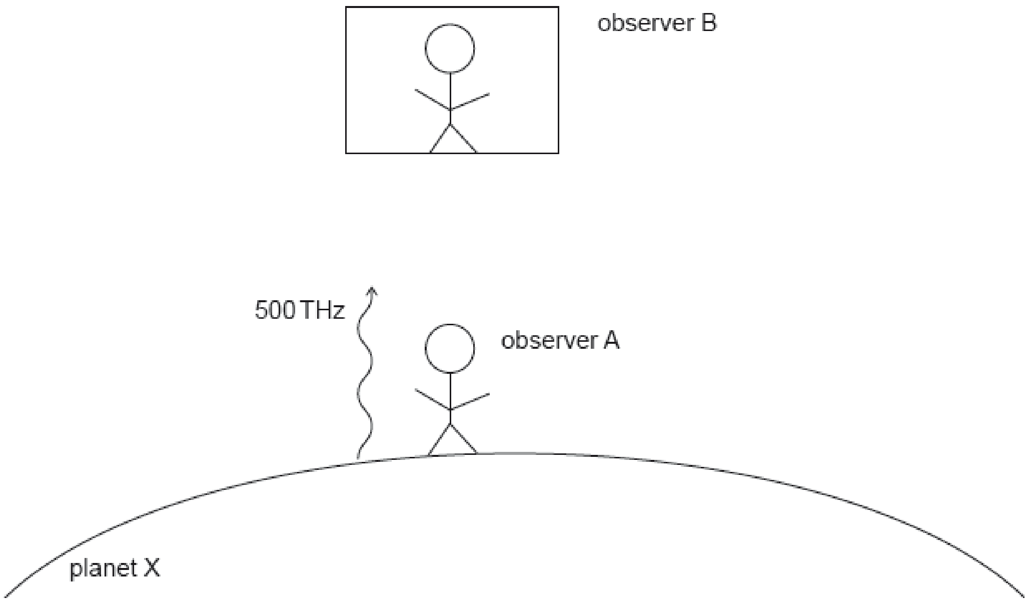
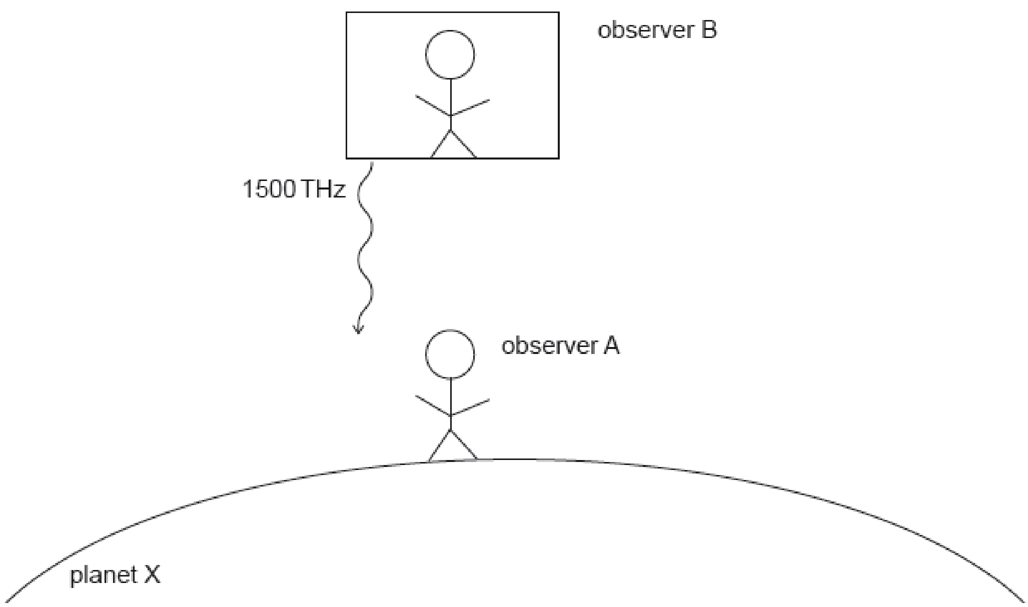
HL Paper 3

An observer A is on the surface of planet X. Observer B is in a stationary spaceship above the surface of planet X.

Observer A sends a beam of light with a frequency 500 THz towards observer B. When observer B receives the light he observes that the frequency has changed by Δf.



Observer B then sends a signal with frequency 1500 THz towards observer A.



a. Calculate the shift in frequency observed by A in terms of Δf. [2]

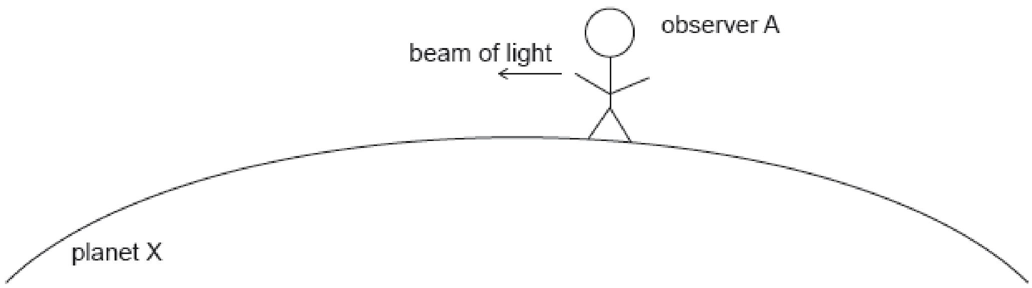
b. Calculate the gravitational field strength on the surface of planet X. [2]

The following data is given:

Δf = 170 Hz.

The distance between observer A and B is 10 km.

c. Observer A now sends a beam of light initially parallel to the surface of the planet. [2]



Explain why the path of the light is curved.

Markscheme

a. $\Delta f \propto f$

therefore the change is «–»$3\Delta f$

[2 marks]

b. g = «$c^2 \frac{\Delta f}{f \Delta h}$ =» $(3 \times 10^8)^2 \frac{170}{5.0 \times 10^{14} \times 10\ 000}$

$g = 3.1$ «ms^{-2}»

*If POT mistake, award **[0]**.*

*Award **[2]** for BCA.*

[2 marks]

c. the mass of the planet warps spacetime around itself

the light will follow the shortest path in spacetime «which is curved»

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The Λ^0 (Lambda) particle decays spontaneously into a proton and a negatively charged pion of rest mass 140 MeV c^{-2}. After the decay, the particles are moving in the same direction with a proton momentum of 630 MeV c^{-1} and a pion momentum of 270 MeV c^{-1}.

a. Determine the rest mass of the Λ^0 particle. [4]

b. Determine, using your answer to (a), the initial speed of the Λ^0 particle. [2]

Markscheme

a. Λ momentum = 900

E_{proton} = «$\sqrt{pc^2 + (mc^2)^2} = \sqrt{630^2 + 938^2} =$» 1130 «MeV»

E_{pion} = «$\sqrt{270^2 + 140^2} =$» 304 «MeV»

so rest mass of Λ = «$\sqrt{(1130 + 304)^2 - 900^2} =$» 1116 «MeV c^{-2}»

b. «$E = \gamma mc^2$ so» γ = « $\frac{1434}{1116}$ =» 1.28

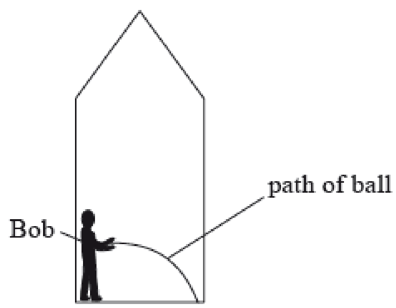
to give 0.64c

Examiners report

a. [N/A]
b. [N/A]

This question is about general relativity.

Bob is standing on the floor of a spaceship and he throws a ball in a direction parallel to the floor.



Between leaving Bob's hand and landing on the floor, the ball follows the path shown.

a. State and explain whether, from the path followed by the ball, Bob can deduce that the spaceship is at rest on the surface of a planet. [3]

b. Outline how the concept of spacetime can be used to explain the [4]

(i) trajectory of the ball if the spaceship is at rest on the surface of a planet.

(ii) nature of a black hole.

c. Calculate the radius that Earth would have to have in order for it to behave as a black hole. The mass of Earth is 6.0×10^{24} kg. [1]

Markscheme

a. no

it could be accelerating (upwards);

because of the principle of equivalence;

that states there is no way that gravitational effects can be distinguished from inertial effects;

b. (i) the planet warps spacetime;

and the ball follows the shortest path in spacetime;

(ii) the black hole causes <u>extreme</u> curvature of spacetime;

any light leaving the surface will be bent back to the surface of the black hole / *OWTTE*;

c. 8.8 mm ***or*** 8.8×10^{-3} m;

Examiners report

a. Generally speaking**,** candidates had much more success with this question.

b. Generally speaking**,** candidates had much more success with this question although the ideas of the shortest path in space time in (b)(i) and of extreme curvature in (b)(ii) were often missed.

c. The substitution was usually done correctly.

The global positioning system (GPS) uses satellites that orbit the Earth. The satellites transmit information to Earth using accurately known time signals derived from atomic clocks on the satellites. The time signals need to be corrected due to the gravitational redshift that occurs because the satellites are at a height of 20 Mm above the surface of the Earth.

a. The gravitational field strength at 20 Mm above the surface of the Earth is about 0.6 N kg^{-1}. Estimate the time correction per day needed to the time signals, due to the gravitational redshift. [3]

b. Suggest, whether your answer to (a) underestimates **or** overestimates the correction required to the time signal. [1]

Markscheme

a. $\frac{\Delta f}{f} = \frac{gh}{c^2}$ so $\Delta f = \frac{0.6 \times 20000000}{\left(3 \times 10^8\right)^2} = 1.3 \times 10^{-10}$

$\frac{\Delta f}{f} = \frac{\Delta t}{t}$

$1.3 \times 10^{-10} \times 24 \times 3600 = 1.15 \times 10^{-5}$ «s» «running fast»

*Award **[3 max]** if for g 0.6 **OR** 9.8 **OR** average of 0.6 and 9.8 is used.*

b. ***ALTERNATIVE 1***

g is not constant through Δ*h* so value determined should be larger

Use ECF from (a)
Accept under or overestimate for SR argument.

ALTERNATIVE 2

the satellite clock is affected by time dilation due to special relativity/its motion

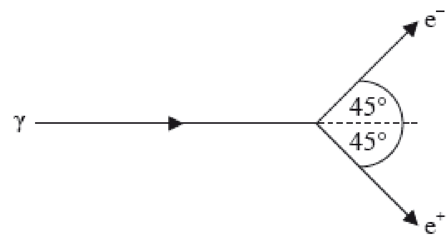
Examiners report

a. [N/A]

b. [N/A]

This question is about pair production and relativistic mechanics.

A γ-photon of energy 2.46 MeV is travelling close to the nucleus of a gold atom. It converts into an electron (e^-) – positron (e^+) pair. Each particle travels at 45° to the original direction of the photon.



Immediately after the conversion, the kinetic energies of the electron and positron are equal. The magnitude of the recoil momentum of the gold nucleus is $0.880\ \mathrm{MeV\,c^{-1}}$ and is in the direction of the photon.

a. Calculate, immediately after the decay, the magnitude of the momentum of the electron. [4]

b. Calculate the value V of the potential difference through which an electron at rest must be accelerated in order to have the same magnitude of momentum as that in (a). [2]

Markscheme

a. momentum of photon $= 2.46\ \mathrm{MeV\,c^{-1}}$;

momentum of electron–positron pair $= (2.46 - 0.880 =)\ 1.58\ \mathrm{MeV\,c^{-1}}$;

$2p_{\text{electron}} \cos 45 = 1.58$;

$p_{\text{electron}} = \left(\frac{1.58}{2\cos 45} =\right)\ 1.12\ \mathrm{MeV\,c^{-1}}$;

b. total energy of electron $= \left(\sqrt{1.12^2 + 0.511^2} =\right)\ 1.23\ \mathrm{MeV}$;

$\mathrm{KE} = \mathrm{eV}\ (= 1.23 - 0.511 = 0.72\ \mathrm{MeV}) \rightarrow V = 0.72\ \mathrm{MV}$;

*Award **[2]** for bald correct answer.*

Examiners report

a. It was rare to see correct solutions to the calculations in this question. Candidates, as in previous years did not seem familiar with handling the units $\mathrm{MeV\,c^{-1}}$ and MeV and became confused.

b. It was rare to see correct solutions to the calculations in this question. Candidates, as in previous years did not seem familiar with handling the units $\mathrm{MeV\,c^{-1}}$ and MeV and became confused.

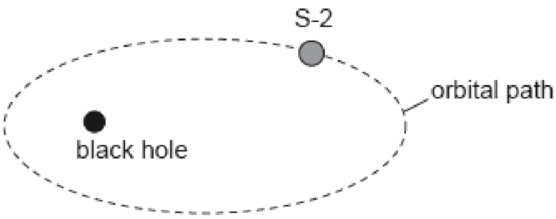
It is believed that a non-rotating supermassive black hole is likely to exist near the centre of our galaxy. This black hole has a mass equivalent to 3.6 million times that of the Sun.

a.i. Outline what is meant by the event horizon of a black hole. [1]

a.ii. Calculate the distance of the event horizon of the black hole from its centre. [2]

Mass of Sun = 2×10^{30} kg

b. Star S-2 is in an elliptical orbit around a black hole. The distance of S-2 from the centre of the black hole varies between a few light-hours and several light-days. A periodic event on S-2 occurs every 5.0 s. [2]



Discuss how the time for the periodic event as measured by an observer on the Earth changes with the orbital position of S-2.

Markscheme

a.i. boundary inside which events cannot be communicated to an outside observer

OR

distance/surface at which escape velocity = *c*

OWTTE

[1 mark]

a.ii. mass of black hole = 7.2×10^{36} «kg»

«$\frac{2GM}{c^2}$ =» 1×10^{10} «m»

[2 marks]

b. wherever S-2 is in orbit, time observed is longer than 5.0 s

when closest to the star S-2 periodic time dilated more than when at greatest distance

Justification using formula or time is more dilated in stronger gravitational fields

[2 marks]

Examiners report

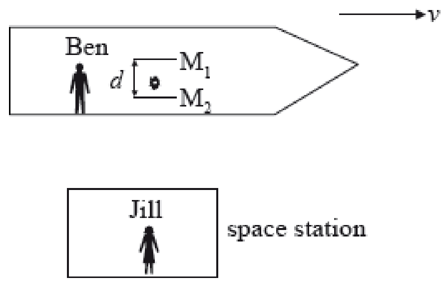
a.i. [N/A]

a.ii. [N/A]

b. [N/A]

This question is about a Galilean transformation and time dilation.

Ben is in a spaceship that is travelling in a straight-line with constant speed *v* as measured by Jill who is in a space station.



Ben switches on a light pulse that bounces vertically (as observed by Ben) between two horizontal mirrors M_1 and M_2 separated by a distance d. At the instant that the mirrors are opposite Jill, the pulse is just leaving the mirror M_2. The speed of light in air is c.

The questions (e) and (f) introduce the concepts of time dilation and length contraction. Discuss how muon decay in the atmosphere provides experimental evidence for these concepts.

Markscheme

Look for these main points however expressed.

muons produced in the upper atmosphere have speeds close to c;

many such muons are observed reaching the surface of Earth;

the half-life as measured in a muon’s frame of reference does not give time for many to reach Earth / means many will have decayed in travelling to Earth;

but the contracted length of the journey/length as measured in a muon’s reference frame means there is sufficient time;

and the dilated time/time measured by Earth observer is also sufficient for muons to reach the surface;

Examiners report

Parts (f) and (g) were HL only with the length contraction usually done correctly in (f). There were, however, many confused ideas in the discussion of muon decay and it‟s bearing on time dilation and length contraction. Rarely were there any attempts to identify the two reference frames involved, that of the muons and that of the Earth. Many candidates as in previous years still have the idea that there is an absolute reference frame and so talk about time going more slowly for moving objects.

The Schwarzschild radius of a black hole is 6.0 x 10^5 m. A rocket is 7.0 x 10^8 m from the black hole and has a clock. The proper time interval between the ticks of the clock on the rocket is 1.0 s. These ticks are transmitted to a distant observer in a region free of gravitational fields.

a. Outline why the clock near the black hole runs slowly compared to a clock close to the distant observer. [2]

b. Calculate the number of ticks detected in 10 ks by the distant observer. [2]

Markscheme

a. this is gravitational time dilation

OR

black hole gives rise to a «strong» gravitational field

clocks in stronger field run more slowly

OR

the clock «signal» is subject to gravitational red-shift

the clock is subject to gravitational red shift

OR

the clock has lost gravitational potential energy in moving close to the black hole

[Max 2 Marks]

b. ***ALTERNATIVE 1 (10 ks is in observer frame):***

$\Delta t' = 10000\sqrt{1-\frac{6.0\times10^{5}}{7.0\times10^{8}}}$

9995.7 so 9995 «ticks»

Allow 9996

*Allow ECF if 10 is used instead of 1000*0

ALTERNATIVE 2 (10 ks is in rocket frame):

$\Delta t = \frac{10000}{\sqrt{1-\frac{6.0\times10^{5}}{7.0\times10^{8}}}}$

10004 «ticks»

Allow ECF if 10 is used instead of 10000

Examiners report

a. [N/A]
b. [N/A]

An electron and a positron have identical speeds but are travelling in opposite directions. Their collision results in the annihilation of both particles and the production of two photons of identical energy. The initial kinetic energy of the electron is 2.00 MeV.

a. Explain, in terms of a conservation law, why two photons need to be created. [1]

b. Determine the speed of the incoming electron. [3]

c. Calculate the energy and the momentum for each photon after the collision. [2]

Markscheme

a. as the total initial momentum is zero, it must be zero after the collision

b. $2 = (\gamma-1)m_0c^2 = (\gamma-1)\ 0.511$

$\gamma = 4.91$

$v = 0.978c$

c. «2 + 2 + 2 × 0.511 = 5.02 MeV» so each photon is 2.51«MeV»

$p = \frac{E}{c} = 2.51$ «MeVc^{-1}»

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A positive pion decays into a positive muon and a neutrino.

$$\pi^+ \rightarrow \mu^+ + v_\mu$$

The momentum of the muon is measured to be 29.8 MeV c^{-1} in a laboratory reference frame in which the pion is at rest. The rest mass of the muon is 105.7 MeV c^{-2} and the mass of the neutrino can be assumed to be zero.

