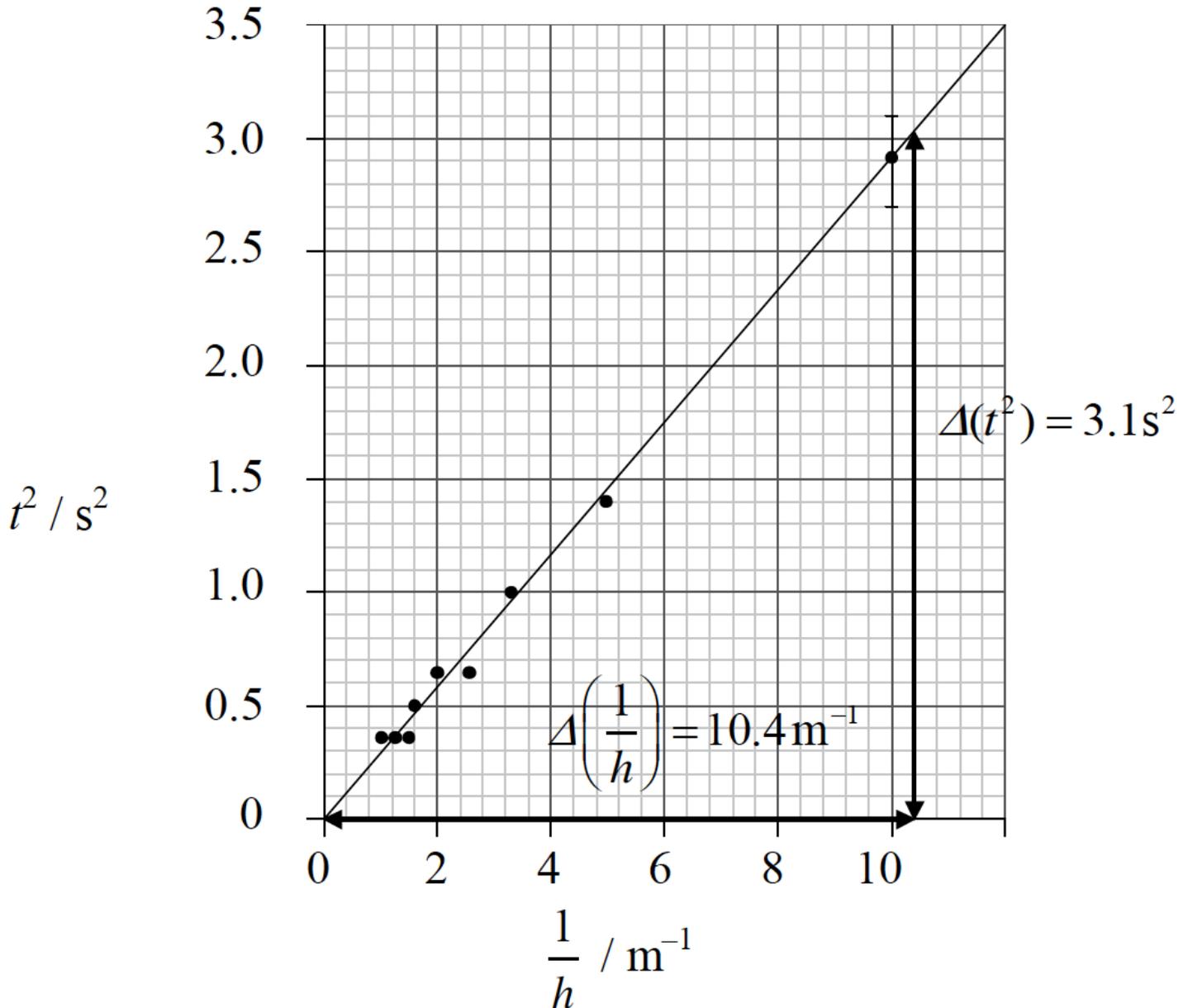
$$\text{percentage uncertainty in } t = \frac{0.1}{0.8} \times 100 = 12.5\%;$$

$$\text{percentage uncertainty in } t^2 = 25\%;$$

$$\text{absolute uncertainty in } t^2 = 0.25 \times 0.8^2 = 0.16 \approx 0.2s^2;$$

answer given to one significant figure;

(iii)



use of gradient triangle over at least half of line;

value of gradient = 0.30; (accept answers in range 0.28 to 0.32)

=  $k^2$  to give  $k=0.55$ ; (accept answers in range 0.53 to 0.57)

or

$$\text{equation of line is } t^2 = \frac{k^2}{h};$$

data values for a point on the line selected;

values substituted into equation to get  $k=0.55$ ; (accept answers in range 0.53 to 0.57)

Award [2] for answers that use a data point not on the best fit line.

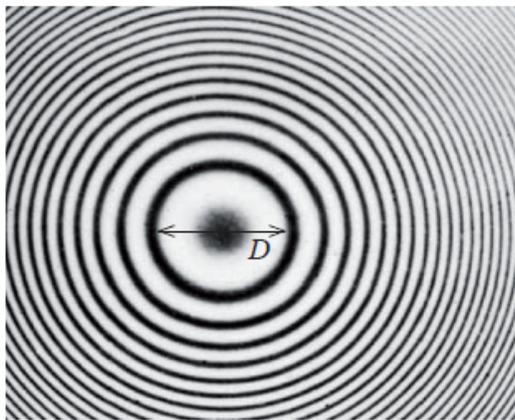
(iv)  $\text{m}^{\frac{1}{2}}\text{s}$ ;

