HL Paper 2

This question is in **two** parts. **Part 1** is about gravitational force fields. **Part 2** is about properties of a gas.

Part 1 Gravitational force fields

a. State Newton's universal law of gravitation. [2]

b. A satellite of mass *m* orbits a planet of mass *M*. Derive the following relationship between the period of the satellite *T* and the radius of its orbit *R* (Kepler's third law). [3]

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

c. A polar orbiting satellite has an orbit which passes above both of the Earth's poles. One polar orbiting satellite used for Earth observation has an orbital period of 6.00 × 10^3s. [8]

Mass of Earth = 5.97 × 10^{24} kg

Average radius of Earth = 6.37 × 10^6 m

(i) Using the relationship in (b), show that the average height above the surface of the Earth for this satellite is about 800 km.

(ii) The satellite moves from an orbit of radius 1200 km above the Earth to one of radius 2500 km. The mass of the satellite is 45 kg.

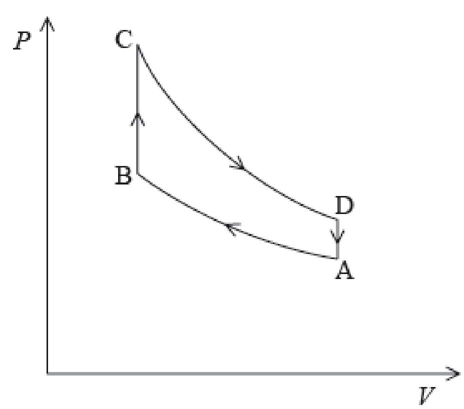
Calculate the change in the gravitational potential energy of the satellite.

(iii) Explain whether the gravitational potential energy has increased, decreased or stayed the same when the orbit changes, as in (c)(ii).

This question is about the thermodynamics of a car engine and the dynamics of the car.

A car engine consists of four cylinders. In each of the cylinders, a fuel-air mixture explodes to supply power at the appropriate moment in the cycle.

The diagram models the variation of pressure *P* with volume *V* for one cycle of the gas, ABCDA, in one of the cylinders of the engine. The gas in the cylinder has a fixed mass and can be assumed to be ideal.



The car is travelling at its maximum speed of $56\ \mathrm{m\,s^{-1}}$. At this speed, the energy provided by the fuel injected into one cylinder in each cycle is 9200 J. One litre of fuel provides 56 MJ of energy.

A car is travelling along a straight horizontal road at its maximum speed of $56\ \mathrm{m\,s^{-1}}$. The power output required at the wheels is 0.13 MW.

A driver moves a car in a horizontal circular path of radius 200 m. Each of the four tyres will not grip the road if the frictional force between a tyre and the road becomes less than 1500 N.

a. At point A in the cycle, the fuel-air mixture is at 18 °C. During process AB, the gas is compressed to 0.046 of its original volume and the pressure increases by a factor of 40. Calculate the temperature of the gas at point B. [1]

b. State the nature of the change in the gas that takes place during process BC in the cycle. [1]

c. Process CD is an adiabatic change. Discuss, with reference to the first law of thermodynamics, the change in temperature of the gas in the cylinder during process CD. [3]

d. Explain how the diagram can be used to calculate the net work done during one cycle. [2]

e. (i) Calculate the volume of fuel injected into one cylinder during one cycle. [3]

(ii) Each of the four cylinders completes a cycle 18 times every second. Calculate the distance the car can travel on one litre of fuel at a speed of $56\ \mathrm{m\,s^{-1}}$.

f. A car accelerates uniformly along a straight horizontal road from an initial speed of $12\ \mathrm{m\,s^{-1}}$ to a final speed of $28\ \mathrm{m\,s^{-1}}$ in a distance of 250 m. The mass of the car is 1200 kg. Determine the rate at which the engine is supplying kinetic energy to the car as it accelerates. [4]

g. (i) Calculate the total resistive force acting on the car when it is travelling at a constant speed of $56\ \mathrm{m\,s^{-1}}$. [5]

(ii) The mass of the car is 1200 kg. The resistive force *F* is related to the speed *v* by $F \propto v^2$. Using your answer to (g)(i), determine the maximum theoretical acceleration of the car at a speed of $28\ \mathrm{m\,s^{-1}}$.

h. (i) Calculate the maximum speed of the car at which it can continue to move in the circular path. Assume that the radius of the path is the same for each tyre. [6]

(ii) While the car is travelling around the circle, the people in the car have the sensation that they are being thrown outwards. Outline how Newton's first law of motion accounts for this sensation.