For the laboratory reference frame

a.i. write down the momentum of the neutrino. [1]

a.ii. show that the energy of the pion is about 140 MeV. [2]

b. State the rest mass of the pion with an appropriate unit. [1]

Markscheme

a.i. «–»29.8 «MeVc^{-1}»

[1 mark]

a.ii. $E_\pi = \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} + p_v c$ ***OR*** E_μ = 109.8 «MeV»

E_π = «$\sqrt{29.8^2 + 105.7^2}$ + 29.8 =» 139.6 «MeV»

Final value to at least 3 sig figs required for mark.

[2 marks]

b. 139.6 MeVc^{-2}

Units required.

Accept 140 MeVc^{-2}.

[1 mark]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]

In an experiment a source of iron-57 emits gamma rays of energy 14.4 ke V. A detector placed 22.6 m vertically above the source measures the frequency of the gamma rays.

a. Calculate the expected shift in frequency between the emitted and the detected gamma rays. [2]

b. Explain whether the detected frequency would be greater or less than the emitted frequency. [2]

Markscheme

a. f = «$\frac{E}{h}=$» $\frac{14\,400\times1.6\times10^{-19}}{6.63\times10^{-34}}$ = «3.475×10^{18} Hz»

Δf = «$\frac{g\times\Delta h\times f}{c^2}\approx$» 8550 «Hz»

[2 marks]

b. «as the photon moves away from the Earth, » it has to spend energy to overcome the gravitational field

since $E = hf$, the detected frequency would be lower «than the emitted frequency»

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

This question is about relativistic dynamics.

A proton is accelerated from rest through a potential difference of 2.5 GV. Determine the momentum of the proton after acceleration.

Markscheme

total energy of proton = eV + rest mass;

([2500+938]MeV=)3438;

$p^2c^2 = \left(E_{\mathrm{tot}}{}^2 = m_0{}^2c^4 =\right) 3438^2 - 938^2$;

$p = 3.3\left(\mathrm{GeVc^{-1}}\right)$ ***or*** 1.76×10^{-18}($\mathrm{kgms^{-1}}$);

NOTE: *The question paper stated the units of potential difference in GeV. Watch for answers stating that the unit of potential difference is V, not eV. For such answers without calculation, award **[1]**.*
*Award **[1]** for correct use of potential difference (2.5GeV) divided by e.*

ie $\frac{2.5\times10^{9}(eV)}{1.6\times10^{19}(\mathrm{C})}=1.56\times10^{28}\,(V)$.

Examiners report

The units of potential difference were incorrectly stated as GeV in this question. The markscheme was adjusted to ensure no candidate was disadvantaged and all examiners were asked to identify any candidates who appeared to have been thrown by the error. On the whole, candidates had interpreted the question with the correct unit of GV.

Correct answers were given by those who worked in logical manner and who clearly stated that total energy of the proton is the sum of kinetic energy and rest mass energy. The derivation of momentum from the formula was not easy for candidates. Candidates with basic arithmetic and algebra failed here. The more able candidates found the momentum in a clear, straightforward way. Looking through the formulas in data-booklet without understanding is not appropriate here. Many candidates confused kinetic energy with total energy.

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a. Outline what is meant by a black hole. [2]

b. An observer views a distant spacecraft that is 23.0 km from the centre of a black hole. The spacecraft contains a clock that ticks once every second and the ticks can be detected by the distant observer. In 2.00 minutes the observer counts 112 ticks of the clock. [3]

Determine the mass of the black hole.

Markscheme

a. region of space with extreme/very large curvature of spacetime

such that light cannot escape the region ***OR*** escape speed within region is > *c*

Do not allow "large" or omission of degree of curvature.

b. time for 1 second spacecraft tick in observer frame =1.07s

$1.07=\frac{1.00}{\sqrt{1-\frac{R_S}{2.3\times10^4}}}$ ***OR*** R_S=2.96×10^3m

$M=\ll\frac{c^2\times2.96\times10^3}{2\times6.67\times10^{-11}}=\gg2.0\times10^{30}$kg

Examiners report

a. [N/A]
b. [N/A]

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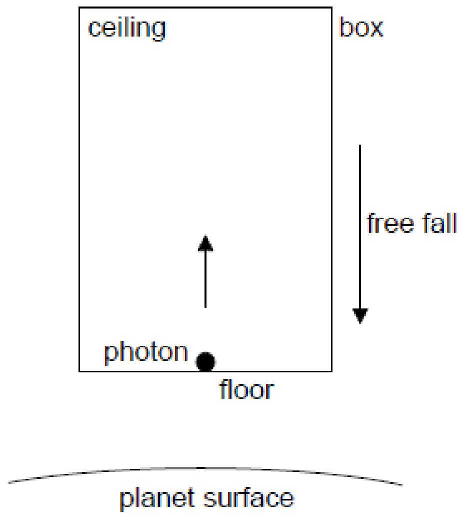
a.i. State what is meant by the event horizon of a black hole. [1]

a.ii. Show that the surface area *A* of the sphere corresponding to the event horizon is given by [1]

$$A=\frac{16\pi G^2M^2}{c^4}.$$

a.iii Suggest why the surface area of the event horizon can never decrease. [1]

b. The diagram shows a box that is falling freely in the gravitational field of a planet. [3]



A photon of frequency *f* is emitted from the floor of the box and is received at the ceiling. State and explain the frequency of the photon measured at the ceiling.

Markscheme

a.i. the surface at which the escape speed is the speed for light

OR

the surface from which nothing/not even light can escape to the outside

OR

the surface of a sphere whose radius is the Schwarzschild radius

Accept distance as alternative to surface.

[1 mark]

a.ii. use of $A = 4\pi R^2$ and $R = \frac{2GM}{c^2}$

«to get $A = \frac{16\pi G^2 M^2}{c^4}$»

[1 mark]

a.iii since mass and energy can never leave a black hole and $A = \frac{16\pi G^2 M^2}{c^4}$

OR

some statement that area is increasing with mass

«the area cannot decrease»

[1 mark]

b. ***ALTERNATIVE 1 —*** *(student/planet frame):*

photon energy/frequency decreases with height
OR
there is a gravitational redshift

detector in ceiling is approaching photons so Doppler blue shift

two effects cancel/frequency unchanged

ALTERNATIVE 2 *– (box frame):*

by equivalence principle box is an inertial frame

so no force on photons

so no redshift/frequency unchanged

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
a.iii. [N/A]
b. [N/A]

This question is about general relativity.

a. Calculate the Schwarzschild radius for an astronomical object of mass 5.0 $\times 10^{30}$ kg. [2]

b. A spaceship is travelling towards the object in (a). The spaceship moves in a straight line such that its distance of closest approach would be about 10^7m. Discuss why the presence of the object in (a) will **not** significantly affect the motion of the spaceship. [2]

c. An observer, when viewing a distant galaxy, sees two images of the galaxy separated by a small angle. A massive star is positioned between the observer and the galaxy. Outline how these observations give support to the theory of general relativity. [3]

Markscheme

a. $\left(R = \frac{2GM}{c^2}\right) \frac{2\times 6.7\times 10^{-11}\times 5.0\times 10^{30}}{9\times 10^{16}}$;

$=7.4\times 10^3$ (m);

b. closest approach much larger than R;

so Newtonian mechanics will apply;

spacetime will not be significantly warped;

c. theory predicts that warping of spacetime will affect light;

the massive star is warping spacetime;

causing the light to bend around it;

*Award **[2 max]** to an answer that addresses the last two marking points.*

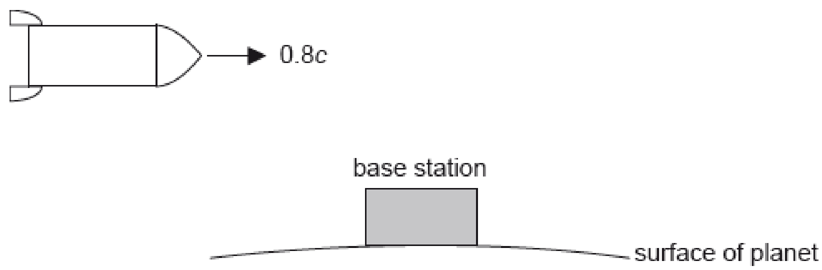
Accept answer to the last two marking points in a form of labelled diagram.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

This question is about relativistic kinematics.

A spacecraft is flying in a straight line above a base station at a speed of 0.8*c*.



Suzanne is inside the spacecraft and Juan is on the base station.

a. State what is meant by an inertial frame of reference. [2]

b.i. A light on the base station flashes regularly. According to Suzanne, the light flashes every 3 seconds. Calculate how often the light flashes according to Juan. [2]

b.ii. While moving away from the base station, Suzanne observes another spacecraft travelling towards her at a speed of 0.8*c*. Suzanne measures the other spacecraft to have a length of 8.00 m. Calculate the proper length of the other spacecraft. [3]

c.i. Suzanne's spacecraft is on a journey to a star. According to Juan, the distance from the base station to the star is 11.4 ly. Show that Suzanne measures the time taken for her to travel from the base station to the star to be about 9 years. [2]

c.ii. Suzanne then returns to the base station at the same speed. The total time since leaving the base station as measured by Suzanne is around 18 years but the total time according to Juan is around 29 years. Explain how it is possible for Suzanne and Juan to have aged by different amounts. [2]

Markscheme

a. a coordinate system;

that is not accelerating / where Newton's first law applies;

b.i. $\gamma = \left[\frac{1}{\sqrt{1-0.8^2}} =\right] 1.67;$

$\Delta t_0 = \left[\frac{3}{1.67} =\right] 1.8 \text{ (s)};$

b.ii. velocity is relative to Suzanne / Suzanne does not measure proper length;

$L_0 = \left(y_L = \frac{5}{3} \times 8 =\right)\ 13;$

m;

Award first marking point even if it is implied.

Award the first marking point if the second mark is awarded.

or

accept 0.8 c with respect to the ground:

$u'_{\mathrm{x}} = \frac{0.8c-[-0.8]c}{1+0.8^2}\ (= 0.976c);$

$\gamma = \frac{1}{\sqrt{1-0.976^2}}\ (= 4.56);$

$l_0 = (4.56 \times 8.0 =)\ 36\ (\mathrm{m});$

***Note**: the final answer for HP3 is different to the SP3.*

c.i. $t = \frac{s}{v} = \frac{11.4}{0.8} = 14.25\ (\mathrm{years});$

$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{14.25}{1.67} = 8.6\ (\mathrm{years});$

Allow ECF from (b).

Accept length contraction with the same result.

c.ii. situation is not symmetrical;

Suzanne must undergo acceleration (when changing direction) but Juan does not;

Examiners report

a. This question required quite high ability to apply relativistic kinematics in standard situations and also explain the twin paradox. Well done by average and better candidates. There was a slight change made to the wording of the question 12(b)(ii) in the published paper and published markscheme in comparison with the wording used in the exam.

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This question is about gravitation.

Planets move in orbits around the Sun. Explain this observation according to

a. Newton's universal law of gravitation. [2]

b. Einstein's theory of general relativity. [2]

Markscheme

a. there is a force of attraction between (the mass of) the Sun and (the mass of) the planet;

which acts as a central/centripetal force for the planet;

b. (mass of the) Sun curves/warps spacetime around it;

planets move in paths of shortest length/geodesic (in this curved/warped spacetime);

Examiners report

a. Rather surprisingly candidates could explain the warping of spacetime and the shortest path followed by a planet but could not explain gravitational force providing centripetal motion.

b. Rather surprisingly candidates could explain the warping of spacetime and the shortest path followed by a planet but could not explain gravitational force providing centripetal motion.

This question is about relativistic kinematics.

a. An observer at rest relative to Earth observes two spaceships. Each spaceship is moving with a speed of 0.85*c* but in opposite directions. The observer measures the rate of increase of distance between the spaceships to be 1.7*c*. [5]

(i) Outline whether this observation contravenes the theory of special relativity.

(ii) Determine, according to an observer in one of the spaceships, the speed of the other spaceship.

b. The observer on Earth in (a) watches one spaceship as it travels to a distant star at a speed of 0.85*c*. According to observers on the spaceship, this journey takes 8.0 years. [9]

(i) Calculate, according to the observer on Earth, the time taken for the journey to the star.

(ii) Outline whether the time interval measured by the observer on Earth is a proper time interval.

(iii) Calculate, according to the observer on Earth, the distance from Earth to the star.

(iv) The observers in the spaceship send a message to Earth halfway through their journey. Determine how long it takes the message to arrive at Earth according to the observers on the spaceship.

Markscheme

a. (i) theory suggests that no object can travel faster than light;

the $1.7c$ is not the speed of a physical object;

so is not in violation of the theory;

(ii) recognition that v is negative relative to u_x;
use of $\frac{0.85c+0.85c}{1+\frac{(0.85c)^2}{c^2}}$;
(0.9869c≈) $0.99c$;
Accept first marking point implied in the second marking point.

b. (i) γ=1.89;

interval on Earth = γ×interval on spaceship;

(interval on Earth 1.90×8 years=)15 years;

*Award **[3]** for a bald correct answer.*

(ii) time interval measured by observer on Earth is not proper because the time interval between the two events is not measured at same place/not the shortest time;

(iii) observer on Earth thinks spaceship has travelled for 15 years;
so distance is $0.85c$×15=12.8≈13 ly;
*Award **[2]** for a bald correct answer.*

or

the spaceship observer observes the distance moved by the Earth=$0.85c$×8.0yr;
proper distance=1.90×$0.85c$×8.0 yr=12.9≈13ly;
*Award **[2]** for a bald correct answer.*

(iv) Earth is at a distance of 4×$0.85c$=3.4ly when signal is emitted;
signal reaches Earth in time T where cT+3.4=$0.85c$T;
T=22.7≈23 years;

Examiners report

a. [N/A]
b. [N/A]

This question is about relativistic momentum and energy.

An electron and a positron travel towards each other in a straight line in a vacuum. A positron is a positively charged electron.

The speed of each particle, as measured by an observer in the laboratory, is 0.85c. The value of the Lorentz factor at this speed is approximately 1.9.

The electron and positron annihilate each other, creating two photons in the process. Each of the photons transfers the same quantity of energy.

a. Calculate the speed of the positron as measured in the frame of reference of the electron. [2]

b.i. Calculate the total energy in the reaction. [1]

b.ii. Outline why two photons must be released in this collision. [2]

b.iii. Determine the frequency of one of the photons. [2]

Markscheme

a. $u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0.85c - [-0.85c]}{1 + [0.85]^2}$;

0.99c;

b.i. $E = \left(2[\gamma m_0 c^2] = 2 \times 1.9 \times 0.511 =\right)\ 1.94\ \mathrm{MeV}$ ***or*** $3.1 \times 10^{-13}\ \mathrm{J}$;

b.ii. total momentum before the collision is zero;

if only one photon is emitted then the total momentum after the collision cannot be zero, otherwise momentum will not be conserved;

b.iii. $f = \frac{E}{h} = \frac{0.5 \times 1.9418 \times 10^6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$; *(allow ECF from (b)(i))*

$f = 2.3 \times 10^{20}\ \mathrm{Hz}$;

Examiners report

a. Well prepared candidates showed a good ability to apply relativistic velocity addition. (b) discriminated well.

b.i. Well prepared candidates showed a good ability to apply relativistic velocity addition. (b) discriminated well.

b.ii. Well prepared candidates showed a good ability to apply relativistic velocity addition. (b) discriminated well.

b.iii. Well prepared candidates showed a good ability to apply relativistic velocity addition. (b) discriminated well.

This question is about muon decay.

Muons are produced in the Earth’s atmosphere at a height of around 10 km above the surface. They then travel at a speed of around 0.98c towards the Earth. The average time for a muon to decay is approximately $2.2\ \mu\mathrm{s}$, according to observers at rest relative to the muon.