# Examiners report

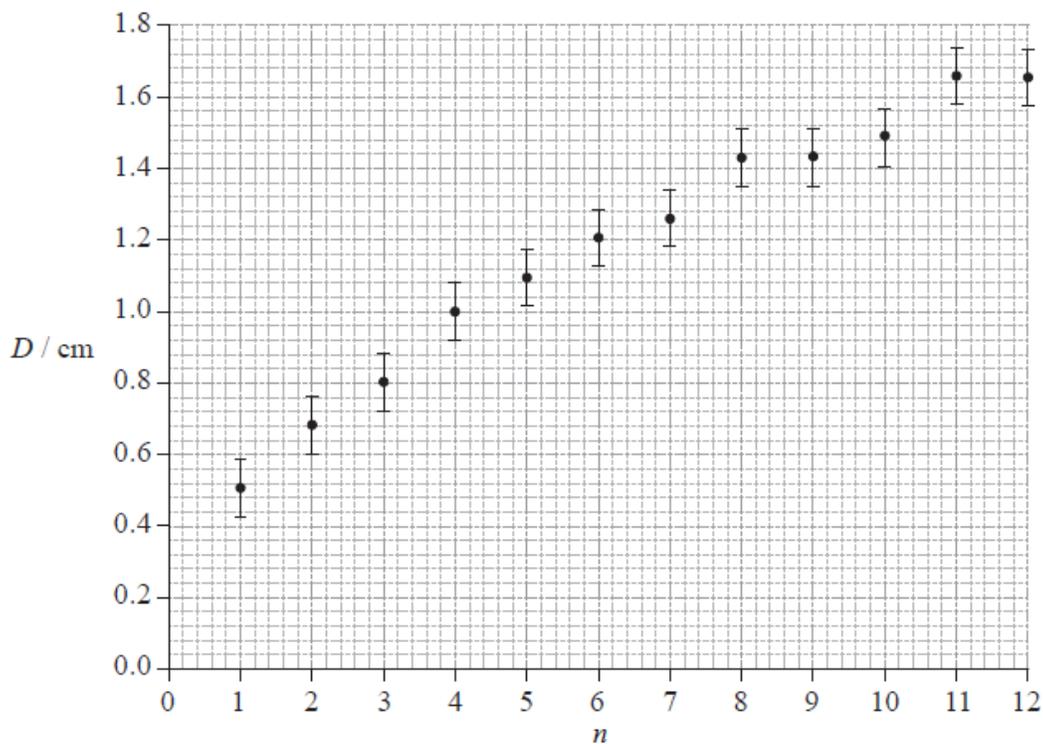
- a. [N/A]
- b. [N/A]

Data analysis question.

The photograph below shows a magnified image of a dark central disc surrounded by concentric dark rings. These rings were produced as a result of interference of monochromatic light.



The graph below shows how the ring diameter  $D$  varies with the ring number  $n$ . The innermost ring corresponds to  $n = 1$ . The corresponding diameter is labelled in the photograph. Error bars for the diameter  $D$  are shown.



a. State **one** piece of evidence that shows that  $D$  is not proportional to  $n$ .

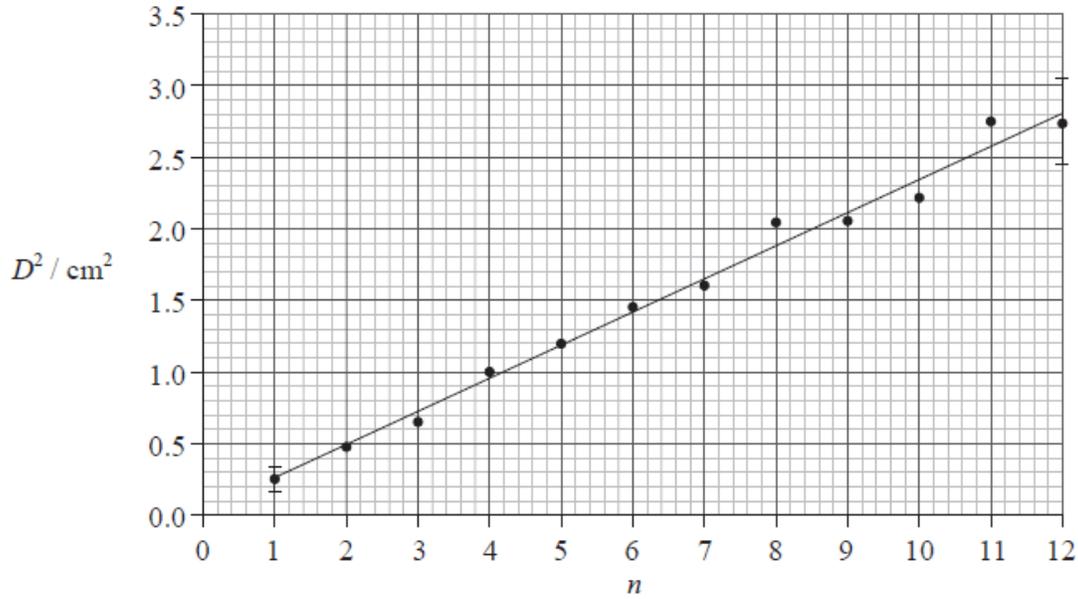
[1]

b. On the graph opposite, draw the line of best-fit for the data points.

[2]

c. Theory suggests that  $D^2 = kn$ .

A graph of  $D^2$  against  $n$  is shown below. Error bars are shown for the first and last data points only.



- Using the graph on page 2, calculate the percentage uncertainty in  $D^2$ , of the ring  $n = 7$ .
- Based on the graph opposite, state **one** piece of evidence that supports the relationship  $D^2 = kn$ .
- Use the graph opposite to determine the value of the constant  $k$ , as well as its uncertainty.
- State the unit for the constant  $k$ .

## Markscheme

a. line of best fit is not straight / line of best fit does not go through origin;

b. smooth curve;

that does not go outside the error bars;

*Ignore extrapolations below  $n=1$ .*

c. (i) absolute uncertainty in diameter  $D$  is  $\pm 0.08\text{cm}$ ;

giving a relative uncertainty in  $D^2$  of  $2 \times \frac{0.08}{1.26} = 0.13$  **or** 13%;

*Award [2] if uncertainty is calculated for a different ring number.*

(ii) it is possible to draw a straight line that passes through the origin (and lies within the error bars);

**or**

the ratio of  $\frac{D^2}{n}$  is constant for all data points;

(iii) gradient =  $k$ ;

calculation of gradient to give 0.23 (*accept answers in range 0.21 to 0.25*);

evidence for drawing or working with lines of maximum and minimum slope;

answers in the form  $k = 0.23 \pm 0.03$ ;

*Accept an uncertainty in  $k$  in range 0.02 to 0.04. First marking point does not need to be explicit.*