This question is in **two** parts. **Part 1** is about fields, electric potential difference and electric circuits. **Part 2** is about thermodynamic cycles.

Part 1 Fields, electric potential difference and electric circuits

a. The magnitude of gravitational field strength *g* is defined from the equation shown below. [4]

$$g = \frac{F_g}{m}$$

The magnitude of electric field strength E is defined from the equation shown below.

$$E = \frac{F_E}{q}$$

For each of these defining equations, state the meaning of the symbols

(i) F_g.

(ii) F_E.

(iii) m.

(iv) q.

b. In a simple model of the hydrogen atom, the electron is regarded as being in a circular orbit about the proton. The magnitude of the electric field strength at the electron due to the proton is E_p . The magnitude of the gravitational field strength at the electron due to the proton is g_p. [3]

Determine the order of magnitude of the ratio shown below.

$$\frac{E_p}{g_p}$$

The gravitational potential due to the Sun at its surface is -1.9×10^{11} J kg^{-1}. The following data are available.

Mass of Earth	$= 6.0 \times 10^{24}$ kg
Distance from Earth to Sun	$= 1.5 \times 10^{11}$ m
Radius of Sun	$= 7.0 \times 10^{8}$ m

a. Outline why the gravitational potential is negative. [2]

b.i. The gravitational potential due to the Sun at a distance r from its centre is V_S. Show that [1]

$$rV_S = \text{constant.}$$

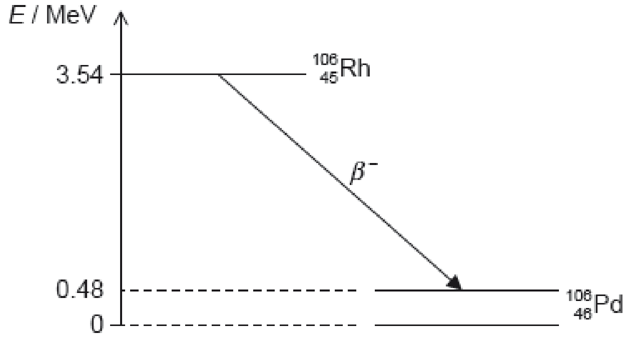
b.ii. Calculate the gravitational potential energy of the Earth in its orbit around the Sun. Give your answer to an appropriate number of significant figures. [2]

b.iii. Calculate the total energy of the Earth in its orbit. [2]

b.iv. An asteroid strikes the Earth and causes the orbital speed of the Earth to suddenly decrease. Suggest the ways in which the orbit of the Earth will change. [2]

c. Outline, in terms of the force acting on it, why the Earth remains in a circular orbit around the Sun. [2]

Rhodium-106 ($^{106}_{45}\text{Rh}$) decays into palladium-106 ($^{106}_{46}\text{Pd}$) by beta minus (β^-) decay. The diagram shows some of the nuclear energy levels of rhodium-106 and palladium-106. The arrow represents the β^- decay.



b. Bohr modified the Rutherford model by introducing the condition $mvr = n\frac{h}{2\pi}$. Outline the reason for this modification. [3]

c.i. Show that the speed *v* of an electron in the hydrogen atom is related to the radius *r* of the orbit by the expression [1]

$$v = \sqrt{\frac{ke^2}{m_e r}}$$

where *k* is the Coulomb constant.

c.ii. Using the answer in (b) and (c)(i), deduce that the radius *r* of the electron's orbit in the ground state of hydrogen is given by the following expression. [2]

$$r = \frac{h^2}{4\pi^2 k m_e e^2}$$

c.iii Calculate the electron's orbital radius in (c)(ii). [1]

d.i. Explain what may be deduced about the energy of the electron in the β^- decay. [3]

d.ii. Suggest why the β^- decay is followed by the emission of a gamma ray photon. [1]

d.iii Calculate the wavelength of the gamma ray photon in (d)(ii). [2]

A planet has radius *R*. At a distance *h* above the surface of the planet the gravitational field strength is *g* and the gravitational potential is *V*.