Many muons are observed to reach the surface of the Earth.

a. Deduce that few muons would be expected to arrive at the surface of the Earth if non-relativistic mechanics are assumed. [2]

b.i. Calculate the average decay time of a muon as observed by an observer on the surface of the Earth. [2]

b.ii. Explain, with a calculation, why many muons reach the surface of the Earth before they have decayed. [2]

Markscheme

a. $t = \left(\frac{d}{v} = \frac{10^4}{0.98 \times 3 \times 10^8} =\right) 3.4 \times 10^{-5}\ \mathrm{s}$;

which is $\frac{3.4 \times 10^{-5}}{2.2 \times 10^{-6}} \approx 15$ decay times;

(so very few muons will reach Earth)

or

$d = vt = 0.98 \times 3 \times 10^8 \times 2.2 \times 10^{-6} = 650\ \mathrm{m}$ in one decay time;

in travelling to Earth, there are $\frac{10000}{650} \approx 15$ decay times;

(so very few muons will reach Earth)

b.i. $\gamma = \sqrt{\frac{1}{1-0.98^2}} = 5.0$;

$t = \gamma t_0 = 5.0 \times 2.2 \times 10^{-6} = 1.1 \times 10^{-5}\ \mathrm{s}$;

b.ii. distance travelled in one half-life is

$d = vt = 0.98 \times 3 \times 10^8 \times 1.1 \times 10^{-5} = 3200\ \mathrm{m}$;

for observers on Earth, the muons are able to travel much further compared with those in (a);

(so many more will be able to reach the Earth before they have decayed)

Examiners report

a. Many well prepared candidates presented good, well structured and clear answers.

b.i. Many well prepared candidates presented good, well structured and clear answers.

b.ii. Many well prepared candidates presented good, well structured and clear answers. Some candidates struggled with (b)(ii).

A proton is accelerated from rest through a potential difference *V* to a speed of 0.86c.

a. Calculate the potential difference *V*. [3]

b. The proton collides with an antiproton moving with the same speed in the opposite direction. As a result both particles are annihilated and two photons of equal energy are produced. [3]

Determine the momentum of one of the photons.

Markscheme

a. $\gamma = 1.96$

$E_k = (\gamma - 1)\ m_0c^2 = 900$ «Me V»

pd ≈ 900 «MV»

*Award **[2 max]** if Energy and Potential difference are not clearly distinguished, eg by the unit.*

[3 marks]

b. energy of proton = γmc^2 = 1838 «Me V»

total energy available = energy of proton + energy of antiproton = 1838 + 1838 = 3676 «Me V»

momentum of a one photon = Total energy / 2c = 1838 «Me Vc^{-1}»

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

An electron is emitted from a nucleus with a total energy of 2.30MeV as observed in a laboratory.

a. Show that the speed of the electron is about 0.98c. [3]

b. The electron is detected at a distance of 0.800 m from the emitting nucleus as measured in the laboratory. [7]

(i) For the reference frame of the electron, calculate the distance travelled by the detector.

(ii) For the reference frame of the laboratory, calculate the time taken for the electron to reach the detector after its emission from the nucleus.

(iii) For the reference frame of the electron, calculate the time between its emission at the nucleus and its detection.

(iv) Outline why the answer to (b)(iii) represents a proper time interval.

Markscheme

a. ***ALTERNATIVE 1***

«rest mass = 0.511 MeV c^{-2}» $\gamma = \frac{2.30}{0.511} = 4.50$

$v = c\sqrt{\frac{\gamma^2-1}{\gamma^2}}$ ***OR*** $3 \times 10^8 \times \left(\frac{4.50^2-1}{4.50^2}\right)^{\frac{1}{2}}$

0.9750c

ALTERNATIVE 2

$\gamma = \ll \frac{1}{\sqrt{1-0.98^2}} = \gg 5.0$

$E = \ll \gamma m_0 c^2 = \gg 4.1 \times 10^{-13}\text{J}$

E=2.6MeV

b. (i) distance $\frac{0.800}{\gamma}$

0.178m

Accept 0.159 for γ = 5.0.

(ii) time=$\frac{0.800}{2.94\times10^8}$

2.74 ns

(iii) $\frac{2.74}{4.5}$ ***OR*** $\frac{0.178}{2.94\times10^8}$

0.608 ns

(iv) it is measured in the frame of reference in which both events occur at the same position
OR
it is the shortest time interval possible

Examiners report

a. [N/A]
b. [N/A]

This question is about spacetime.

a. Describe what is meant by spacetime. [2]

b. State the shape of the path in spacetime of a body [2]

(i) moving at constant velocity.

(ii) orbiting the Earth.

c. Explain how spacetime is used to describe the gravitational attraction between Earth and a satellite orbiting the Earth. [2]

Markscheme

a. a coordinate system;

in which time is at right angles to space coordinates / consisting of three dimensions of space and one of time;

b. (i) straight line;

(ii) curve/circle/ellipse;

c. (general relativity suggests that) Earth warps spacetime;

the satellite follows shortest path in spacetime which is an orbit about Earth;

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

This question is about relativistic mechanics.

A proton, after acceleration from rest through a potential difference *V,* has momentum 1600MeVc^{-1}.

Calculate the value of *V.*

Markscheme

$E=\sqrt{p^2c^2+(mc^2)^2}=\sqrt{1600^2+938^2}\approx 1855\,(\mathrm{MeV})$;

ΔE=1855-938=917(MeV);

potential difference, $V\approx$920(MV); *(this marking point must be explicit)*

*Award **[3]** for a bald correct answer.*

Examiners report

In any question with units expressed in terms of c there is potential for confusion. However many candidates are now able use the relativistic energy - momentum equation correctly. (a) was well answered by many.

This question is about relativistic energy and momentum.

a. A proton is accelerated from rest through a potential difference *V*. The proton reaches a speed of 0.970c. Determine the value of *V*. [3]

b. Calculate, after acceleration for the proton in (a), its [2]

(i) mass.

(ii) momentum.

Markscheme

a. $V=(\gamma-1)938\times10^6$;

$\gamma=\left(\frac{1}{\sqrt{1-(0.970)^2}}\right)=4.11$;

$V=(3.11\times938\times10^6=)\ 2.92\times10^9\mathrm{V}$ ***or*** 2.92GV;

b. (i) $3.86\times10^3\mathrm{MeVc^{-2}}$ ***or*** $3.86\mathrm{GeVc^{-2}}$;

(ii) $3.74\times10^3\mathrm{MeVc^{-1}}$ ***or*** $3.74\mathrm{GeVc^{-1}}$;

Examiners report

a. [N/A]
b. [N/A]

This question is about relativistic energy and momentum.

a. A proton is accelerated from rest through a potential difference *V*. After acceleration the mass of the proton is equal to four times its rest mass. [3]

Determine the value of *V*.

b. For the proton in (a) calculate, after acceleration, its [3]

(i) speed.

(ii) momentum.

Markscheme

a. $V = [\gamma - 1]\, 938 \times 10^6$;

γ=4;

V=(3×938×10^6=)2.81×10^9 (V) ***or*** 2.81 (GV);

or

eV=[γ −1]mc^2;
γ=4;
$V = \left(\frac{3\times1.67\times10^{-27}\times9\times10^{16}}{1.6\times10^{-19}}\right) = 2.81 \times 10^9\ (\mathrm{V})$;
*Award **[3]** for a bald correct answer.*

b. (i) recognize that $4 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$;

to give *v*=0.968c ***or*** 2.90×10^8(ms^{-1});

*Allow **[2 max]** ECF for wrong γ taken from (a).*

*Award **[2]** for a bald correct answer.*

(ii) $p = (\gamma m_0 v =)\, 1.9 \times 10^{-18}\ (\mathrm{kgms}^{-1})$ ***or*** 3.63×10^3($MeVc^{-1}$) or 3.63($GeVc^{-1}$);
Watch for ECF from (a) or (b)(i).

Examiners report

a. [N/A]
b. [N/A]

This question is about the equivalence principle and black holes.

a. State the principle of equivalence. [1]

b. The gravitational field strength near the surface of a neutron star is 1.2 ×$10^{13}$$Nkg^{-1}$. A light ray is emitted from a stationary probe at a height of 250 m above the surface. The frequency of the light measured in the probe is 4.8×10^{14} Hz. [4]

(i) Determine the frequency of the light received at the surface of the star according to an observer at the surface.

(ii) Describe how gravitational red-shift leads to the concept of gravitational time dilation.

c. General relativity predicts the existence of black holes. [3]

(i) State what is meant by a black hole.

(ii) Suggest **two** ways in which a black hole may be detected.

Markscheme

a. a frame of reference that is freely falling in a gravitational field is equivalent to an inertial frame of reference far from all masses;

or

inertial and gravitational effects are indistinguishable;

or

a frame of reference accelerating in empty space is equivalent to a frame of reference at rest in a gravitational field;

b. (i) $\left(\frac{\Delta f}{f_0} = \frac{g\Delta h}{c^2}\text{andso}\right) \Delta f = \left(\frac{1.2\times10^{13}\times250\times4.8\times10^{14}}{9\times10^{16}} =\right) 0.16 \times 10^{14}\text{Hz};$

$f = \left(4.8 \times 10^{14} + 0.16 \times 10^{14} =\right) 5.0 \times 10^{14}\text{Hz};$

ECF applies only if the wrong value of Δf is added to f_0.

(ii) as light travels away from a massive body it is red-shifted/frequency decreases/time period increases;
hence closer to the body the time period is shorter / time runs slower (than higher up) – this is (equivalent to) time dilation;

c. (i) a point of infinite curvature / a point of infinite space / a singularity of spacetime / a region from which nothing can escape / escape velocity ≥c;

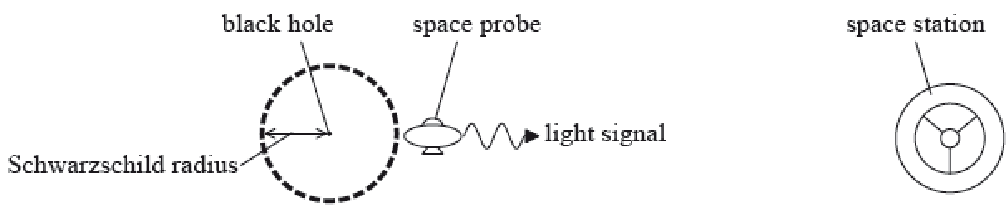
(ii) matter falling into the black hole radiates;
the (gravitational) influence on other objects;
by observing its gravitational lensing effect;
emission of Hawking radiation;

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

This question is about black holes.

A space probe is stationary in the gravitational field of a black hole.



The mass of the black hole is 4.5×10^{31} kg. The space probe is emitting a pulse of blue light at a time interval of 1.0 seconds as measured on the space probe. The light is received by an observer on a distant space station that is stationary with respect to the space probe.

a. Explain why the light reaching the space station will be red-shifted. [3]

b. The time between the pulses as measured by the observer on the distant space station is found to be 1.5 s. Calculate the distance of the space probe from the black hole. [3]

Markscheme

a. the principle of equivalence predicts photon energy decreases as it moves against a g field;

this energy is given by $E = hf$;

hence as E decreases, f must also decrease;

or

photon is subject to extreme warping of spacetime;

under these conditions a distant observer observes dilation of the photon period / *OWTTE*;

increase in time period is equal to decrease in photon frequency;

b. $R_S = \frac{2GM}{c^2} = \frac{2\times6.67\times10^{-11}\times4.5\times10^{31}}{9\times10^{16}} = 66700\text{ m};$

$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{R_S}{r}}} = 1.5 = \frac{1.0}{\sqrt{1-\frac{66700}{r}}};$

$r = 1.2\times10^5\text{ m};$

Examiners report

a. The stronger candidates clearly explained gravitational red-shift in a black hole gravitational field.

b. The stronger candidates clearly explained gravitational red-shift in a black hole gravitational field. In (b), some candidates applied formulae from the Data Booklet; the best candidates also explained the formula.

This question is about relativistic mechanics.

a. Calculate the potential difference through which a proton, starting from rest, must be accelerated for its mass–energy to be equal to three times its rest mass energy. [3]

b. Calculate the momentum of the proton after acceleration. [3]

Markscheme

a. $3m_pc^2$ = kinetic energy gain+m_pc^2 ;

kinetic energy gain = $2\times938c^2(\text{MeVc}^{-2})$;

1900MV;

b. $9m_p^2c^4=p^2c^2+m_p^2c^4$;

$p^2c^2=8m_p^2c^4$;

$p=2700\text{MeVc}^{-1}$;

or

$\gamma=3$;
to give $v=0.943c$;
$p=\gamma m_p v=3\times938\times0.943=2700\text{MeVc}^{-1}$;

Examiners report

a. [N/A]
b. [N/A]

This question is about relativistic mechanics.

A proton, after acceleration from rest through a potential difference *V*, has momentum 1600MeVc^{-1}.

Calculate the speed of the proton after acceleration.

Markscheme

$\gamma=\frac{1855}{938}=1.977$ ***or*** $\gamma-1=\frac{917}{938}$;

$\frac{v}{c}=\sqrt{1-\frac{1}{\gamma^2}}=0.86\text{c}$ ***or*** $v=\frac{1600}{938\times1.997}\text{c}=0.86\text{c}$;

*Award **[2]** for a bald correct answer.*
Watch for ECF from (a).

Examiners report

in (b) gamma was often obtained correctly to find V - but many more got lost. Surprisingly no useful 'Pythagorean triangles' were seen which make understanding this topic much easier.

---

a. Describe what is meant by spacetime. [2]

b. Outline how the concept of spacetime accounts for the [5]

(i) orbiting of Earth about the Sun.

(ii) nature of a black hole.

Markscheme

a. spacetime is a (4 dimensional) coordinate system/a coordinate system used to locate events;

consisting of three space coordinates and one time coordinate;

b. (i) the mass of the Sun causes the spacetime around it to be warped;

objects follow the shortest path/geodesic between points in spacetime;

for the Earth this an elliptical/circular path about the Sun;

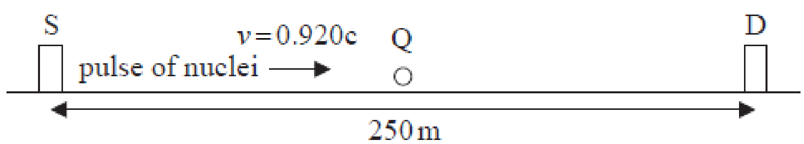
(ii) black holes produce a region of spacetime with extreme curvature; } *(must indicate the curvature is very large for this mark)*
such that not even radiation can escape from the influence of a black hole;

Examiners report

a. [N/A]
b. [N/A]

This question is about relativistic kinematics.

A short pulse containing many nuclei of a radioactive isotope is emitted from a source S in a laboratory. The nuclei have speed v = 0.920c as measured with respect to the laboratory.



The pulse arrives at a detector D. The detector is 250 m away as measured by an observer in the laboratory.

a. Calculate the time it takes the pulse to travel from S to D, according to [3]

(i) an observer in the laboratory.

(ii) an observer Q moving along with the pulse.

b. Calculate the distance between the source S and the detector D according to observer Q. [1]

c. A particular nucleus in the pulse decays by emitting an electron in the same direction as that of the nucleus. The speed of the electron [2]

measured in the laboratory is 0.985c.

Calculate the speed of the electron as measured by observer Q.

d. The laboratory observer and observer Q agree that by the time the pulse arrives at D, half of the nuclei in the pulse have decayed. [2]

Outline, without further calculation, how this is evidence in support of time dilation.

Markscheme

a. (i) 9.1×10^{-7} s;

(ii) the gamma factor is $\gamma = \left(\frac{1}{\sqrt{1-0.920^2}} =\right) 2.55;$
and so $t' = \left(\frac{9.1\times10^{-7}}{2.55} =\right) 3.5\times10^{-7}\text{s};$

b. 98(m);

Allow ECF from gamma factor in (a)(ii).

c. $u' = \left(\frac{u-v}{1-\frac{uv}{c^2}} =\right) \frac{0.985c-0.920c}{1-\frac{0.985c\times0.920c}{c^2}}$;

u'=0.693c;

d. both observers measure different values for the half-life;

and the two half-lives are related by the gamma factor;

Examiners report

a.

b.

c.

d.

A lambda Λ^0 particle at rest decays into a proton p and a pion π^- according to the reaction

$$\Lambda^0 \rightarrow \text{p} + \pi^-$$

where the rest energy of p = 938 MeV and the rest energy of π^- = 140 MeV.

The speed of the pion after the decay is 0.579*c*. For this speed γ = 1.2265. Calculate the speed of the proton.

Markscheme

pion momentum is $\gamma mv = 1.2265 \times 140 \times 0.579 = 99.4$ «MeV c^{-1}»

use of momentum conservation to realize that produced particles have equal and opposite momenta

so for proton $\gamma v = \frac{99.4}{938} = 0.106c$

solving to get *v* = 0.105*c*

Accept pion momentum calculation using $E^2 = p^2c^2 + m^2c^4$.

*Award **[2 max]** for a non-relativistic answer of v = 0.0864c*

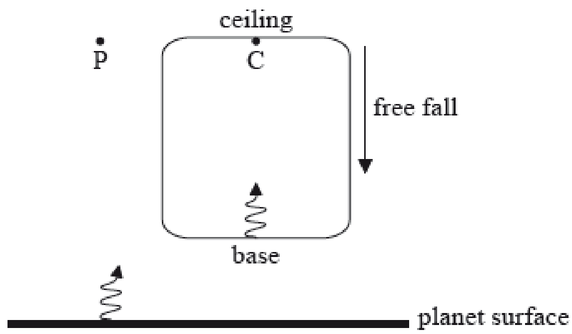
[4 marks]

Examiners report

[N/A]

This question is about general relativity.

The diagram shows monochromatic light of frequency f_0 being emitted from the base of a box towards an observer C at the ceiling of the box. The box is in free fall above the surface of the planet. Light of the same frequency is also emitted from the surface of the planet towards an observer P at rest above the surface of the planet.



The frequency of the light as measured by C is f_{C} and the frequency of the light as measured by P is f_{P}.