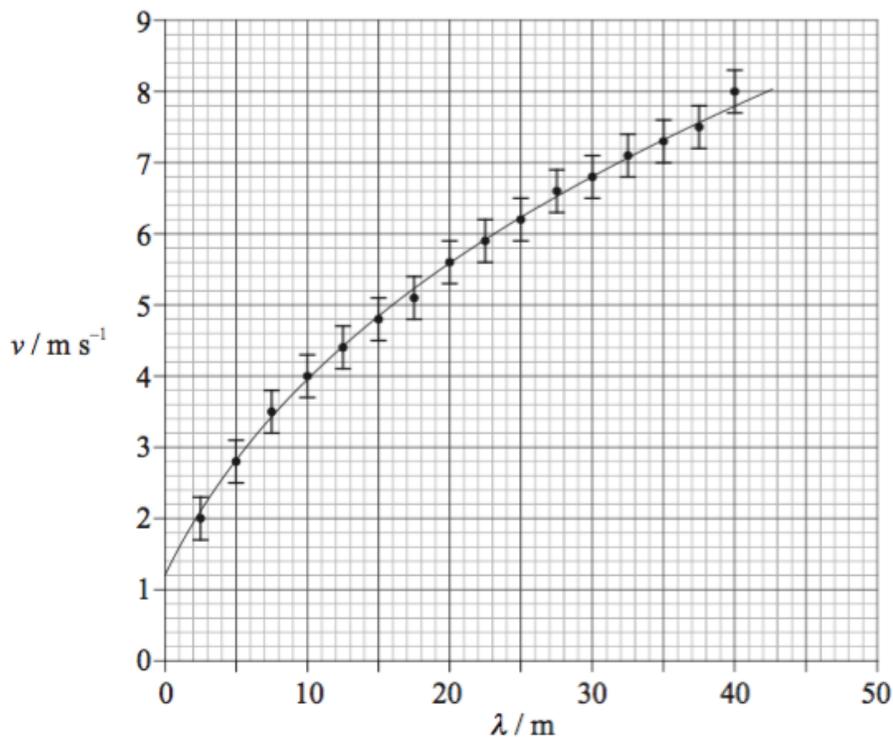
(iv)  $\text{cm}^2$ ;

# Examiners report

- a. [N/A]
- b. [N/A]
- c. [N/A]

Data analysis question.

The speed  $v$  of waves on the surface of deep water depends only on the wavelength  $\lambda$  of the waves. The data gathered from a particular region of the Atlantic Ocean are plotted below.



The uncertainty in the speed  $v$  is  $\pm 0.30 \text{ m s}^{-1}$  and the uncertainty in  $\lambda$  is too small to be shown on the diagram.

State, with reference to the graph,

a. (i) why  $v$  is not directly proportional to  $\lambda$ .

[2]

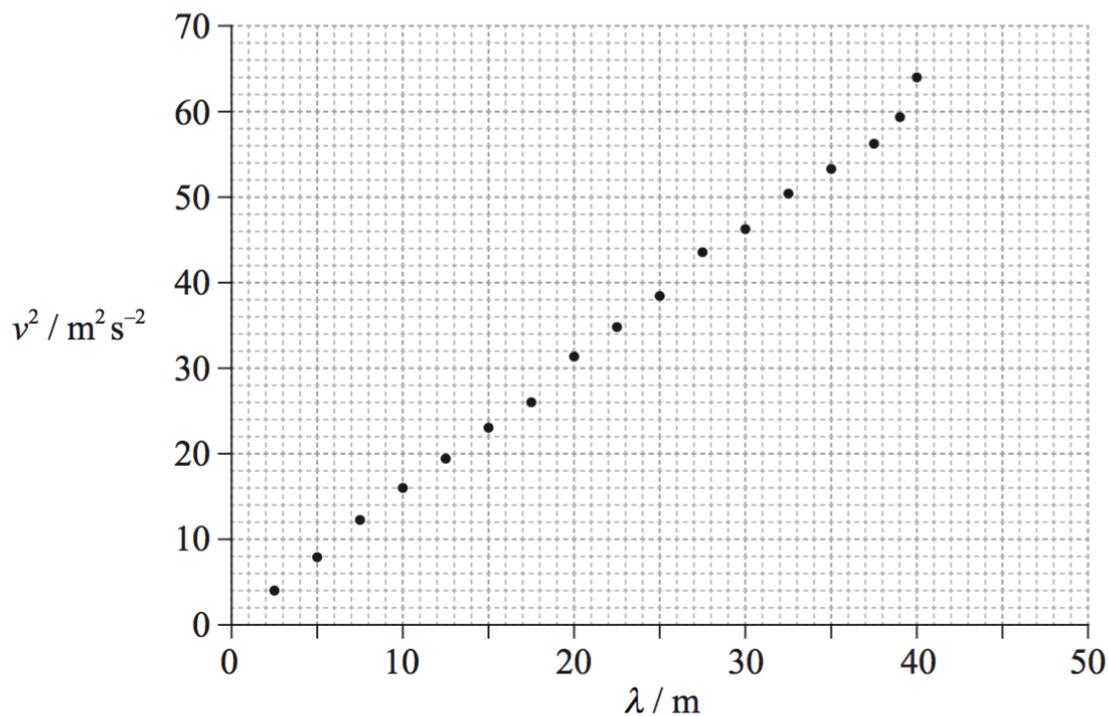
(ii) the value of  $v$  for  $\lambda=39\text{m}$ .

b. It is suggested that the relationship between  $v$  and  $\lambda$  is of the form

[9]

$$v = a\sqrt{\lambda}$$

where  $a$  is a constant. To test the validity of this hypothesis, values of  $v^2$  against  $\lambda$  are plotted below.



- (i) Use your answer to (a)(ii) to show that the absolute uncertainty in  $v^2$  for a wavelength of 39 m is  $\pm 5 \text{ m}^2 \text{ s}^{-2}$ .
- (ii) The absolute uncertainty in  $v^2$  for a wavelength of 2.5 m is  $\pm 1 \text{ m}^2 \text{ s}^{-2}$ . Using this value and the value in (b)(i), construct error bars for  $v^2$  at the data points for  $\lambda = 2.5 \text{ m}$  and 39 m.
- (iii) State why the plotted data in (b)(ii) suggest that it is likely that  $v$  is proportional to  $\sqrt{\lambda}$ .
- (iv) Use the graph opposite to determine the constant  $a$ .
- (v) Theory shows that  $a = \sqrt{\frac{k}{2\pi}}$ . Determine a value for  $k$ .