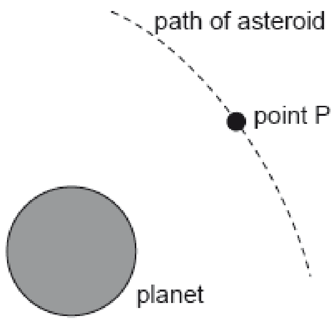
a.i. State what is meant by gravitational field strength. [1]

a.ii. Show that $V = -g(R + h)$. [2]

a.iii Draw a graph, on the axes, to show the variation of the gravitational potential *V* of the planet with height *h* above the surface of the planet. [2]

b. A planet has a radius of 3.1×10^6 m. At a point P a distance 2.4×10^7 m above the surface of the planet the gravitational field strength is 2.2 N kg^{-1}. Calculate the gravitational potential at point P, include an appropriate unit for your answer. [1]

c. The diagram shows the path of an asteroid as it moves past the planet. [3]

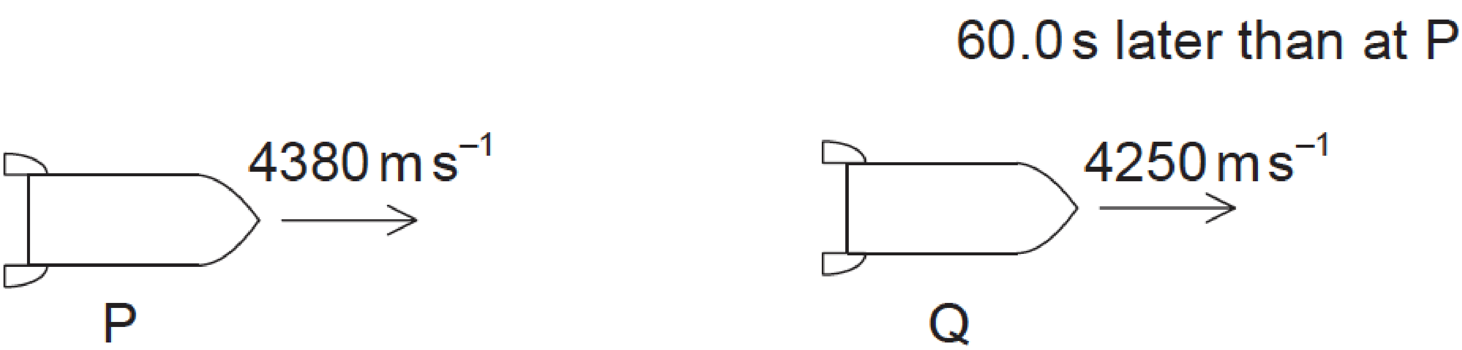


When the asteroid was far away from the planet it had negligible speed. Estimate the speed of the asteroid at point P as defined in (b).

d. The mass of the asteroid is 6.2×10^{12} kg. Calculate the gravitational force experienced by the **planet** when the asteroid is at point P. [2]

Part 2 Motion of a rocket

A rocket is moving away from a planet within the gravitational field of the planet. When the rocket is at position P a distance of 1.30×10^7m from the centre of the planet, the engine is switched off. At P, the speed of the rocket is $4.38 \times 10^3 ms^{-1}$.