State and explain whether the frequencies f_{C} and f_{P} are less than, equal to or greater than f_0.

b. (i) f_{C} [5]

(ii) f_{P}

c. Newton explained the motion of a planet around the Sun in terms of a force of gravitation between the Sun and the planet. Describe how Einstein's theory of general relativity explains the motion of the planet around the Sun. [2]

Markscheme

b. (i) the box is equivalent to an inertial frame of reference (far from any masses);

so $f_{\mathrm{C}} = f_0$;

(ii) the energy of a photon is hf_{P};

light loses energy as it rises;

so $f_{\mathrm{P}} < f_0$;

or

P's frame of reference is equivalent to a frame accelerating upwards (far from any masses);

P moves away from the emitted light / the "space" between P and original source increases;

so (by the Doppler effect) $f_{\mathrm{P}} < f_0$;

c. the mass of the Sun bends the spacetime around it;

particles follow paths of least length/geodesics (when no forces act on them);

paths of least length/geodesics are curves in the bent spacetime;

Examiners report

b. The equivalence principle was usually stated, but not always in unambiguous terms. In (b) the two situations were often confused. Candidates were less sure of the frequency in the free fall situation and usually did not recognise the box as an inertial frame. Many were able to explain why the frequency received by P was less than f_0. Most were able to refer to the sun 'bending' spacetime. Fewer referred to geodesics or the fact that they corresponded to the circular/elliptical orbit of a planet.

c. The equivalence principle was usually stated, but not always in unambiguous terms. In (b) the two situations were often confused. Candidates were less sure of the frequency in the free fall situation and usually did not recognise the box as an inertial frame. Many were able to explain why the frequency received by P was less than f_0. Most were able to refer to the sun 'bending' spacetime. Fewer referred to geodesics or the fact that they corresponded to the circular/elliptical orbit of a planet.

This question is about relativistic mechanics.

a. Show that the speed *v* of a particle of total energy *E* and momentum *p* is given by the following equation. [2]

$$v = \frac{pc^2}{E}$$

b. Determine, using the answer in (a), the speed of a particle whose rest mass is zero. [2]

Markscheme

a. combined use of *p* = *γmv* and *E*=*γmc*^{2};

eliminate the mass and gamma factor by, for example, dividing to get $\frac{p}{E} = \frac{v}{c^2}$;
to get the result
Accept going backwards from given result to reach correct formulae.

b. for a particle with zero rest mass, *E*=*pc*;

and so $v = \left(\frac{pc^2}{pc} =\right) c$;

*Award **[1]** for "zero rest mass particles are photons and so v = c".*

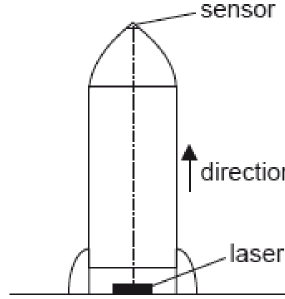
Examiners report

a.

b.

This question is about general relativity.

A spacecraft is at rest on the surface of a distant planet. A laser beam is fired from the base of the spacecraft to a sensor at the top of the spacecraft.



The spacecraft has a height of 112 m. The laser beam emitted from the source has a frequency of 4.52×10^{14} Hz and the sensor detects a shift in the frequency of the laser beam of 3.20 Hz.

a.i. Show that the gravitational field strength at the surface of the planet is about $5.7\ \mathrm{N\,kg^{-1}}$. [2]

a.ii. Discuss the shift in frequency of the laser beam. [2]

b. The spacecraft leaves the planet with an acceleration of $5.7\ \mathrm{m\,s^{-2}}$. The same experiment is carried out by firing the laser beam from the base of the spacecraft to the sensor at the top of the spacecraft. Compare the shift in frequency of the laser beam with that detected in (a). [2]

Markscheme

a.i. $\frac{3.20}{4.52 \times 10^{14}} = \frac{g \times 112}{c^2}$;

$g = 5.69\ (\mathrm{N\,kg^{-1}})$; *(needs to show three significant figures)*

a.ii. as photon moves away from the surface of the planet it gains gravitational potential energy;

$E = hf$ the frequency has become lower (to compensate for this change);

b. the equivalence principle states that it is impossible to distinguish between an accelerating reference frame and a gravitational field;

therefore the frequency observed will be the same;

Accept combined effect if spacecraft still in the gravitational field.

Examiners report

a.i. Part (a)(i) and (b) were done well. Discussion about the shift in frequency in (a)(ii) was difficult as many candidates did not mention potential energy or other equivalents in the discussion.

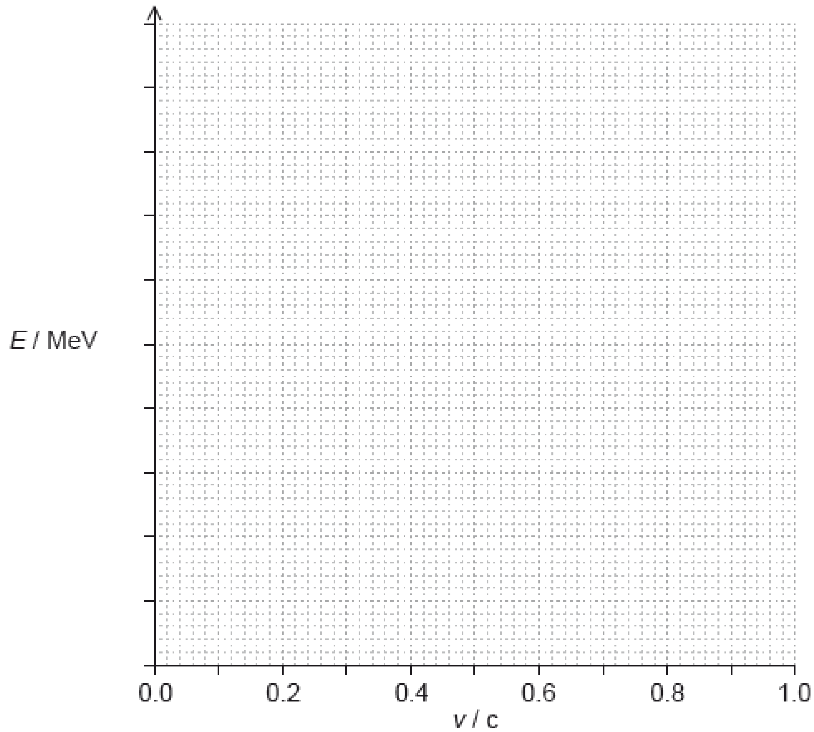
a.ii. Part (a)(i) and (b) were done well. Discussion about the shift in frequency in (a)(ii) was difficult as many candidates did not mention potential energy or other equivalents in the discussion.

b. Part (a)(i) and (b) were done well. Discussion about the shift in frequency in (a)(ii) was difficult as many candidates did not mention potential energy or other equivalents in the discussion.

This question is about mass and energy.

The positive kaon K^+ has a rest mass of $494\ \mathrm{MeV\,c^{-2}}$.

a.i. Using the grid, sketch a graph showing how the energy of the kaon increases with speed. [2]



a.ii. The kaon is accelerated from rest through a potential difference so that its energy becomes three times its rest energy. Calculate the potential difference through which the kaon was accelerated. [2]

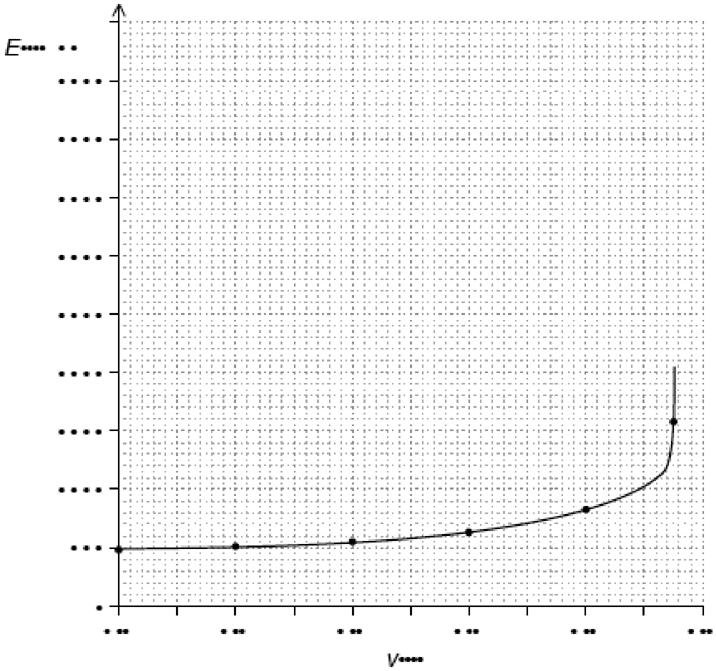
b. The neutral kaon is unstable and one of its possible modes of decay is [2]

$$K^0 \rightarrow \pi^0 + \pi^0.$$

The π^0 has a rest mass of $135\ \mathrm{MeV\,c^{-2}}$. The K^0 has a rest mass of $498\ \mathrm{MeV\,c^{-2}}$. The K^0 is at rest before it decays. The two π^0 particles move apart in opposite directions along a straight line. Determine the momentum of **one** of the π^0 particles.

Markscheme

a.i.



graph starting at $E = 494$ when $\frac{v}{c} = 0$ ***or*** a roughly horizontal line drawn until at least 0.4 c;

rises sharply/becomes asymptotic as $\frac{v}{c}$ approaches 1;

a.ii. $E_K = 2 \times 494 \text{ MeV} = 988 \text{ (MeV)}$;

potential difference $= 988 \times 10^6 \text{ (V)}$ ***or*** $1 \times 10^9 \text{ (V)}$;

b. $pc = \left(\sqrt{\left[\frac{E}{2}\right]^2 - m_0^2c^4} =\right) \sqrt{\left[\frac{498}{2}\right]^2 - 135^2}$;

$p = 209 \text{ (MeV c}^{-1}\text{)}$;

Examiners report

a.i. Discriminated well between the best and average candidates, in part (a)(ii). Many weaker candidates did not distinguish between energy and kinetic energy of the particle, and often also forgot to calculate the potential difference; part (b) proved difficulty for majority of candidates, even if some very clear and good answers are among the answers of better candidates.

a.ii. Discriminated well between the best and average candidates, in part (a)(ii). Many weaker candidates did not distinguish between energy and kinetic energy of the particle, and often also forgot to calculate the potential difference; part (b) proved difficulty for majority of candidates, even if some very clear and good answers are among the answers of better candidates.

b. Discriminated well between the best and average candidates, in part (a)(ii). Many weaker candidates did not distinguish between energy and kinetic energy of the particle, and often also forgot to calculate the potential difference; part (b) proved difficulty for majority of candidates, even if some very clear and good answers are among the answers of better candidates.

This question is about relativistic mechanics.

A proton is accelerated from rest through a potential difference of 1.5 GV.

Calculate, for the accelerated proton, the

a. total energy. [2]

b. momentum. [2]

c. speed. [2]

Markscheme

a. change in total energy/kinetic energy is 1.5 GeV;

total energy is 1.5+0.938=2.4(GeV) ***or*** 3.8×10^{-10} (J) ***or*** 3.9×10^{-10} (J);

*Award **[2]** for a bald correct answer.*

b. $pc\left(=\sqrt{E^2-[mc^2]^2}\right)=\sqrt{2.4^2-[0.938]^2}$; *(allow ECF from (a))*

p=2.2 ***or*** 2.3($GeVc^{-1}$) ***or*** 1.2×10^{-18} ($kgms^{-1}$);

*Award **[2]** for a bald correct answer.*

c. $y=\left(\frac{2.44\text{GeV}}{0.938\text{GeV}}\right)=2.6;$

$u=\frac{p}{\gamma m_0}=\frac{2.25}{2.6\times0.938}=0.92\text{c};$ *(allow ECF from (a) and (b))*

*Award **[2]** for a bald correct answer.*

or

$\left(p=\gamma m_0 u=\gamma m_0 c^2\frac{u}{c^2}\Rightarrow\right)u=\frac{pc}{E}c;$

$=\left(\frac{2.2}{2.4}c=\right)0.92c;$ (allow ECF from (a) and (b))

*Award **[2]** for a bald correct answer.*

Examiners report

a. In part (a) the KE was usually easily identified and added to the proton rest energy.

b. Part (b): In any question with units expressed in terms of MeV and c there is enormous potential for confusion. However an increasing number of candidates are able use the relativistic energy – momentum equation ($E^2=(mc^2)^2+p^2c^2$) correctly as they realise that it becomes a Pythagorean $E^2=m^2+p^2$ when using the simpler units. The commonest mistake was to try to make use of the value of “c” in the calculation instead of just sticking with the values given.

c. In (c) gamma was frequently found correctly and converted to a speed, but ECF was often necessary.

This question is about relativistic mechanics.

A rho meson (ρ) decays at rest in a laboratory into a pion (π^+) and an anti-pion (π^-) according to

$\rho \to \pi^+ + \pi^-$.

The rest masses of the particles involved are:

$m_{\pi^+} = m_{\pi^-} = 140 \text{ MeV c}^{-2}$

$m_\rho = 770 \text{ MeV c}^{-2}$

a. (i) Show that the initial momentum of the pion is 360 MeV c^{-1}. [6]

(ii) Show that the speed of the pion relative to the laboratory is 0.932c.

(iii) Calculate, in MeV c^{-2}, the mass that has been converted into energy in this decay.

b. The pion (π^+) emits a muon in the same direction as the velocity of the pion. The speed of the muon is 0.271c relative to the pion. Calculate the speed of the muon relative to the laboratory. [2]

Markscheme

a. (i) use of $E^2 = (\text{mc}^2)^2 + p^2c^2$;

by conservation of energy, total energy of pion is $\frac{770}{2} = 385 \text{ MeV}$;

$385^2 = 140^2 + p^2c^2$; *(award **[3]** immediately if this marking point is seen)*

Solving for momentum gives the answer p = 359 $MeV\ c^{-1}$ ≈ 360 $MeV\ c^{-1}$.

Answer is given, marks are for correct working only.

No ECF if wrong energy used.

(ii) $\gamma = \frac{385}{140}\ (= 2.75)$;

hence $v = \sqrt{1 - \frac{1}{\gamma^2}}c = \sqrt{1 - \frac{1}{2.75^2}}c\ (= 0.932c)$;

Answer given, award marks for working only.

Watch for ECF from (a)(i) or first marking point.

(iii) $(770 - 2 \times 140) = 490 \text{ MeV c}^{-2}$;

b. $u = \left(\frac{u'+v}{1+\frac{u'v}{c^2}} =\right) \frac{0.932\text{c}+0.271\text{c}}{1+\frac{0.932\text{c}\times 0.271\text{c}}{c^2}}$;

$u = 0.960\text{c}$;

*Award **[2]** for a bald correct answer.*

Allow working which does not mention c.

Examiners report

a. (i) In any question with units expressed in terms of MeV and c there is enormous potential for confusion. However a reasonable number are able to use the relativistic energy - momentum equation $\left(E^2 = (mc^2)^2 + p^2c^2\right)$ correctly. The most common mistake was to try to make use of the value of 'c' in the calculation instead of just sticking with the values given. In (a)(ii) few could determine gamma. (iii) was much easier.

b. (b) required use of the relativistic velocity addition formula. Quite a few performed velocity subtraction.

This question is about relativistic dynamics.

A proton is accelerated from rest by a potential difference *V* and reaches a speed of 2.5×10^{8} m s^{-1}. Calculate *V*.

Markscheme

$\gamma m_0c^2=m_0c^2+eV$;

$\gamma=1.81$;

7.7×10^{8}(V) ;

Examiners report

This was poorly done although there were some very good answers. A significant minority of candidates used the non-relativistic equation for kinetic energy

---

This question is about rest mass and relativistic energy.

a. (i) Define the *rest mass* of a particle. [2]

(ii) The rest mass of a particle is said to be an invariant quantity. State, with reference to special relativity, what is meant by the term invariant.

b. In a thought experiment, two particles X and Y, each of rest mass 380 MeVc^{-2}, are approaching each other head on. [5]

The speed of X and of Y is 0.60 c relative to a laboratory.

(i) Calculate the momentum of X in the frame of reference in which Y is at rest.

(ii) As a result of the collision a single particle Z is formed. Determine the rest mass of Z. The gamma factor for a speed of 0.60 c is 1.25.

Markscheme

a. (i) the mass of an object in its rest frame / the mass as measured by an observer at rest with respect to the body;

(ii) a quantity that is the same for all observers/reference frames;

b. (i) speed of X relative to Y is $\left(\frac{0.60c-(-0.60c)}{1+0.60^2}\right) = 0.882c;$

gamma factor at this speed is $\gamma = \frac{1}{\sqrt{1-0.882^2}} = 2.12;$

momentum is then $p=\gamma mv= 2.12\times380\times0.882c=710\text{MeV}c^{-1}$;

*Award **[3]** for a bald correct answer between 700MeVc^{-1} and 713MeVc^{-1} due to rounding.*

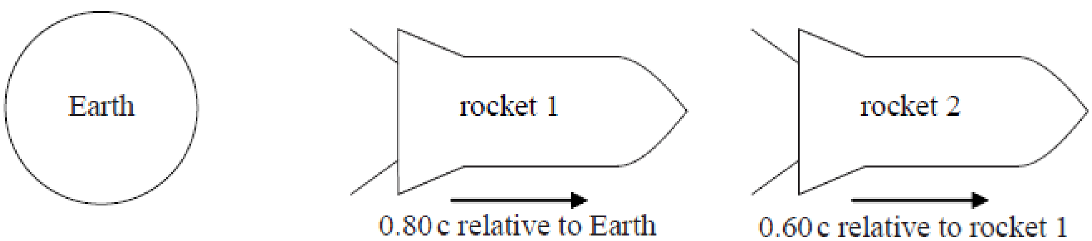
(ii) $Mc^2 = 2\times\gamma mc^2 = 2\times\frac{5}{4}\times 380;$
$\Rightarrow M=950\text{MeV}c^{-2}$;
*Award **[2]** for a bald correct answer.*

Examiners report

a. [N/A]
b. [N/A]

This question is about relative velocities and energy at relativistic speeds.