## Markscheme

a. (i) the graph is not linear/a straight line (going through the error bars) / does not go through origin **[1]**

(ii)  $7.7 \text{ ms}^{-1}$ ; (**N.B.** line is drawn for candidate, answer must be correct) **[1]**

b. (i) % uncertainty in  $v = \left(\frac{0.3}{7.7}\right) 3.9\%$ ;

doubles 3.9% (allow ECF from (a)(ii)) to obtain % uncertainty in  $v^2 (=7.8\%)$ ;

absolute uncertainty ( $=\pm[0.078 \times 59.3]$ ) = 4.6;

( $=\pm 5 \text{ m}^2 \text{ s}^{-2}$ )

**or**

calculates overall range of possible value as 7.4–8.0; (allow ECF)

squares values to yield range for  $v^2$  of 54.8 to 64; (allow ECF)

so error range becomes 9.2 hence  $\pm 4.6$ ; (must see this value to 2 sig fig or better to award this mark)

(ii) correct error bars added to first point ( $\pm 1/2$  square) and last-but-one point ( $\pm 2.5$  squares); (*judge by eye*)

(iii) a straight-line/linear graph can be drawn that goes through origin;

(iv) uses triangle to evaluate gradient; { (*triangle need not be shown if read-offs clear, read-offs used must lie on candidate's drawn line*) to arrive at gradient value of  $1.5 \pm 0.2$ ; (*unit not required*)

recognizes that gradient of graph is  $a^2$  and evaluates  $a = 1.2 \pm 0.2 (\text{m}^{1/2} \text{s}^{-1})$ ;

**or**

candidate line drawn through origin and one data point read;

correct substitution into  $v^2 = a^2 \lambda$ ; ( $a^2$  does not need to be evaluated for full credit)

$a = 1.2 \pm 0.2 (\text{m}^{1/2} \text{s}^{-1})$ ;

Award [2 max] if line does not go through origin – allow  $1/2$  square.

Award [1 max] if one or two data points used and no line drawn.

(v)  $k = 9.4 \text{ms}^{-2}$ ; (allow ECF from (b)(iv))

## Examiners report

a. (i) Most were able to identify one (of several reasons) why the proportionality did not apply.

(ii) Almost all could state the value at the required point to a sensible accuracy.

b. (i) Many fully understood the simple treatment of combination of errors and arrived at a correct and well-explained solution.

(ii) The error bars were usually correctly drawn, however in a small number of cases, candidates drew the same length bar for both points (usually using the value for the upper data point).

(iii) Unlike in (b) the reasons for proportionality were usually incomplete on this occasion and few candidates scored the mark. The fact that the line goes through the origin was often ignored.

(iv) This question was done poorly; the work of many candidates was very disappointing here. Only about half the candidates attempted to draw a straight line on the graph (they were told to “Use the graph”) and simply used two points on the graph without reference to a line. This gained little credit as the candidate gave no evidence at all that the chosen pair of points both lay on the line. Candidates then often compounded this by quoting  $a^2$  as the answer to the question, failing to recognize that a square root was required.

(v) Most candidates were able to take their derived  $a$  (correct or not) and evaluate  $k$  however the unit of  $k$  was usually ignored.

Data analysis question.

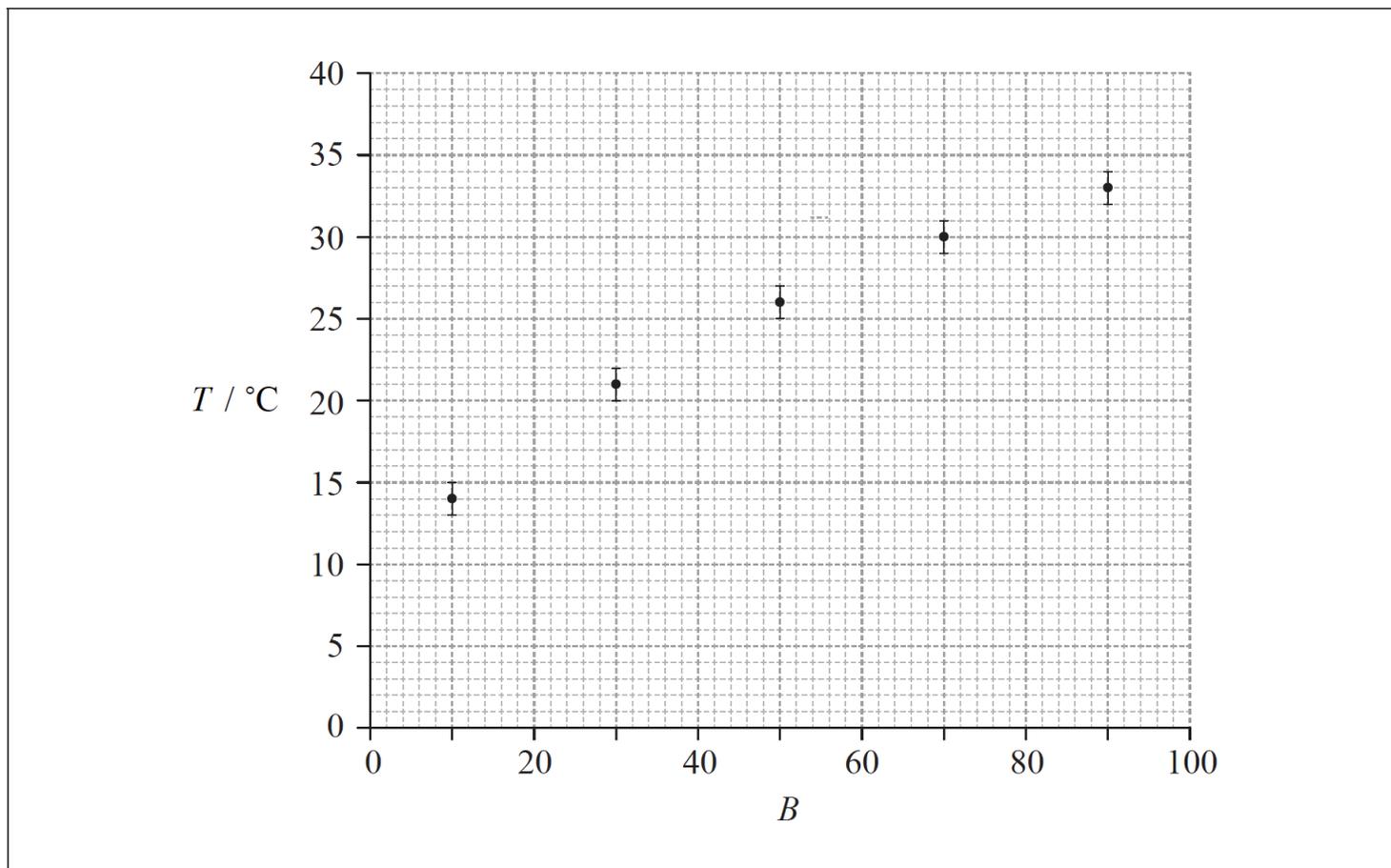
Connie and Sophie investigate the effect of colour on heat absorption. They make grey paint by mixing black and white paint in different ratios. Five identical tin cans are painted in five different shades of grey.