At a time of 60.0 s later, the rocket has reached position Q. The speed of the rocket at Q is $4.25 \times 10^3 ms^{-1}$. Air resistance is negligible.

c. Outline, with reference to the energy of the rocket, why the speed of the rocket is changing between P and Q. [2]

d. Estimate the average gravitational field strength of the planet between P and Q. [2]

e. (i) An object is a distance *r* from the centre of a planet. Show that the minimum speed required to escape the gravitational field is equal to [5]

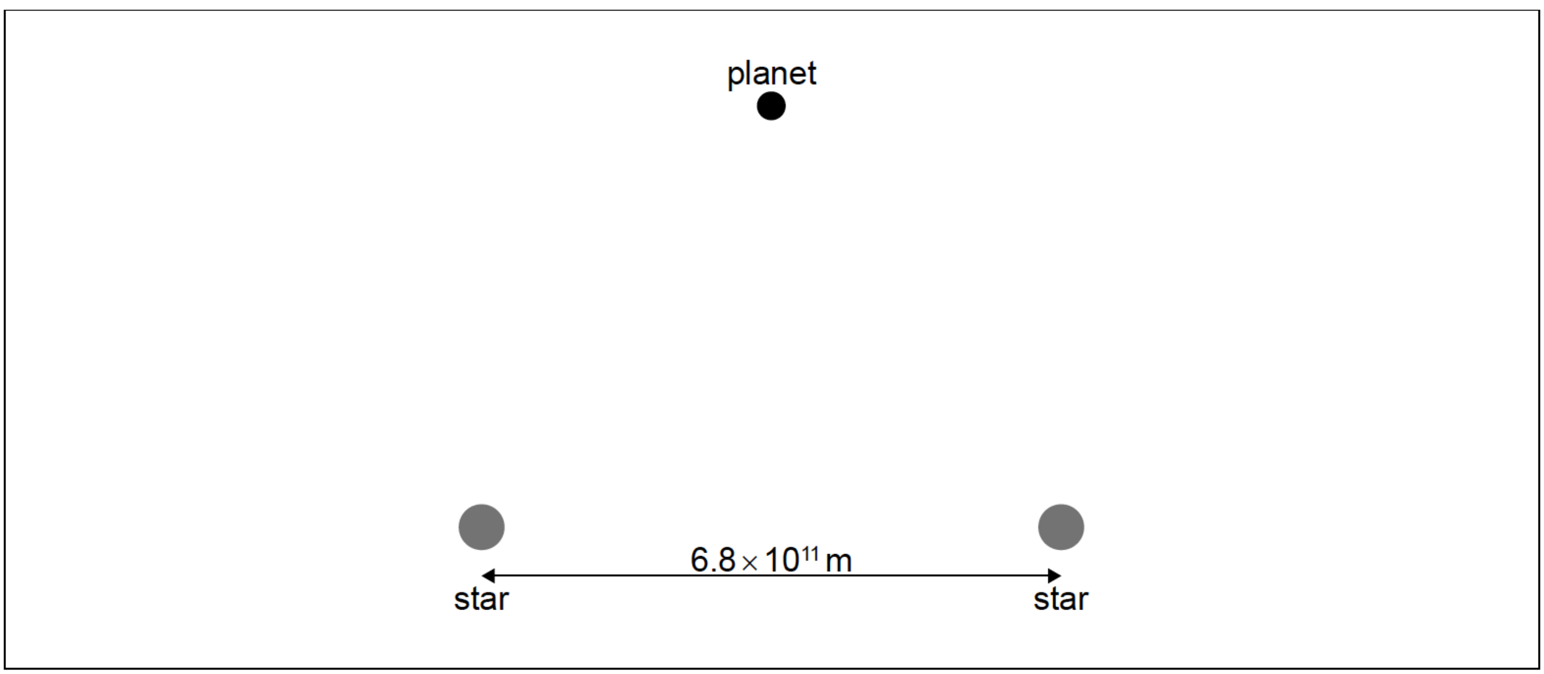
$$\sqrt{2g'r}$$

where g' is the gravitational field strength at distance *r* from the centre of a planet.