Two identical rockets are moving along the same straight line as viewed from Earth. Rocket 1 is moving away from the Earth at speed 0.80 c **relative to the Earth** and rocket 2 is moving away from rocket 1 at speed 0.60 c **relative to rocket 1**.



a. Calculate the velocity of rocket 2 relative to the Earth, using the [3]

(i) Galilean transformation equation.

(ii) relativistic transformation equation.

b. Comment on your answers in (a). [2]

c. The rest mass of rocket 1 is 1.0×10^3kg. Determine the **relativistic** kinetic energy of rocket 1, as measured by an observer on Earth. [2]

Markscheme

a. (i) $u'_x = u_x + v = 0.60c + 0.80c = 1.40c$;

(ii) $u_x{}'\left(=\frac{u_x+v}{1+\frac{u_xv}{c^2}}\right) = \frac{0.60c+0.80c}{1+\frac{0.60c\times0.80c}{c^2}};$
$\left(=\frac{1.40c}{1.48}\right) = 0.95c;$
*Award **[1]** for answers that use v = –0.80 c to get an answer of –0.38 c.*

b. the answer to (a)(i) exceeds *c* / the answer to (a)(ii) does not exceed *c*;

hence the Galilean transformation is not valid / the relativistic transformation must be used / *OWTTE*;

c. $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1=\frac{0.80^2 c^2}{c^2}}} = 1.7;$

$E_K = [\gamma - 1]\, m_0 c^2 = [1.667 - 1] \times 1.0 \times 10^3 \times \left[3.0 \times 10^8\right]^2;$

$=6.0\times10^{19}$J

*Award **[0]** for answers that use E_k = ½ mv^2 to get an answer of 2.9×10^{19} J.*

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

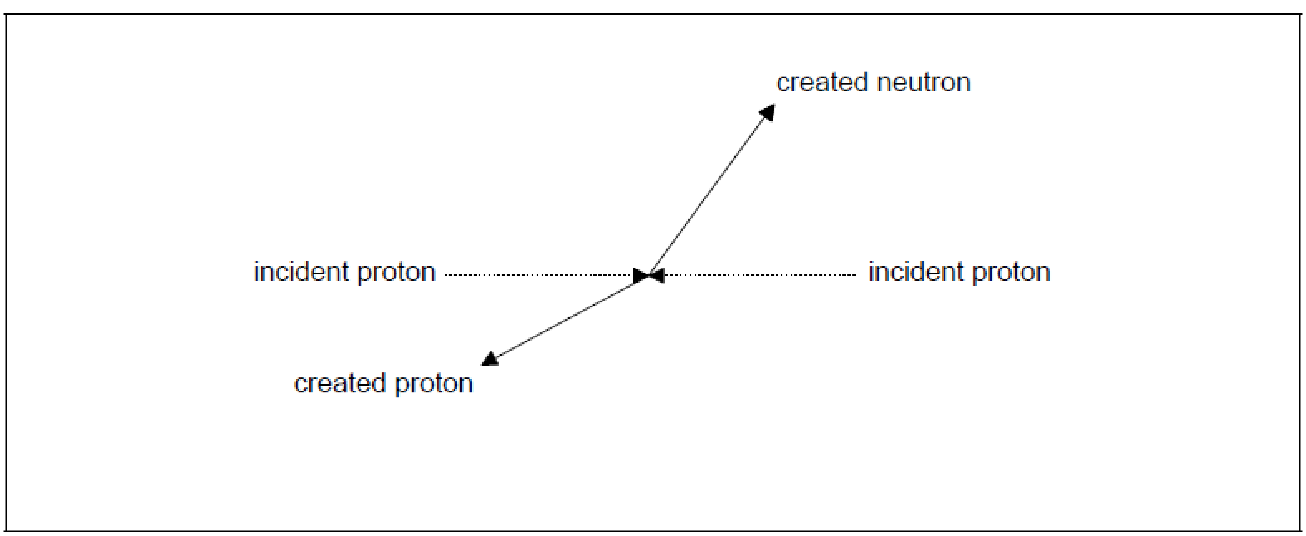
Two protons, travelling in opposite directions, collide. Each has a total energy of 3.35 GeV.

As a result of the collision, the protons are annihilated and three particles, a proton, a neutron, and a pion are created. The pion has a rest mass of 140 MeV c^{-2}. The total energy of the emitted proton and neutron from the interaction is 6.20 GeV.

a. Calculate the gamma (γ) factor for one of the protons. [1]

b.i. Determine, in terms of MeV c^{-1}, the momentum of the pion. [3]

b.ii. The diagram shows the paths of the incident protons together with the proton and neutron created in the interaction. On the diagram, draw the path of the pion. [1]



Markscheme

a. γ «$= \frac{3350}{938}$» = 3.37

[1 mark]

b.i. energy of pion = (3350 × 2) – 6200 = 500 «MeV»

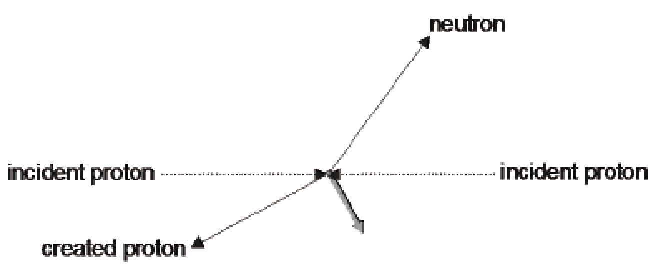
$500^2 = p^2c^2 + 140^2$

p = 480 «MeV c^{-1}»

[3 marks]

b.ii. path of pion constructed in direction around 4–5 o'clock by eye

eg:



[1 mark]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]

This question is about spacetime.

Explain, with reference to the warping of spacetime, the gravitational attraction between Earth and the Sun.

Markscheme

spacetime is warped by matter;

moving objects follow the shortest path/metric/geodesic between two points in spacetime;

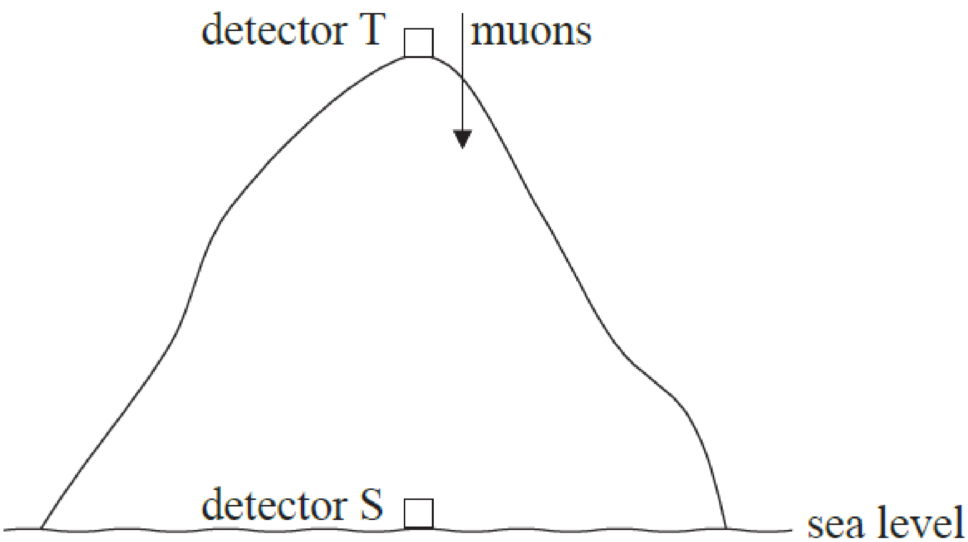
the shortest path for Earth is a (closed) curve around Sun (this is what we call gravitational attraction);

Examiners report

A few candidates gave very good answers. Most candidates gained the first marking point by referring to mass distorting/warping spacetime, but then there were many vague descriptions of the Earth "falling into the well" of the Sun rather than the Earth following a geodesic through spacetime.

This question is about muon decay experiments.

Muons created high in the atmosphere move vertically down towards the surface of Earth. A muon detector T is placed on top of a mountain and another, detector S, is placed at sea level.



Detector T detects 570 muons per hour. In the rest frame of the muons their half-life is 1.5 μs. According to an observer, at rest on the mountain, the muons take 6.0 μs to travel from detector T to detector S.

a. Show that, if the muons move at non-relativistic speed, the number of muons detected at sea level would be approximately 36 per hour. [2]

b. The muons in (a) move toward the surface of Earth with a relativistic speed of 0.968c. [4]

(i) Determine the half-life of the muons according to the observer at rest on the mountain.

(ii) The number of muons observed at detector S is 285 per hour. Explain, using your answers to (a) and (b)(i), how this observation provides evidence for time dilation.

Markscheme

a. the number of half-lives that go by until muons make it to sea level is $\frac{6.0}{1.5} = 4$;

and so the number of muons per hour would be $\frac{570}{2^4} = \frac{570}{16}(= 35.6)$;

(≈36)

Answer given, reward correct working only.

b. (i) *y*= 3.985;

so the dilated half-life is 3.985×1.5 = 5.977 ≈ 6.0(μs);

(ii) 285 muons per hour represents a half-life of 6.0(μs);
this time is four times greater than in the muon frame, and so confirms time dilation;
Allow similar arguments explaining that the two different half-lives or count-rates are proof of time dilation.

Examiners report

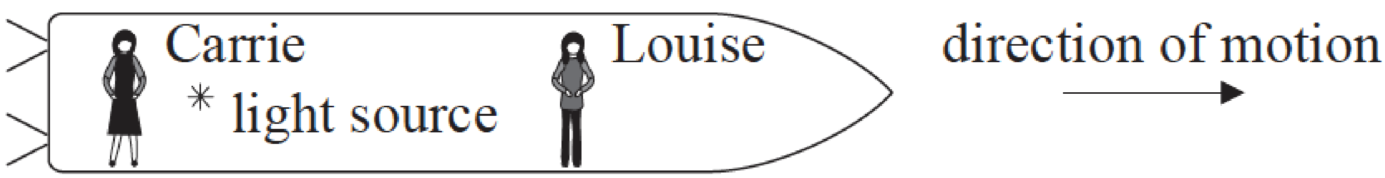
a. (a) provided an easy two marks for the majority, as did (b)(i) for the muon half-life in the Earth rest-frame.

b. (a) provided an easy two marks for the majority, as did (b)(i) for the muon half-life in the Earth rest-frame. In (b)(ii) not all candidates referred to their previous answers, as directed, in explaining the evidence for time dilation. There is uncertainty about the meaning of the words 'dilated time' and who perceives it. Candidates need to know that **dilated** means larger/longer and refers to the longer elapsed time on two <u>different</u> clocks in the observers frame compared to the shorter elapsed 'proper time' on a single, relatively moving clock. In other words, the relatively moving clock's elapsed time (between two events at which this clock is present) is always less than that between the observer's two clocks separated in space. This is a very difficult concept and candidates really need to try to get to grips with it. Without reference to clocks it is almost impossible to understand time dilation or simultaneity. Spacetime diagrams are helpful again here. Due to this misunderstanding there were many references to 'time running slow **for** the muons'. This is incorrect. Muon's time runs slow **for** the Earth observers. The difference is subtle but important.

This question is about relativity.

Carrie is in a spaceship that is travelling towards a star in a straight-line at constant velocity as observed by Peter. Peter is at rest relative to the star.

Carrie and Louise, two observers in a spaceship, view a light source placed close to Carrie. When the spaceship is travelling at a constant velocity, they both measure the frequency of the light source and obtain identical values.



The magnitude of the velocity of the spaceship increases.

State and explain any changes to the frequency of the light source, as measured by Louise, that occur during the acceleration.

Markscheme

speed of light is independent of speed of source;

so (because of acceleration) source appears to move away from Louise / *OWTTE*;

<u>Doppler</u> effect predicts a red-shift;

Louise measures a lower frequency;

or

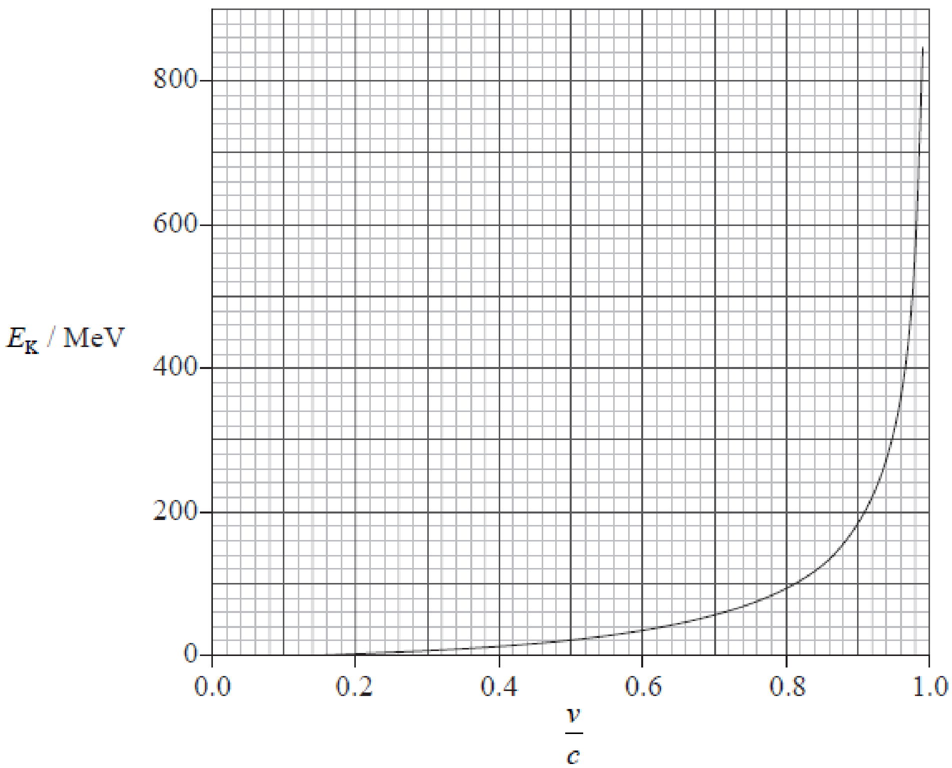
accelerating spaceship is equivalent to being at rest in a gravitational field / *OWTTE*;
photons leaving the source therefore gain potential energy and lose kinetic energy;
since $E=hf$;
Louise will measure a lower frequency;

Examiners report

[N/A]

This question is about mass and energy.

a. The graph shows the variation with the fraction $\frac{v}{c}$, of the kinetic energy E_K of a particle, where v is the speed of the particle. [4]



Determine, using the value E_K =360 MeV from the graph, the rest mass of the particle in MeV c^{-2}.

b. Determine, using data from the graph, the potential difference required to accelerate the particle in (a) from a speed of 0.63c to a speed of 0.96c. The charge of the particle is +*e*. [2]

Markscheme

a. v=0.96c when E_K=360(MeV);

calculation of gamma factor $\gamma = \frac{1}{\sqrt{1-0.96^2}} = 3.571$;

so that $m = \frac{E_K}{(\gamma-1)c^2} = \frac{360}{2.571c^2}$;

m=140MeVc^{-2};

b. change in kinetic energy is 360-40 = 320MeV;

so voltage required is 320MV ; *(unit needed)*

MV needed for second marking point.

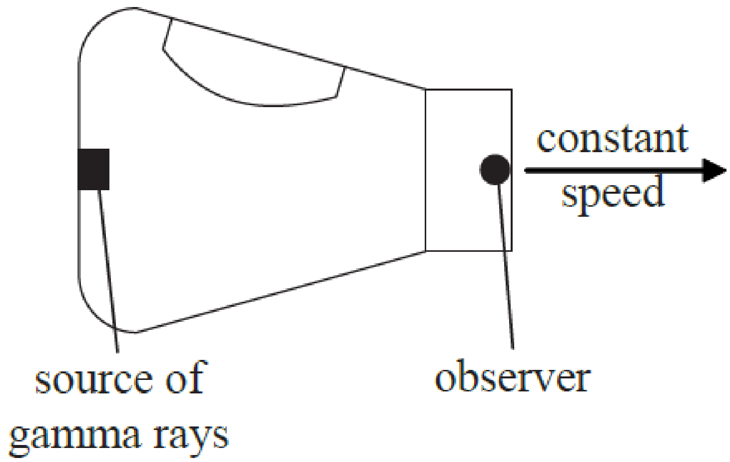
Examiners report

a.

b.

This question is about gravitational red-shift.

An observer is in a space capsule moving at a constant speed in the absence of gravitational fields.



A monochromatic source of gamma rays is fixed to the rear wall of the capsule. An observer at the other end of the capsule measures the frequency of the gamma rays.