10% black paint    30% black paint    50% black paint    70% black paint    90% black paint

Connie and Sophie put an equal amount of water at the same initial temperature into each can. They leave the cans under a heat lamp at equal distances from the lamp. They measure the temperature increase of the water,  $T$ , in each can after one hour.

- a. Connie suggests that  $T$  is proportional to  $B$ , where  $B$  is the percentage of black in the paint. To test this hypothesis, she plots a graph of  $T$  against  $B$ , as shown on the axes below. The uncertainty in  $T$  is shown and the uncertainty in  $B$  is negligible. [6]



- State the value of the absolute uncertainty in  $T$ .
- Comment on the fractional uncertainty for the measurement of  $T$  for  $B=10$  and the measurement of  $T$  for  $B=90$ .
- On the graph opposite, draw a best-fit line for the data.
- Outline why the data do not support the hypothesis that  $T$  is proportional to  $B$ .

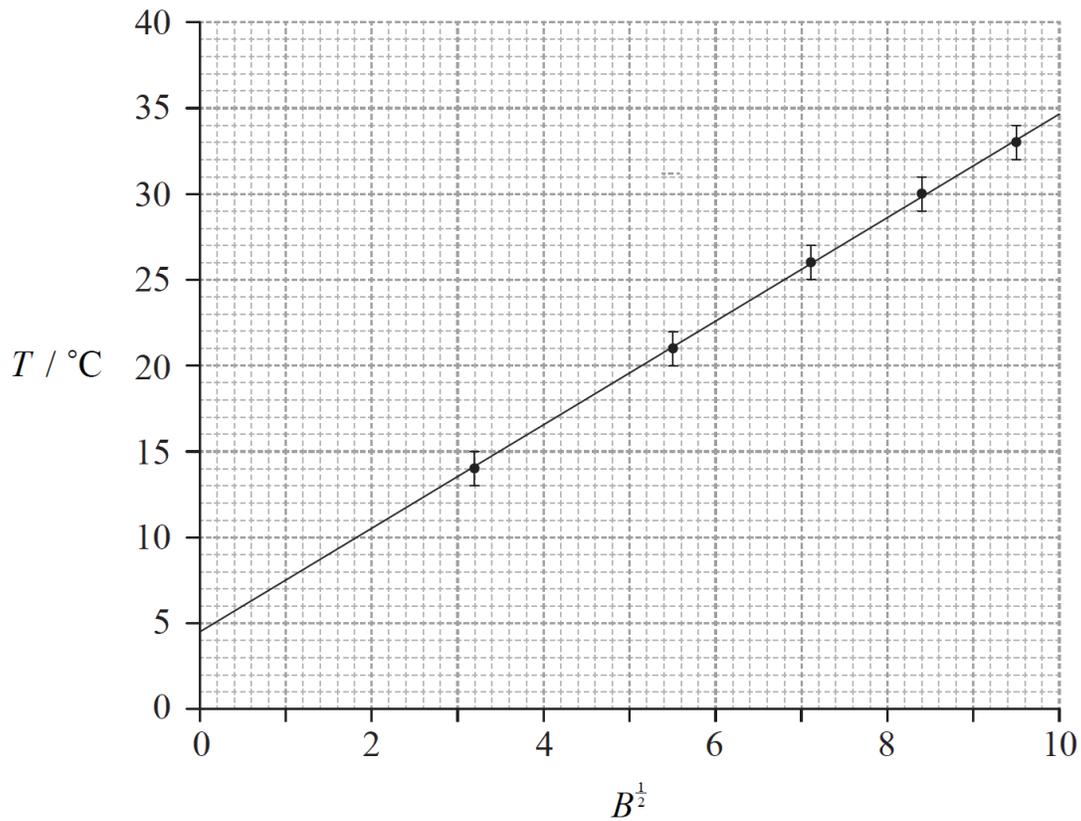
- b. Sophie suggests that the relationship between  $T$  and  $B$  is of the form

[5]

$$T = kB^{\frac{1}{2}} + c$$

where  $k$  and  $c$  are constants.

To test whether or not the data support this relationship, a graph of  $T$  against  $B^{\frac{1}{2}}$  is plotted as shown below. The uncertainty in  $T$  is shown and the uncertainty in  $B^{\frac{1}{2}}$  is negligible.



(i) Use the graph to determine the value of  $c$  with its uncertainty.

(ii) State the unit of  $k$ .

## Markscheme

a. (i) ( $\pm$ ) 1 ( $^\circ\text{C}$ );

(ii) absolute uncertainty is the same for the two points;  
since  $T$  is higher at  $B = 90$  (*stated or shown*), relative uncertainty is smaller;