(ii) Discuss, using a calculation, whether the rocket at P can completely escape the gravitational field of the planet without further use of the engine.

f. A space station is in orbit at a distance *r* from the centre of the planet in (e)(i). A satellite is launched from the space station so as just to escape from the gravitational field of the planet. The launch takes place in the same direction as the velocity of the space station. Outline why the launch velocity relative to the space station can be less than your answer to (e)(i). [1]

The diagram shows a planet near two stars of equal mass *M*.



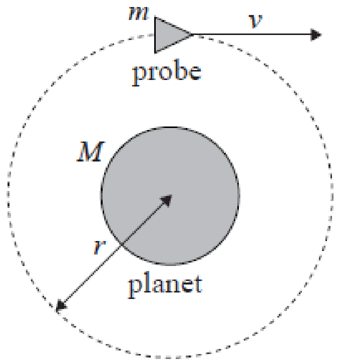
Each star has mass $M = 2.0\times10^{30}$ kg. Their centres are separated by a distance of 6.8×10^{11} m. The planet is at a distance of 6.0×10^{11} m from each star.

a. On the diagram above, draw **two** arrows to show the gravitational field strength at the position of the planet due to each of the stars. [2]

b. Calculate the magnitude and state the direction of the resultant gravitational field strength at the position of the planet. [3]

This question is about a probe in orbit.

A probe of mass *m* is in a circular orbit of radius *r* around a spherical planet of mass *M*.



(diagram not to scale)

a. State why the work done by the gravitational force during one full revolution of the probe is zero. [1]

b. Deduce for the probe in orbit that its [4]

(i) speed is $v = \sqrt{\frac{GM}{r}}$.

(ii) total energy is $E = -\frac{GMm}{2r}$.

c. It is now required to place the probe in another circular orbit further away from the planet. To do this, the probe's engines will be fired for a very short time. [2]

State and explain whether the work done on the probe by the engines is positive, negative **or** zero.

This question is in **two** parts. **Part 1** is about solar radiation and the greenhouse effect. **Part 2** is about orbital motion.

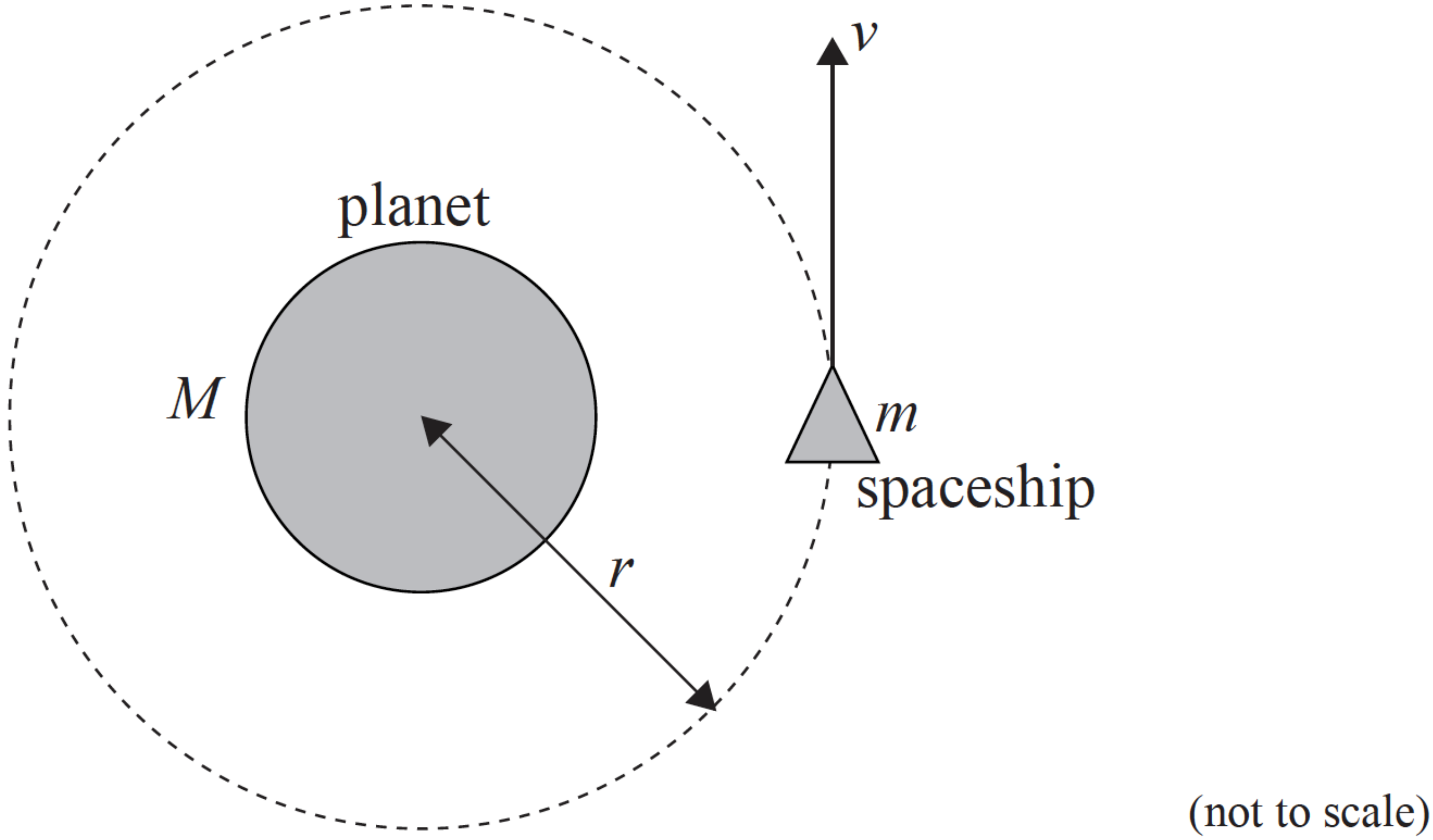
Part 1 Solar radiation and the greenhouse effect