The capsule now starts to decelerate.

a. Deduce the change in the frequency of the gamma rays, as measured by the observer, when the deceleration begins. [3]

b. Outline, with reference to the principle of equivalence, how the situation in (a) relates to the concept of gravitational red-shift. [2]

Markscheme

a. the speed of light is constant / the speed of light does not depend on the speed of the source (relative to the observer);

the gamma source appears to be moving towards the observer / *OWTTE;*

so the frequency will be blue-shifted / have higher frequency / frequency will increase;

b. principle of equivalence relates acceleration (of frame of reference) to (frame in a) gravitational field / *OWTTE;*

situation is same (*ie* blue-shifted) as source falling under gravity towards observer;

Examiners report

a. Was very poorly done. Candidates had an understanding of photons gaining/losing energy and so changing frequency when travelling through a gravitational field, but had great difficulty understanding the same phenomenon by thinking in terms of acceleration and the Doppler effect. Many candidates tried to do a calculation even though no numerical information was given. It should be noted that the command term deduce means "Reach a conclusion from the information given" and so a calculation is not usually required.

b. In (b) There were many good statements of the equivalence principle, but most candidates had difficulty relating it to the situation in (a).

This question is about black holes.

Sirius B has a mass of 2.0×10^{30} kg.

Calculate the minimum density required for Sirius B to become a black hole in the future.

Markscheme

$R_S = \frac{2GM}{c^2} = 2.96\,(\mathrm{km})$;

$\rho=1.8\times10^{19}(\mathrm{kgm^{-3}})$;

Examiners report

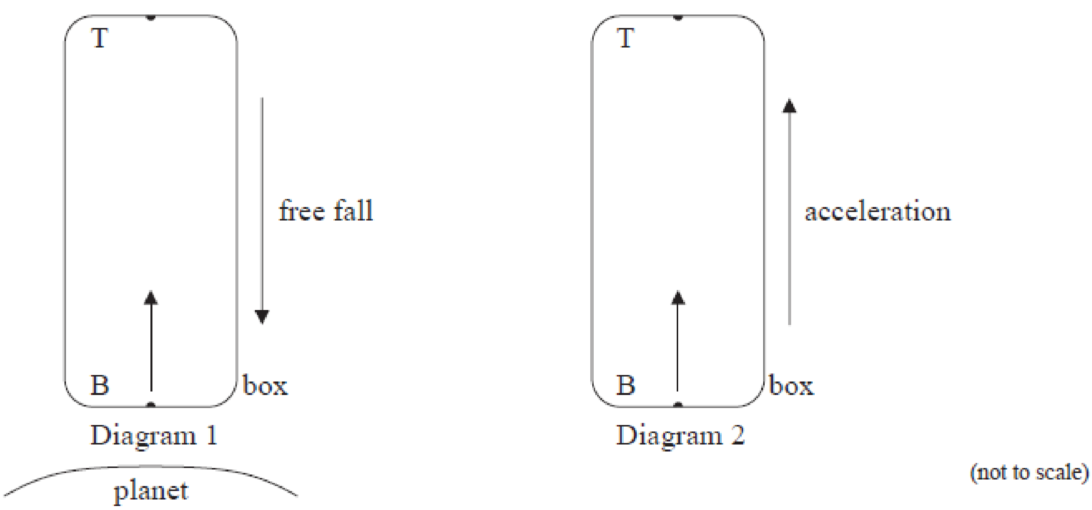
Many candidates correctly calculated the Schwarzschild radius but an amazing number failed to calculate the density given the mass and radius.

Many candidates did not know how to calculate the volume of a sphere.

This question is about the equivalence principle.

a. State the equivalence principle. [1]

b. The diagram shows two identical boxes in two different states of motion. [4]



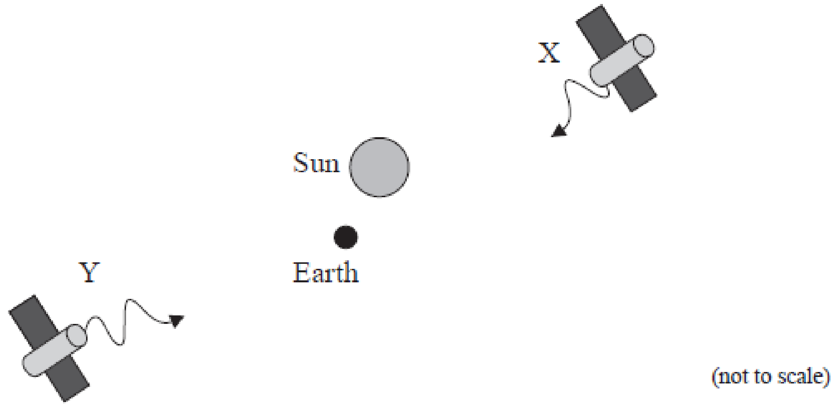
In **diagram 1** the box is in free fall close to the surface of a planet. In **diagram 2** the box is accelerating in a region of space far from other masses.

A ray of monochromatic light is emitted from the base B of each box and is received at the top T of each box.

Observers at B measure the frequency of the emitted light to be f_0.

State and explain, for each state of motion in diagram 1 and 2, if the frequency of light measured by an observer at T will be less than, equal to **or** greater than f_0.

c. Radio signals, sent at the same time from Earth, reflect off two satellites X and Y as shown. The satellites are at the same distance from Earth. [2]



The signal from Earth to satellite X and the reflected signal pass close to the Sun.

Compare, using the theory of general relativity, the arrival times at Earth of the signal from X and the signal from Y.

Markscheme

a. inertial/acceleration effects are indistinguishable from gravitational effects / a freely falling frame of reference in a gravitational field is equivalent to an inertial frame of reference / an accelerating frame of reference in outer space is equivalent to a frame of reference at rest in a gravitational field;

b. 1: $f = f_{0;}$

because the frame of reference is equivalent to an inertial frame of reference;

2: $f < f_{0;}$
the frame of reference is equivalent to a frame of reference at rest in a gravitational field and so light is gravitationally red-shifted;

or

$f < f_{0;}$
observer at T will observe the source as though it was moving away from him and so will measure a Doppler red-shifted frequency;

c. the signal from satellite X will arrive after that from satellite Y / there will be a time delay in the arrival of signal X;

because the X signal undergoes gravitational time dilation/bends/curves (in the field of the Sun);

Examiners report

a.

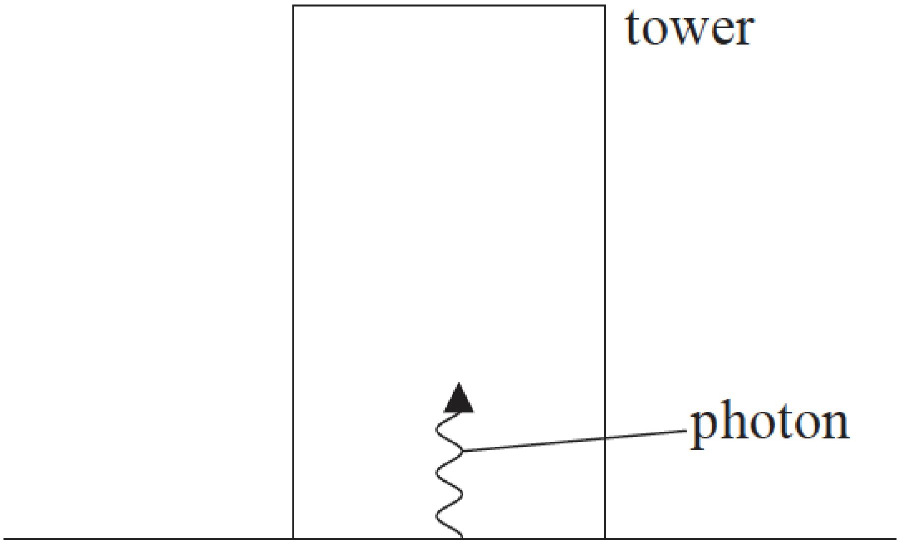
b.

c.

This question about general relativity.

a. State the principle of equivalence. [1]

b. A gamma-ray photon is emitted from the base of a tower towards the top of the tower. [5]



(i) Explain, using the principle of equivalence, why the frequency of the photon as measured at the top of the tower is less than that measured at the base of the tower.

(ii) The frequency of the photon at the base is 3.5×10^{18} Hz and the tower is 23 m high. Determine the shift Δf in the frequency of the photon at the top of the tower.

(iii) Suggest, using your answer to (b)(ii), why the photon frequency must be measured very precisely for this experiment to be successful.

Markscheme

a. inertial and gravitational effects are indistinguishable / a freely falling frame in a gravitational field is equivalent to an inertial frame far from all masses / an accelerating frame is equivalent to a frame at rest in a gravitational field;

b. (i) *The question does not specifically state the location of the tower so allow any of the explanations below.*

(the principle of equivalence predicts) photon energy decreases as it moves against a *g* field;

this energy is given by $E=hf$;

hence as E decreases, f must also decrease;

or

the tower is equivalent to a frame accelerating upwards;
the top of the tower is moving away from the light emitted from the base;
and so by the Doppler effect/red-shift the frequency at the top will be less;

or

in freely falling tower the frequency at the top and bottom would be the same;
an outside observer sees the top moving towards the light emitted from the base and so (by the Doppler effect) expects a blue-shift;
for the frequency to be the same at the top the light moving upwards must suffer an equal red-shift;

(ii) 8.8×10^{3}Hz / 8.9×10^{3}Hz / 9.0×10^{3}Hz;

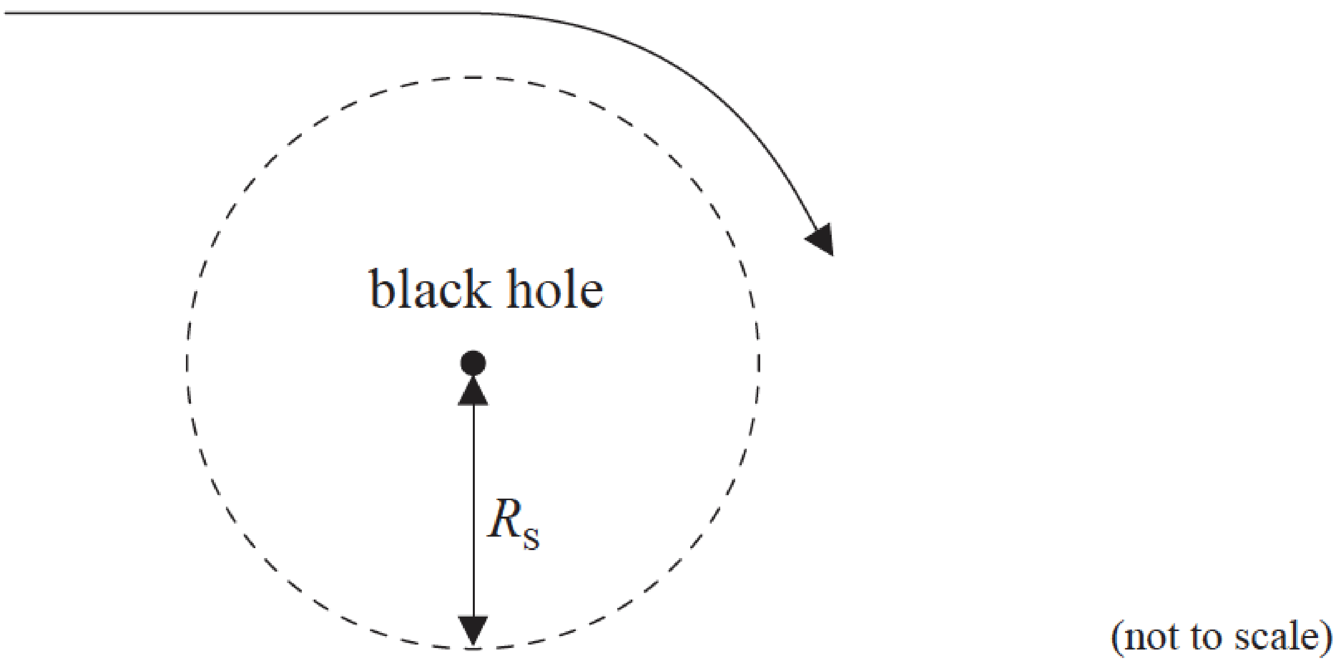
(iii) $\frac{\Delta f}{f}\left(=\frac{8.8\times10^{3}}{3.5\times10^{18}}\right)\approx10^{-15}$ / the shift is very small compared to the original frequency / the new frequency differs from the original in the 15th decimal place;

Examiners report

a. [N/A]
b. [N/A]

This question is about black holes.

The diagram shows the path of a light ray in the space around a black hole.



The radius of the dotted circle is the Schwarzschild radius of the black hole.

a. Define the *Schwarzschild radius* of a black hole. [1]

b. Explain, using the concept of spacetime, why the path of the light ray is straight at distances far from the black hole and curved when near the black hole. [3]

Markscheme

a. the distance from the black hole where the escape speed is the speed of light / the distance from the black hole inside which nothing can escape;

b. light travels along – geodesics/paths of shortest length;

geodesics/light paths – follow the curvature of spacetime / *OWTTE*;

spacetime is more curved nearer/less curved further from a black hole;

Examiners report

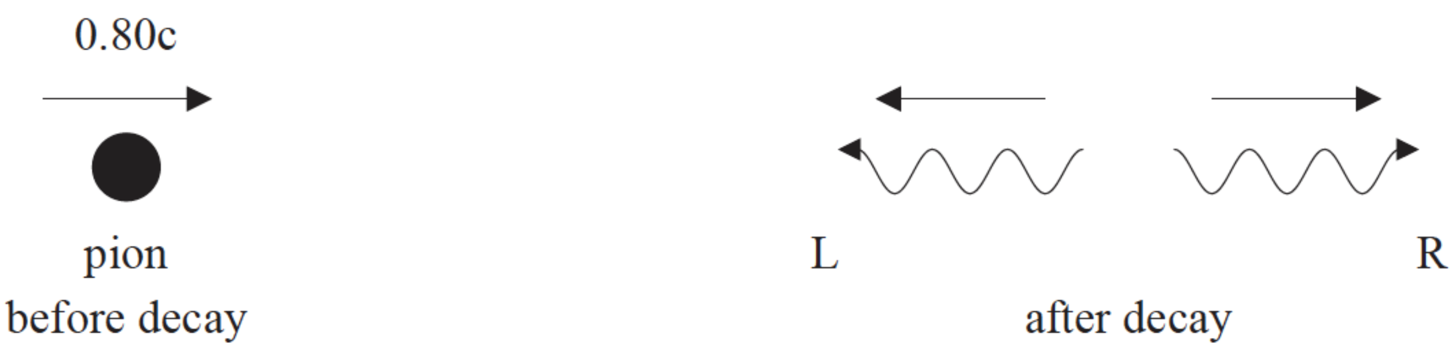
a. [N/A]
b. [N/A]

This question is about relativistic mechanics.

a. In an experiment at CERN in 1964, a neutral pion moving at a speed of 0.99975c with respect to the laboratory decayed into two photons. The speed of each photon was measured with respect to the laboratory. [2]

Describe how the result of this experiment provided support for special relativity.

b. In another experiment, a neutral pion moving at 0.80c relative to a laboratory decayed into two photons as shown in the diagram. [6]



Photon R moved in the direction of the pion and photon L in the opposite direction. The rest mass of the pion is 135 MeVc^{-2}.

According to a laboratory observer,

(i) determine the total energy of the pion in MeV.

(ii) determine the momentum of the pion, in MeVc^{-1}.

(iii) state and explain which photon, R **or** L, has the greater energy.

Markscheme

a. special relativity rests on the postulate that the speed of light (*c*) is independent of the speed of its source / speed of light is constant;

both photons were measured to have a speed equal to c with respect to the lab thus verifying the postulate;

b. (i) the gamma factor is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3} = 1.67;$
so the total energy of the pion is $\left(\frac{5}{3} \times 135\right) = 225\text{MeV};$

(ii) $p = (\gamma m v) = \frac{5}{3} \times 135 \times 0.80c;$
=180MeVc^{-1};

or

use of $E^2 = p^2c^2 + m^2c^4$
$p = \sqrt{(225^2 - 135^2)};$
=180MeVc^{-1};

(iii) (since the momentum of a photon is $\frac{E}{c}$) by <u>momentum conservation</u>
$\frac{E_R}{c} - \frac{E_L}{c} = 180\text{MeVc}^{-1};$
hence the right photon has the greater energy;

or

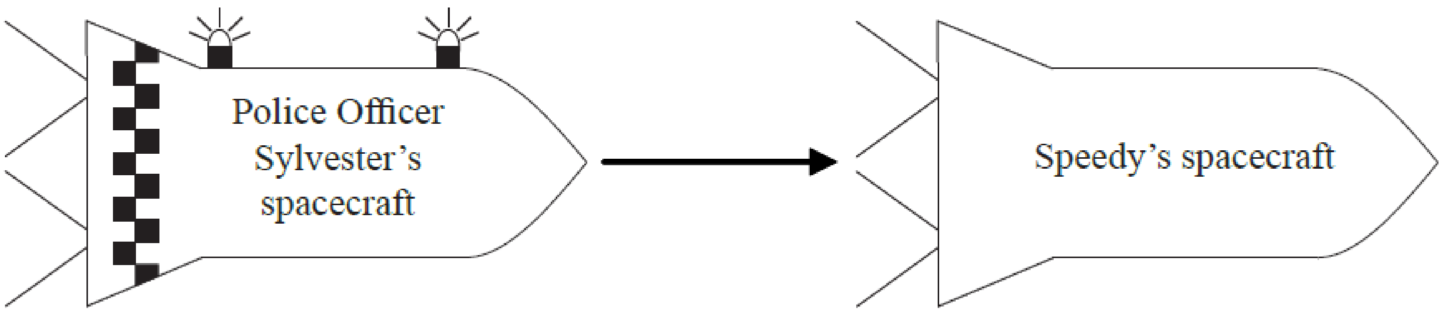
the total momentum before the decay is directed towards the right;
by <u>momentum conservation</u> the momentum after the decay is also to the right, hence the right photon has the greater energy;
Award ***[1]*** *for "right photon has greater energy" with wrong or missing explanation.*

Examiners report

a. [N/A]
b. [N/A]

This question is about relativistic kinematics.