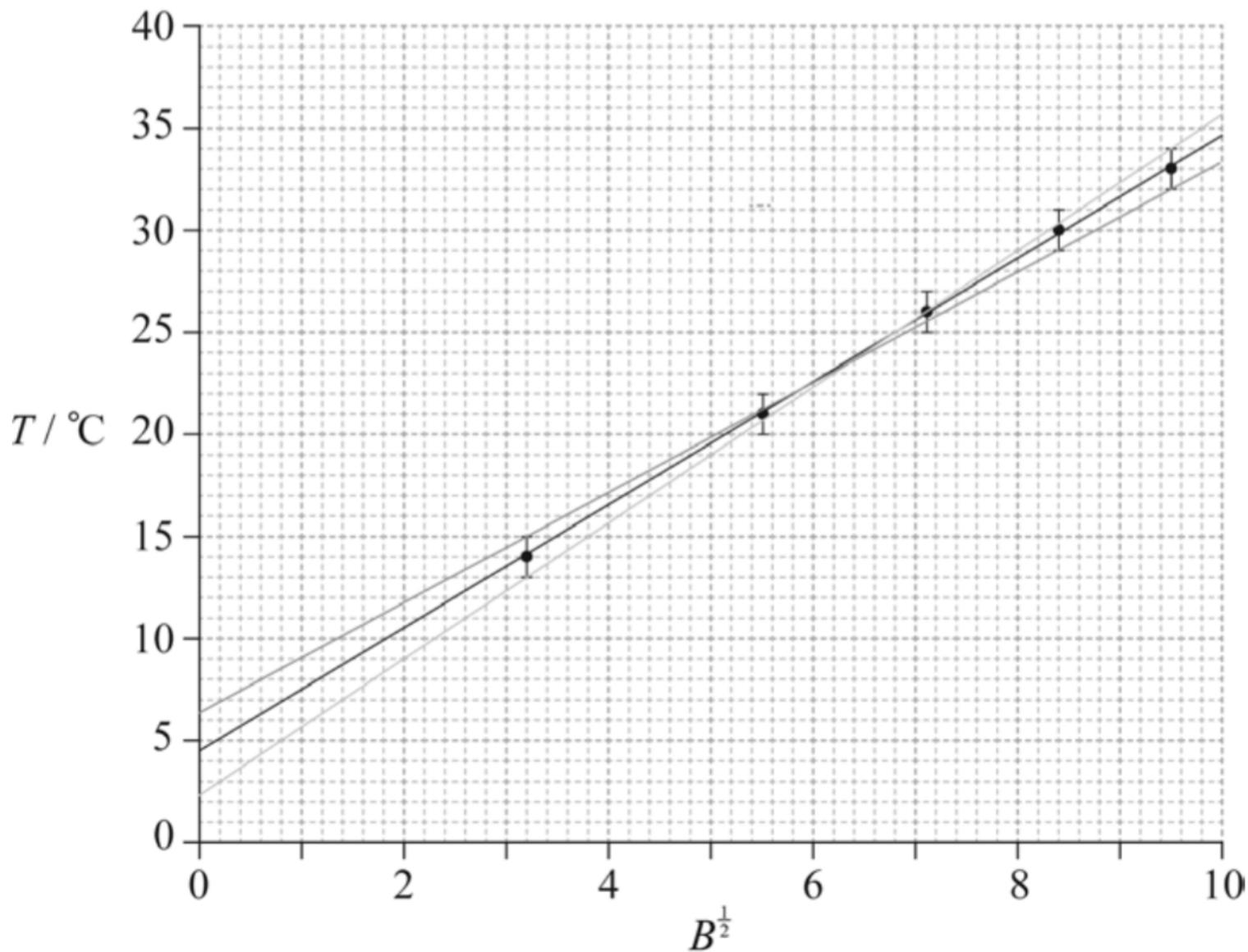
*or*

fractional uncertainties are  $0.07/\frac{1}{14}$  /7% for  $B=10$  and  $0.03/\frac{1}{33}$  /3% for  $B=90$ ;  
fractional uncertainty is smaller for  $B=90$ ;

(iii) smooth curve passing through all error bars;  
*Do not allow thick or hairy or doubled lines, or lines where the curvature changes abruptly.*

(iv) the line is not straight/is a curve/does not have a constant gradient/is not linear;  
it does not pass through the origin/(0, 0)/zero;

b. (i)



intercept read as 4.7; (ignore significant figures, allow range of 4.5 to 4.9)

two worst fit lines drawn through extremes of error bars;

uncertainty found from worst fit lines;

uncertainty rounded to 1 significant digit expressed in the form as  $\pm$  (value)

and intercept rounded to same precision;

Award [4] for a statement of  $5 \pm 2$  and lines drawn.

(ii)  $^\circ\text{C}$  ;

## Examiners report

a. ai) Most candidates could accurately read the absolute uncertainty from an error bar. The only mistake made was by those who wrote  $\pm 2$ .

aii) Most were able to calculate the fractional uncertainties, but too often the figures were not compared.

aiii) This was mainly answered well, with few straight lines.

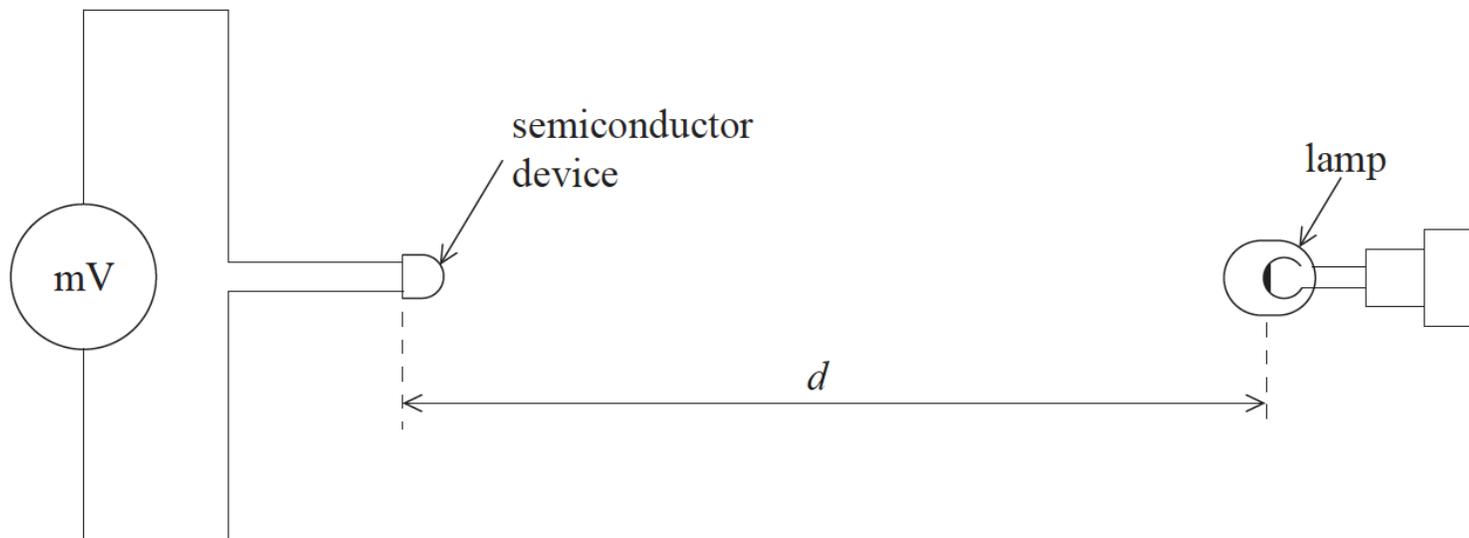
aiv) Most recognized the shape of the line required for proportionality.

b. bi) Few candidates appreciated the importance of using the best and worst fit lines in finding an uncertainty from the line of best-fit. Many candidates could not state the uncertainty and value to an appropriate precision.

bii) Most candidates successfully identified the unit.