The following data are available.

Quantity	Symbol	Value
Radius of Sun	R	7.0×10^{8} m
Surface temperature of Sun	T	5.8×10^{3} K
Distance from Sun to Earth	d	1.5×10^{11} m
Stefan-Boltzmann constant	σ	5.7×10^{-8} W m^{-2} K^{-4}

Part 2 Orbital motion

A spaceship of mass *m* is moving at speed *v* in a circular orbit of radius *r* around a planet of mass *M*.



a. State the Stefan-Boltzmann law for a black body. [2]

b. Deduce that the solar power incident per unit area at distance *d* from the Sun is given by [2]

$\frac{\sigma R^2 T^4}{d^2}$.

c. Calculate, using the data given, the solar power incident per unit area at distance *d* from the Sun. [2]

d. State **two** reasons why the solar power incident per unit area at a point on the surface of the Earth is likely to be different from your answer in (c). [2]

e. The average power absorbed per unit area at the Earth's surface is $240 Wm^{-2}$. By treating the Earth's surface as a black body, show that the average surface temperature of the Earth is approximately 250K. [2]

f. Explain why the actual surface temperature of the Earth is greater than the value in (e). [3]

h. (i) Identify the force that causes the centripetal acceleration of the spaceship. [4]

(ii) Explain why astronauts inside the spaceship would feel "weightless", even though there is a force acting on them.

i. Deduce that the speed of the spaceship is $v = \sqrt{\frac{GM}{r}}$. [2]

j. The table gives equations for the forms of energy of the orbiting spaceship. [4]

Form of Energy	Equation
Kinetic	$E_K = \frac{GMm}{2r}$
Gravitational potential	$E_P = -\frac{GMm}{r}$
Total (kinetic + potential)	$E = -\frac{GMm}{2r}$

The spaceship passes through a cloud of gas, so that a small frictional force acts on the spaceship.

(i) State and explain the effect that this force has on the total energy of the spaceship.

(ii) Outline the effect that this force has on the speed of the spaceship.