Speedy is in a spacecraft being chased by Police Officer Sylvester. In Officer Sylvester's frame of reference, Speedy is moving directly towards Officer Sylvester at 0.25c.



Officer Sylvester switches on the blue flashing lamps on his police spacecraft.

(i) Calculate, assuming that a Galilean transformation applies to this situation, the value of the speed of the light that Speedy would measure using the flashing lamps.

(ii) Speedy measures the speed of the light emitted by the flashing lamps. Deduce, using the relativistic addition of velocities, that Speedy will obtain a value for the speed of light equal to *c*.

Markscheme

(i) 1.25*c*

(ii) $v' = \frac{c-v}{1-\frac{cv}{c^2}}$;
$= \frac{0.75c}{1-0.25}$;
shows that fraction = *c*;
*Award **[2 max]** if signs incorrect.*

Examiners report

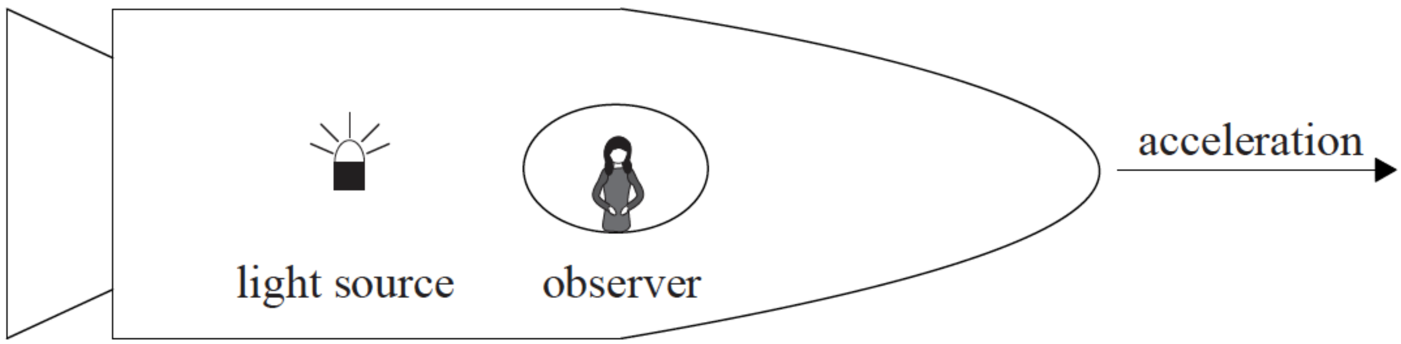
Most candidates answered (b)(i) correctly. Many candidates refused to put an answer greater than c because that is impossible, missing the point of the question (that the Galilean transformation gives a wrong answer so the relativistic transformation is required).

In (b)(ii) most candidates got the signs wrong and so scored two marks out of three.

This question is about general relativity.

a. State the principle of equivalence. [1]

b. An observer in a spaceship moving at constant speed measures the frequency f_0 of light emitted by a source. The spaceship now accelerates to the right. [5]



The observed frequency changes to f.

(i) Outline why, during the acceleration, f is less than f_0.
(ii) Explain how the result outlined in (b)(i) leads to the deduction that time dilates near a planet.

Markscheme

a. a frame of reference accelerating in outer space is equivalent to a frame of reference at rest in a gravitational field / an inertial frame of reference in outer space is equivalent to a freely falling frame of reference in a (uniform) gravitational field;

*Award **[0]** for only "gravitational and inertial mass are equivalent".*

b. (i) light source appears to be moving away from the observer;

so there is a red-shift (according to the Doppler effect);

or

spaceship (by equivalence) can be regarded as (at rest) in a gravitational field;
photons lose energy in reaching observer (so frequency must be reduced);

(ii) the planet has a gravitational field;
so (by equivalence) the situation is as though light source is near a planet;
f is still observed to be less than f_0 / period of the light can be taken as unit of time;
this can be interpreted as an increase in the time for emission of one wavelength / increase in the period (*ie* time is dilated);

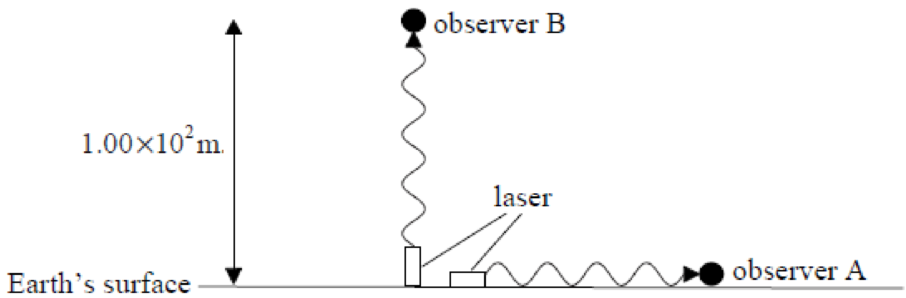
Examiners report

a. The principle of equivalence is generally well understood by the candidates. However, the majority of candidates wrote general statements (not wrong) but not in a sufficiently clear sequence. There were many vague statements about gravity and inertia in (a), which was not in response to the question of "state the principle...".

b. In (b)(ii), some candidates did not realize that the question remained focused on the principle of equivalence. Good answers to this question required a deep understanding of the principle.

This question is about gravitational red-shift.

Two identical lasers are situated on the surface of the Earth. One is directed horizontally towards observer A, who measures the frequency to be 4.62×10^{14}Hz. The other is directed vertically upwards towards observer B, who is at a height of 1.00×10^{2}m.

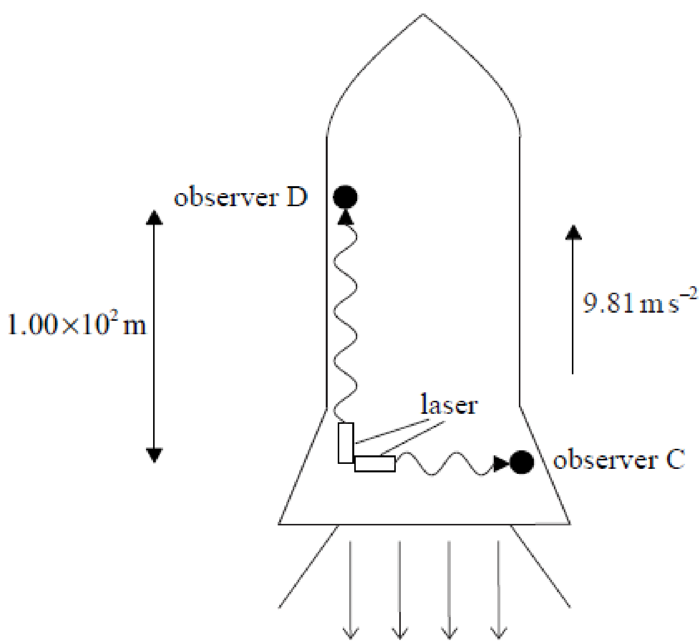


a. (i) State how the frequency as measured by observer B compares with the frequency as measured by observer A. [4]

(ii) Calculate the difference in frequency between the laser light as measured by observers A and B.

(iii) State **one** assumption that you made in (a)(ii).

b. The lasers are now placed on a spaceship, which is accelerating upwards at a constant rate of 9.81ms^{-2}, far away from any other masses as shown below. The distance of observer D from the laser is 1.00×10^{2}m. Observer C is at the bottom of the spaceship. [2]



Explain, with reference to the equivalence principle, the frequencies measured by observers C and D, as compared to observers A and B.

Markscheme

a. (i) observer B measures a lower frequency;

(ii) $\frac{\Delta f}{f} = \frac{g\Delta h}{c^2} \Rightarrow \Delta f = \frac{4.62\times10^{14}\times9.81\times1.00\times10^{2}}{\left[3.00\times10^{8}\right]^{2}}$;

Δf=5.04Hz;

Accept use of g = 10 ms^{-2} *to get f = 5.13 Hz.*

(iii) assume that *g* is constant over the height interval;

b. the equivalence principle states that it is impossible to distinguish between an accelerating reference frame and a gravitational field;

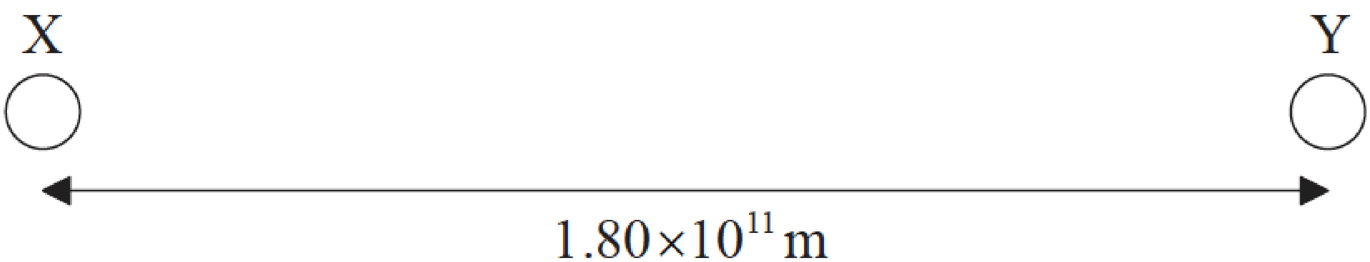
therefore the frequency measured by observer D will be lower than that measured by observer C (by 5.04Hz) / observer C measures the same value as A, observer D measures the same frequency as B / *OWTTE*;

Examiners report

a. [N/A]
b. [N/A]

This question is about time dilation and relativistic mass.

a. Two space stations X and Y are at rest relative to each other. The separation of X and Y as measured in their frame of reference is 1.80×10^{11}m. [1]



State what is meant by a frame of reference.

b. A radio signal is sent to both space stations in (a) from a point midway between them. On receipt of the signal a clock in X and a clock in Y are each set to read zero. A spaceship S travels between X and Y at a speed of 0.750c as measured by X and Y. In the frame of reference of S, station X passes S at the instant that X's clock is set to zero. A clock in S is also set to zero at this instant. [9]

(i) Calculate the time interval, as measured by the clock in X, that it takes S to travel from X to Y.

(ii) Calculate the time interval, as measured by the clock in S, that it takes S to travel from X to Y.

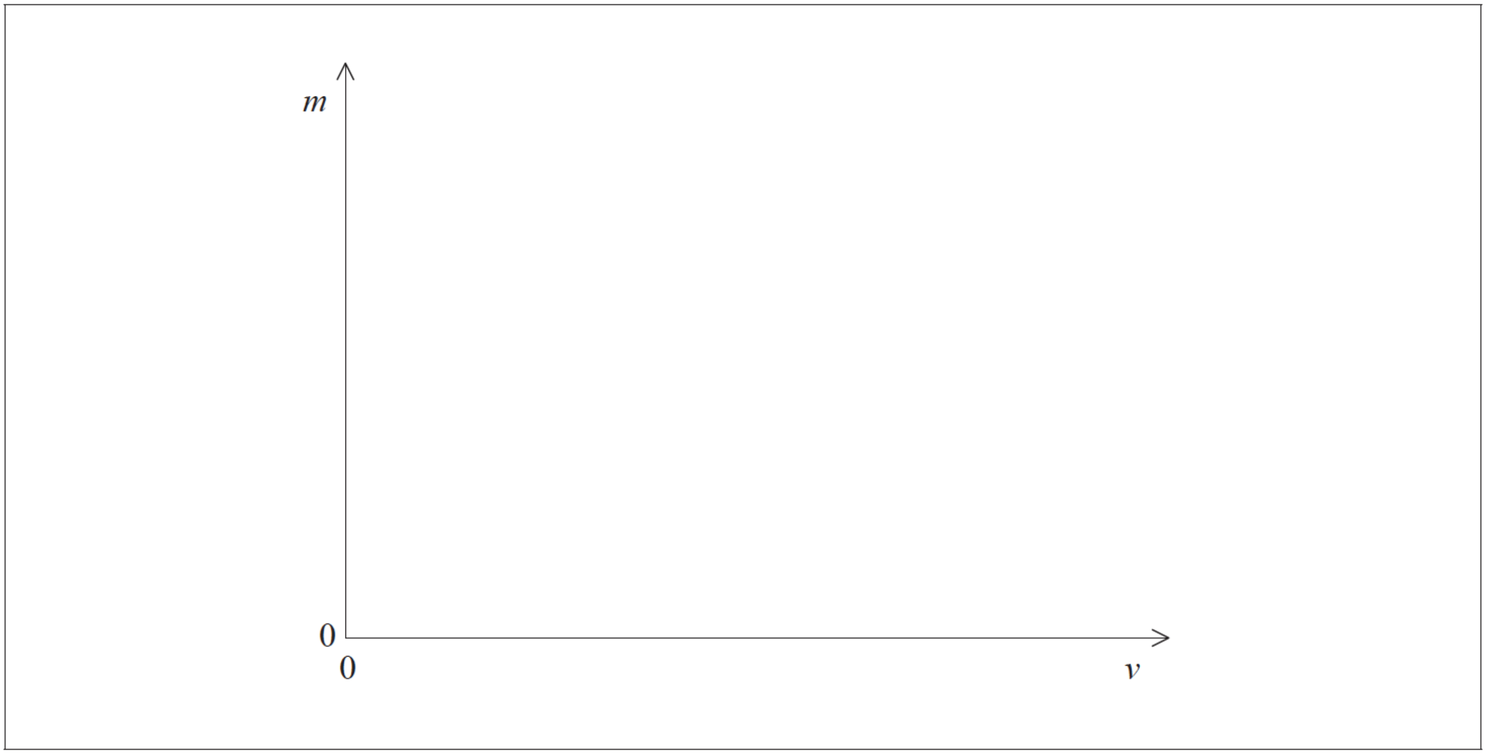
(iii) Explain whether the clock in X **or** the clock in S measures the proper time.

(iv) Explain why, according to S, the setting of the clock in X and the setting of the clock in Y does not occur simultaneously.

c. The spaceship S in (b) is moving with speed 0.750c as measured by X and Y and has a total energy of 2.72×10^{20}J as measured by X and Y. [4]

(i) Determine the rest mass of spaceship S.

(ii) Using the axes, sketch a graph to show how the mass m of spaceship S changes with its speed v. Your graph should identify the rest mass $m=m_0$ and the speed $v=c$.



d. Muons are produced in the upper atmosphere of Earth and travel towards the surface of Earth where they are detected. Explain how, with reference to the situation described in (b), the production and detection of muons provide evidence for time dilation. [3]

Markscheme

a. a set of coordinates that can be used to locate events/position of objects;

b. (i) $\frac{1.80\times10^{11}}{0.750\times3\times10^{8}}$;

=800(s);

*Award **[2]** for a bald correct answer.*

(ii) $\gamma = \left(\frac{1}{\sqrt{1-0.750^2}} =\right) 1.51;$

$\text{time} = \left(\frac{800}{1.51} =\right) 530\,(\text{s});$

Watch for ECF from (b)(i) or first marking point in (b)(ii).
*Award **[2]** for a bald correct answer.*

(iii) only S's clock measures proper time;
because S's clock is at both events / events occur at same place in S's frame;

(iv) according to S, Y moves towards/X moves away from the radio signal;
the signal travels at the same speed/at the speed of light in each direction;
therefore according to S's clock the signal reaches Y before it reaches X/X after reaching Y;

or

S's frame is different/moving relative to the X and Y frame;
the two events/arrival of signals are separated in space;
so if simultaneous for XY, cannot be simultaneous for S;