Data analysis question.

A particular semiconductor device generates an emf, which varies with light intensity. The diagram shows the experimental arrangement which a student used to investigate the variation with distance  $d$  of the emf  $\epsilon$ . The power output of the lamp was constant. (The power supply for the lamp is not shown.)



The table shows how  $\epsilon$  varied with  $d$ .

$d / \text{cm}$	$\epsilon / \text{mV}$
19.1	5.5
18.0	6.0
16.0	8.6
14.0	11.9
12.0	19.7
10.0	37.5

a. Outline why the student has recorded the  $\epsilon$  values to different numbers of significant digits but the same number of decimal places. [2]

b. On looking at the results the student suggests that  $\epsilon$  could be inversely proportional to  $d$ . He proceeds to multiply each  $d$  value by the corresponding value of  $\epsilon$ . [5]

(i) Explain why this procedure can be used to disprove the student's suggestion but it cannot prove it.

(ii) Using the data for  $d$  values of 19.1 cm, 16.0 cm and 10.0 cm discuss whether or not  $\varepsilon$  is inversely proportional to  $d$ .

c. The graph shows some of the data points with the uncertainty in the  $d$  values.

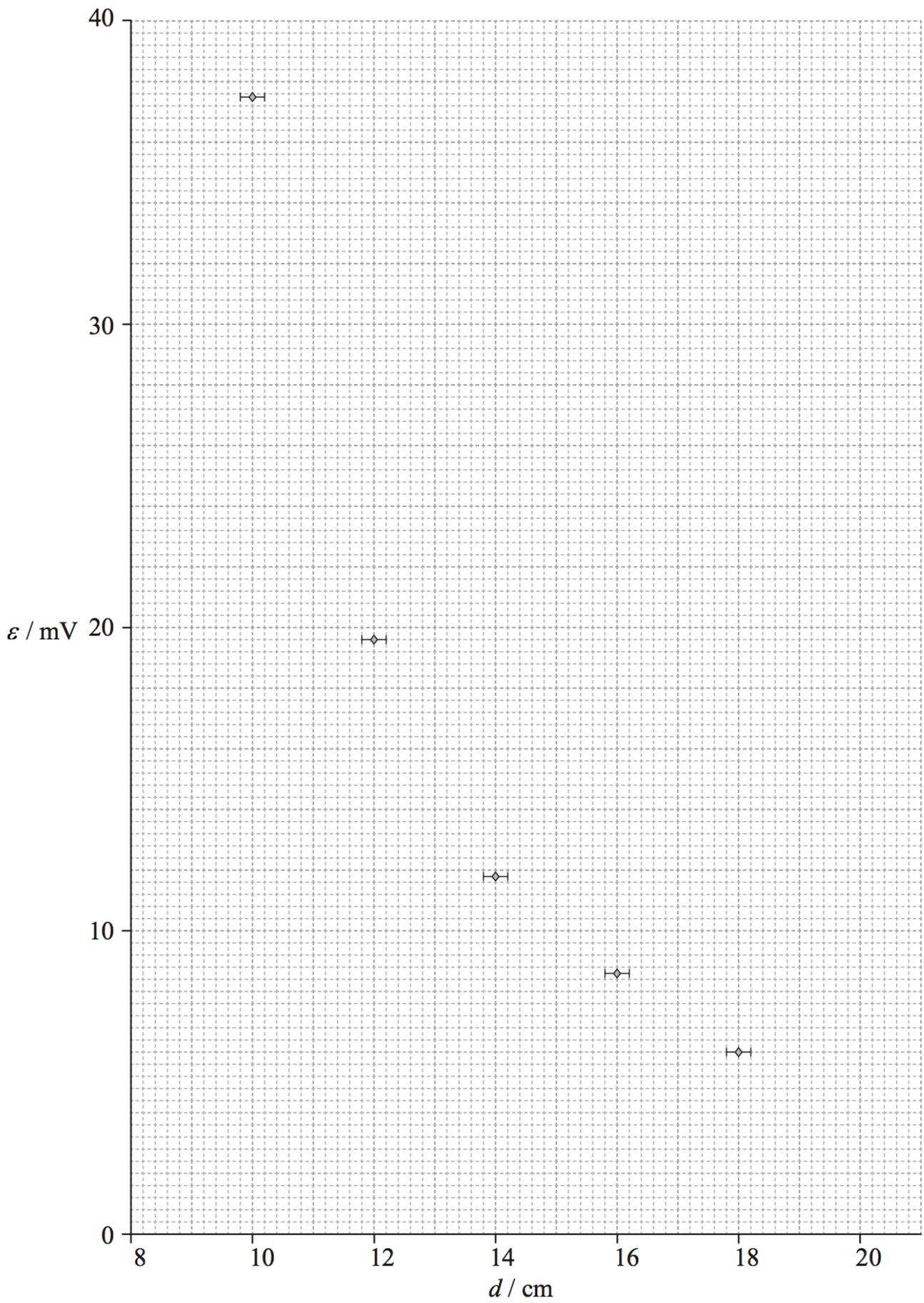
[3]

On the graph

(i) draw the data point corresponding to the value of  $d=19.1$  cm.

(ii) assuming that there is a constant absolute uncertainty in measuring all values of  $d$ , draw the error bar for the data point in (c)(i).

(iii) sketch the line of best-fit for all the plotted points.



d. All values of  $\varepsilon$  have a percentage uncertainty of  $\pm 3\%$ . Calculate the percentage uncertainty in the product  $d\varepsilon$  for the value of  $d=18.0\text{cm}$ .

[2]

## Markscheme

a. reference to meter/instrument;

reference to constant accuracy/precision;

*Award [2] for "voltmeter measures to 0.1 V".*

b. (i) looking for constant value;

clear deviation means not/unlikely to be valid;

close to constant only means possibility of validity;

(ii) two of 105, 138, 375 correct;

third value correct;

products so far apart clearly not inversely proportional;

**or**

attempts to show that  $\frac{d_1}{\varepsilon_2} \neq \frac{d_2}{\varepsilon_1}$  **or**  $\frac{d_1}{d_2} \neq \frac{\varepsilon_2}{\varepsilon_1}$  for two pairs of values;

third pair of values used;

ratios so far apart clearly not inversely proportional;

c. (i) point plotted  $\pm 1/2$  small square;

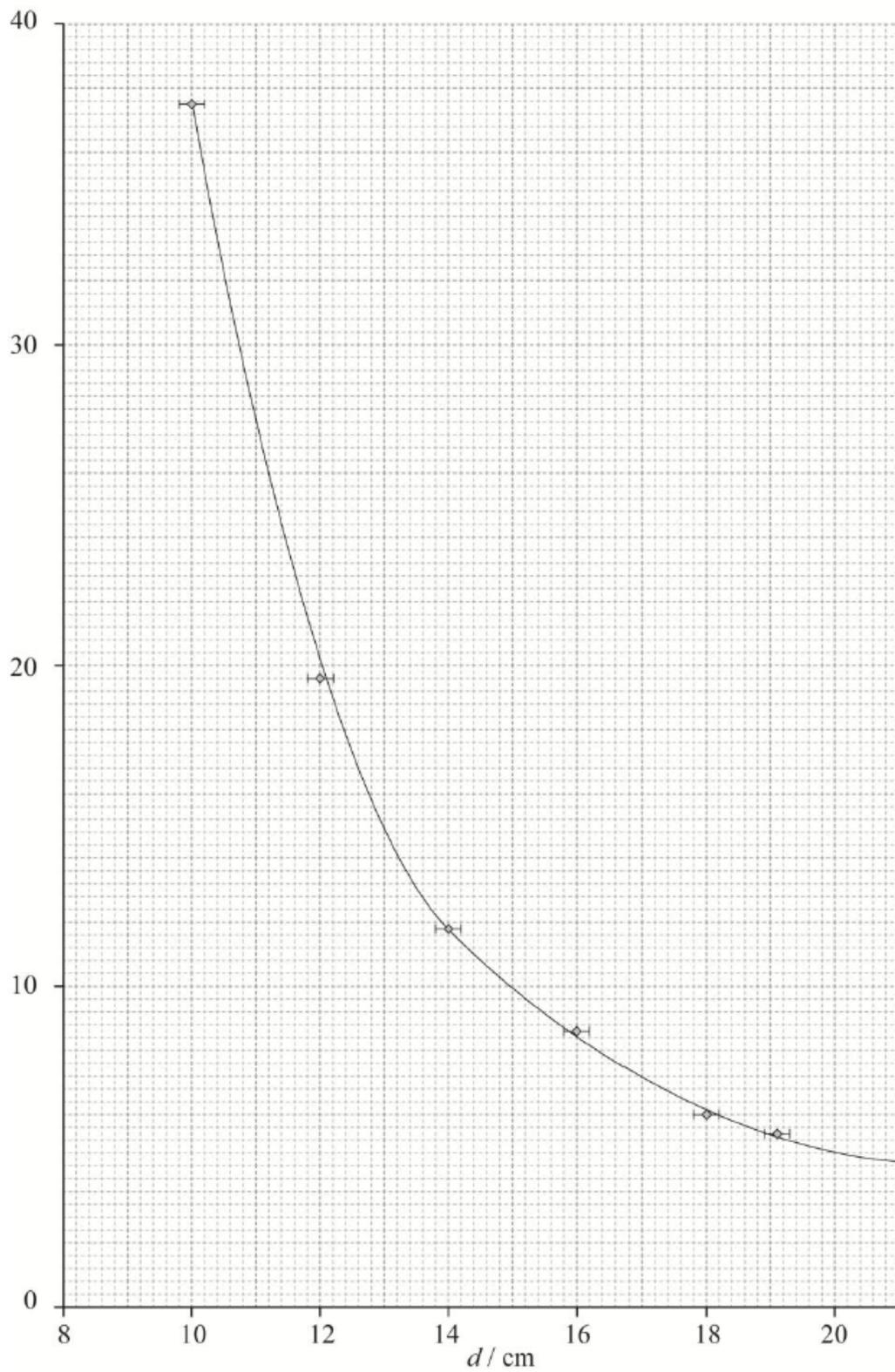
*See graph for position.*

*1.5 small squares down +5.5 small squares across from previous plotted point.*

(ii) symmetrical error bar, 1 small square in each direction  $\pm 1/2$  small square;

(iii) single smooth curve within each error bar;

*Do not condone multiple, hairy or unduly thick lines.*



d. % uncertainty in  $d$  value  $\left( = \frac{0.2\text{cm}}{18\text{cm}} \right) = 1/1.1\%$ ;

% uncertainty in  $d\varepsilon$  product = 4/4.1%;

Allow ECF from wrong absolute error in  $d$ .

## Examiners report

- a. Many candidates failed to recognise that the number of decimal places is a reflection of the precision of a piece of equipment – in this case the millivoltmeter. Using different number of significant figures simply indicates that the reading is larger or smaller but it will be to the same precision. A sizeable proportion of candidates believed the number of decimal places was something that they could choose in an arbitrary manner.
- b. (i) Most candidates failed to realise that the result of multiplying a series of corresponding values of  $\epsilon$  and  $d$  only needed to show different values for the equation to be disproved but that all possible values would need to be taken prove it (clearly an impossibility).
- (ii) By performing the task in (i) most candidates showed that there was too large a discrepancy between the three sets of products to suggest that the equation was viable.
- c. (i) Most candidates were able to correctly plot the data point despite there being a relatively difficult scale division.
- (ii) The majority of candidates drew an appropriate error bar.
- (iii) Many failed to take sufficient care when sketching the line of best fit. Lines of best fit are not always straight and it is important that candidates practise drawing curves in preparation for examinations. It was common to see multiple lines, some of which did not pass through the horizontal part of the error bar (the vertical edges being irrelevant and of arbitrary length).
- d. Most candidates were able to find the percentage uncertainty in the  $d$  value or correctly added the two percentage values; many were unable to do both.
-