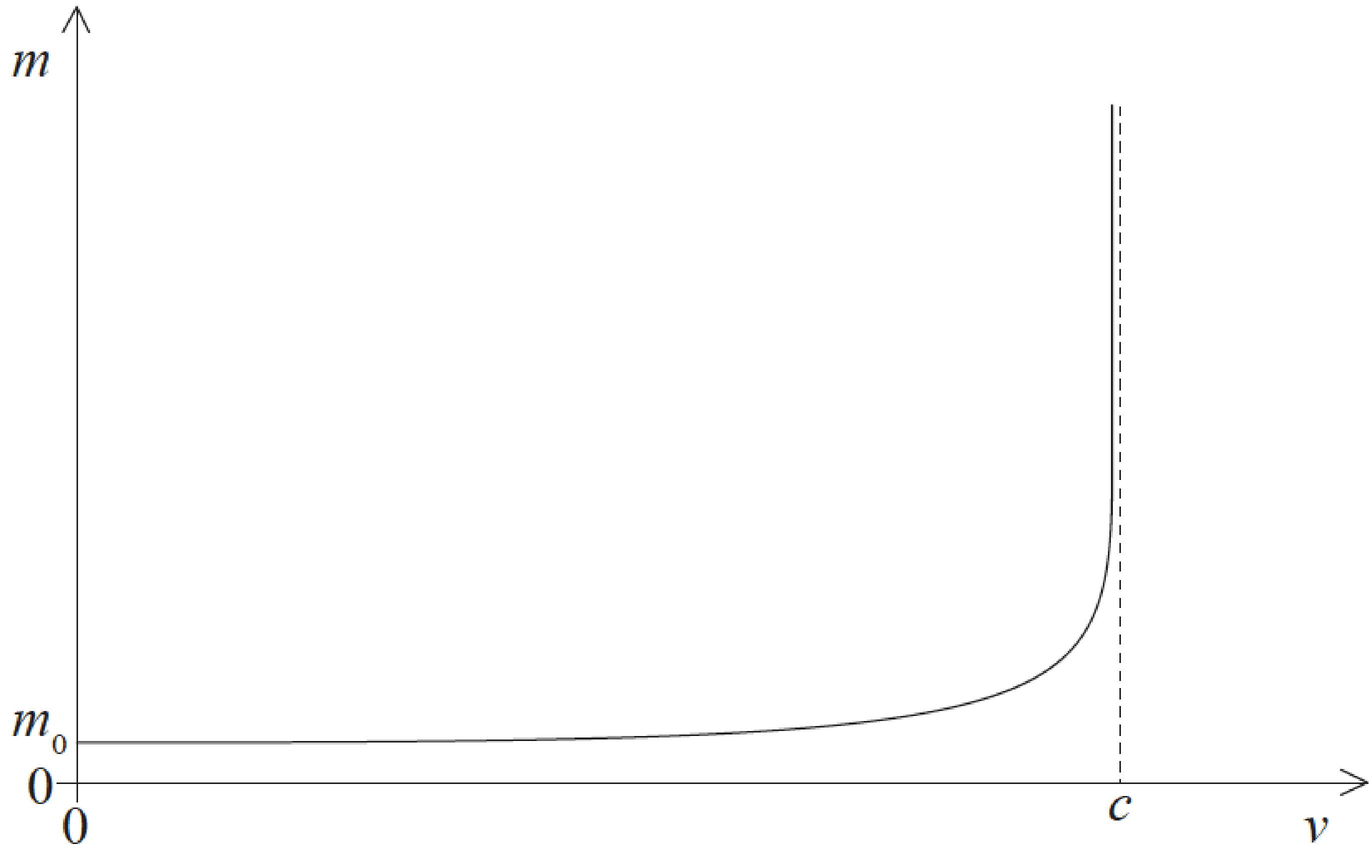
c. (i) $\text{restmass} = \frac{\text{totalenergy}}{\gamma c^2}$; *(allow ECF for γ from 12(b)(ii))*

$= \left(\frac{2.72\times10^{20}}{1.51\times9\times10^{16}} =\right) 2.0\times10^3\,(\text{kg})$;

*Award **[1 max]** if gamma is not used and answer is 3000 (kg).*

*Award **[2]** for a bald correct answer.*

(ii)



general shape showing asymptote to $v=c$;

non-zero value for $m=m_0$;

d. the muons are equivalent to S and the Earth to X,Y;

without time dilation most muons would decay before reaching the Earth's surface;

from Earth frame, with time dilation, the (proper) half-life of muons becomes dilated/larger;

from muon frame, with time dilation, the (proper) journey time is less than that on Earth; } *(do not award marking point for arguments just based on length contraction)*

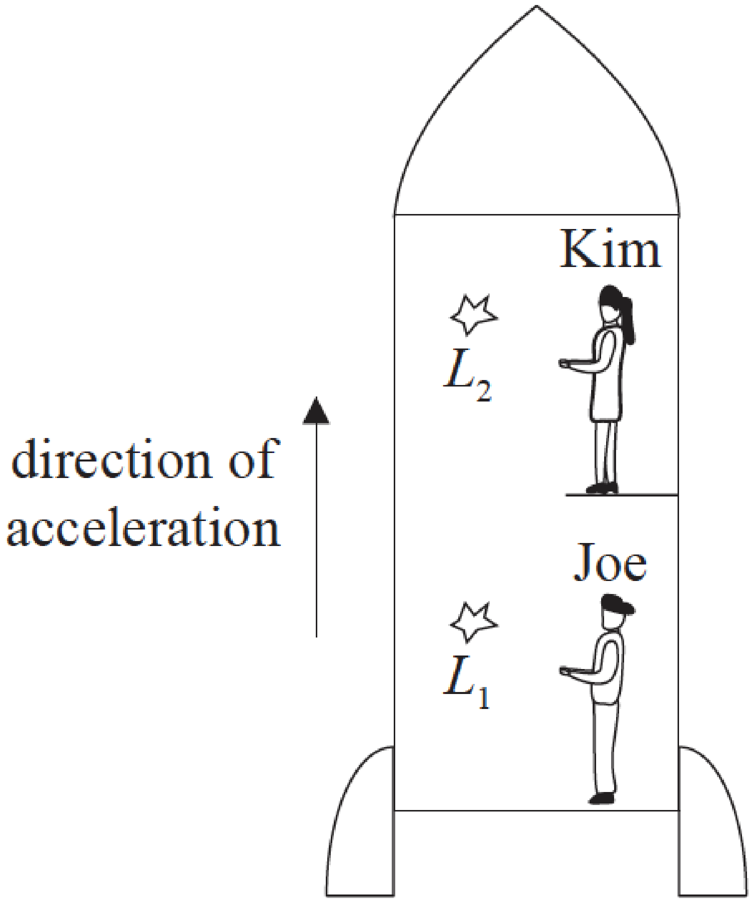
fewer muons decay/more muons survive than expected without time dilation; } *(award marking point even if no explanation is given)*

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

This question is about the principle of equivalence and red-shift.

Joe and Kim are travelling in a spaceship.



Joe is next to a light source L_1 and Kim is next to an identical light source L_2.

The acceleration of the spaceship is zero. Kim measures the frequency of the light from L_1 to be the same as the frequency of the light from L_2.

a. Outline why, if the spaceship now accelerates, Kim will measure the light from L_1 to be red-shifted with respect to the light from L_2. [3]

b. Suggest, with reference to Einstein's principle of equivalence, how your answer to (a) leads to the idea that a clock near a massive body runs more slowly than a clock in free space. [2]

Markscheme

a. *Look for an argument along these lines:*

the speed of light is independent of speed of source;

because of the direction of acceleration;

to Kim it will appear as if L_1 is moving away from her;

light from L_1 will therefore be Doppler shifted toward the red end of the spectrum;

or

accelerating spaceship is equivalent to being at rest in a gravitational field / *OWTTE*;
photons leaving the source therefore gain potential energy and lose kinetic energy;
since *E*=*hf*;

Kim will measure a lower frequency;

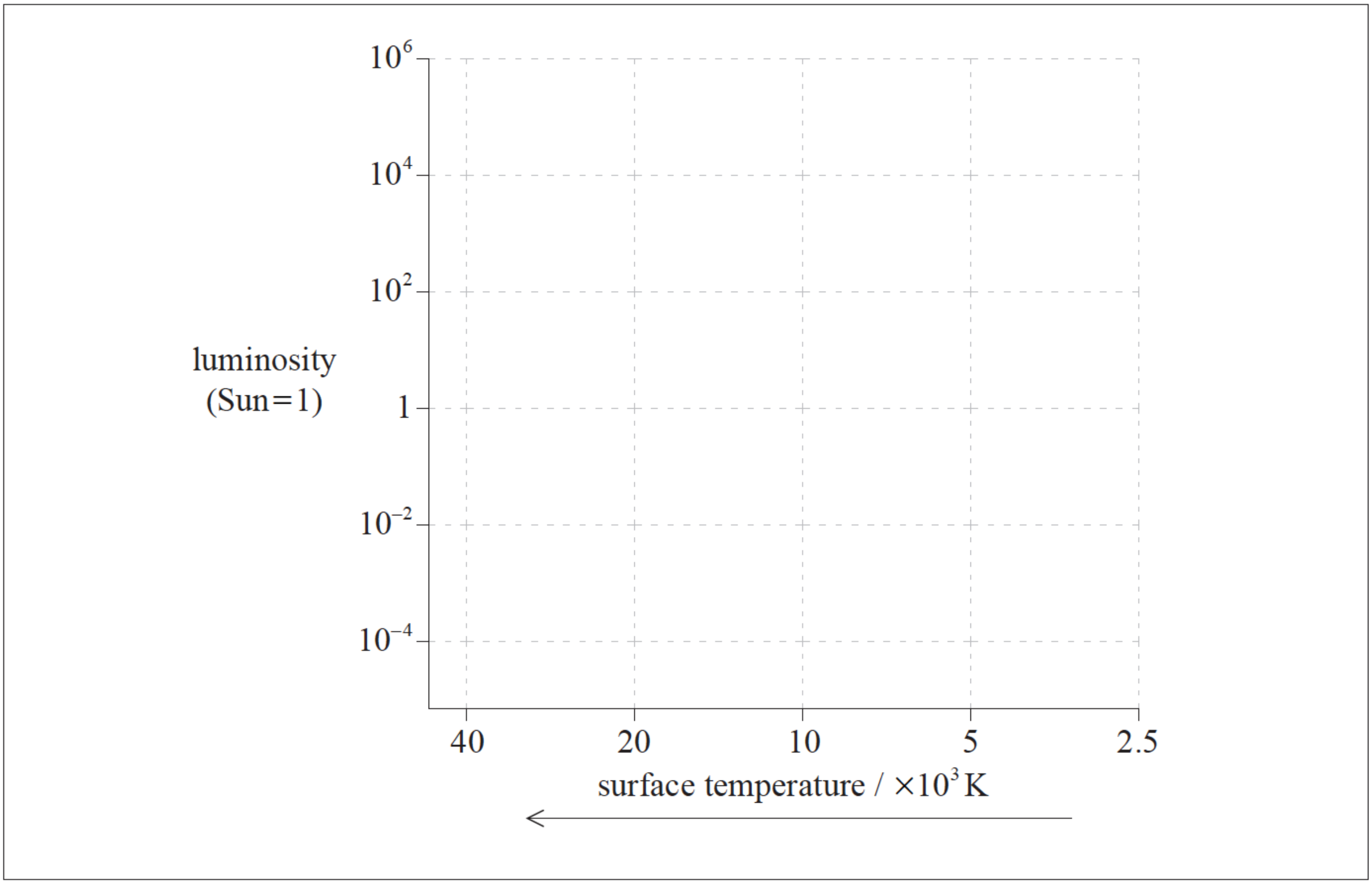
b. according to the equivalence principle the accelerating spaceship is equivalent to a frame of reference at rest in a (uniform) gravitational field / can be regarded as being at rest on the surface of a planet/large mass / *OWTTE*;

if light is red shifted from L_1 then this implies period of the light source is longer therefore time is slower / *OWTTE*;

Examiners report

a. [N/A]
b. [N/A]

This question is about stellar distances and stellar properties.



d. According to Vladimir, a clock at rest in the railway carriage will appear to run slower than a clock at rest beside him. However, according to Natasha, Vladimir's clock will run slower than a clock at rest beside her. [4]

(i) Outline how this time dilation phenomenon leads to the "twin paradox" in which one of the twins embarks on a return journey to a distant star at a speed close to that of light whilst the other twin remains on Earth.

(ii) State the reason behind the resolution of the paradox.

e. Evidence for time dilation comes from the decay of muons. A pulse of muons produced by cosmic radiation in the upper atmosphere of Earth travels to Earth with a speed of 0.96c as measured by an observer at rest on the surface of Earth. The half-life of the muons, as measured in the frame of reference in which the muons are at rest, is 3.1×10^{-6}s. [3]

(i) Determine for the muons, the distance that Earth will have travelled towards them after half of the muons in the pulse have decayed.

(ii) Calculate for the Earth observer, the distance that the muon pulse will have travelled towards Earth after half of the muons in the pulse have decayed.

f. Suggest how your answers to (e)(i) and (e)(ii) provide evidence that supports the theory of special relativity. [3]

Markscheme

d. (i) *Look for an argument along these lines:*

on the return of the travelling twin according to the twin on Earth the travelling twin will have aged very little compared to himself/herself;

however, since time dilation is symmetric it could be the twin on Earth who has done the least aging;

experiment suggests that it is the travelling twin who ages the least;

(ii) because of the accelerations undergone by the travelling twin the situation is not symmetric / travelling twin is not in the same inertial frame of reference/changes inertial frame of reference;

e. (i) $(0.96\times3.0\times10^{8}\times3.1\times10^{-6})$=890m;

(ii) $\gamma = \frac{1}{\sqrt{1-(0.96)^2}}$;
distance = (3.57×890=) ***or*** $(3.57\times0.96\times3.0\times10^{8}\times3.1\times10^{-6}=)$ 3200m;

f. using the laboratory half-life, most of the muons would have decayed before reaching Earth;

however many muons are detected at the surface;

showing that the half-life is dilated / to the muons the distance travelled is contracted;

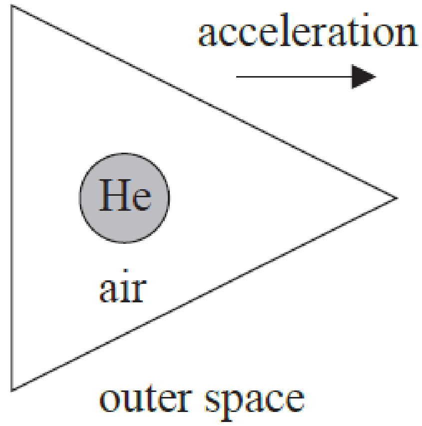
Examiners report

d. [N/A]
e. [N/A]
f. [N/A]

This question is about general relativity.

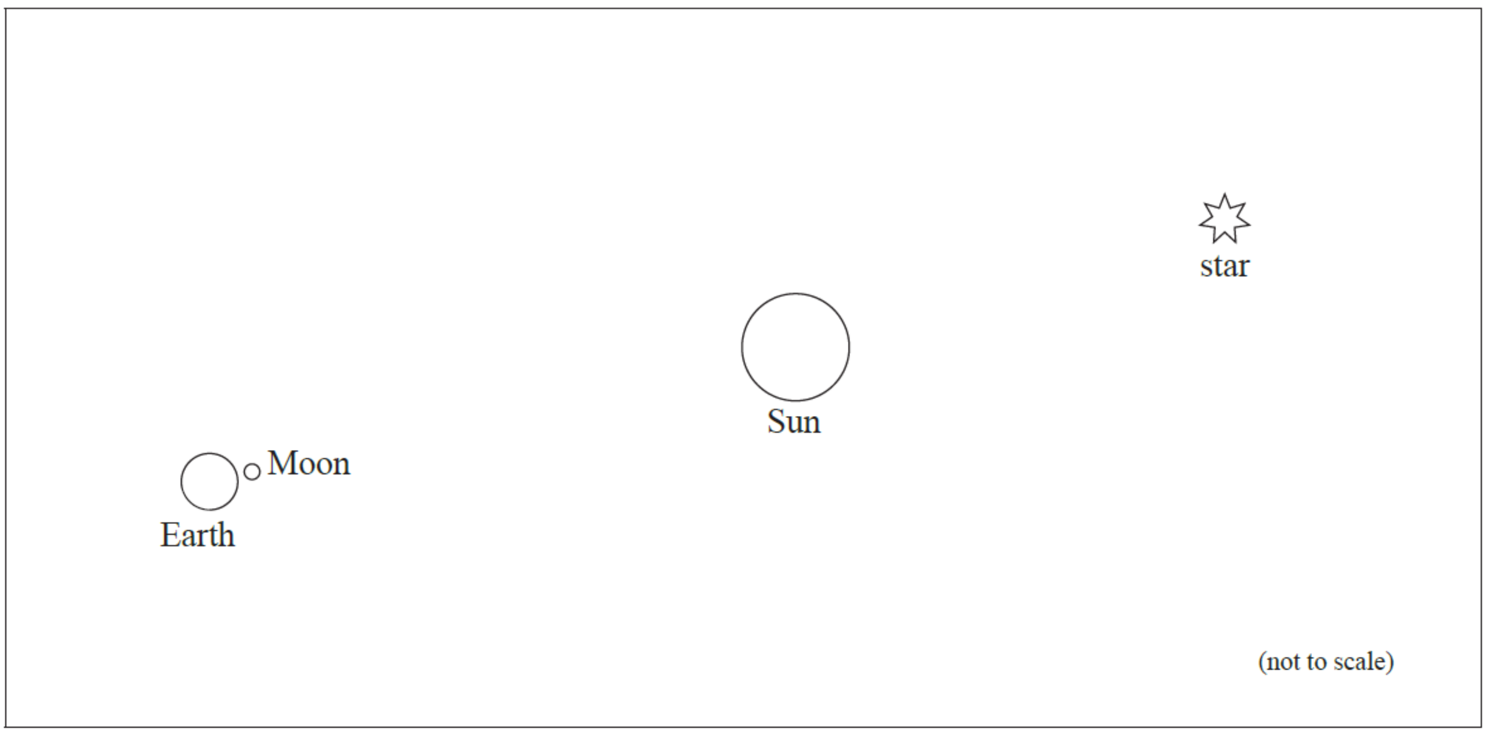
a. State the equivalence principle. [2]

b. A helium filled balloon is floating in air inside a spacecraft in outer space. The spacecraft begins to accelerate to the right. [3]



Explain, with reference to the equivalence principle, the motion, if any, of the helium balloon relative to the spacecraft.

c. In an experiment, to verify the bending of light as it passes close to the Sun, the position of a star was measured during a total solar eclipse. [5]



(i) Explain why the measurement of the star's position was made during a total solar eclipse.

(ii) On the diagram above, draw lines to determine the apparent position of the star as seen from Earth.

(iii) State what other measurement must be made in order to determine the angle by which rays from the star are bent by the Sun.

(iv) The angle of bending of a light ray from the star that just grazes the Sun's surface is θ. State and explain the effect, if any, on θ if the Sun were to be replaced by another star of equal radius but larger mass.

Markscheme

a. a frame of reference accelerating in (outer space) is equivalent to a frame of reference at rest in a gravitational field / gravitational effects are indistinguishable from inertial effects;

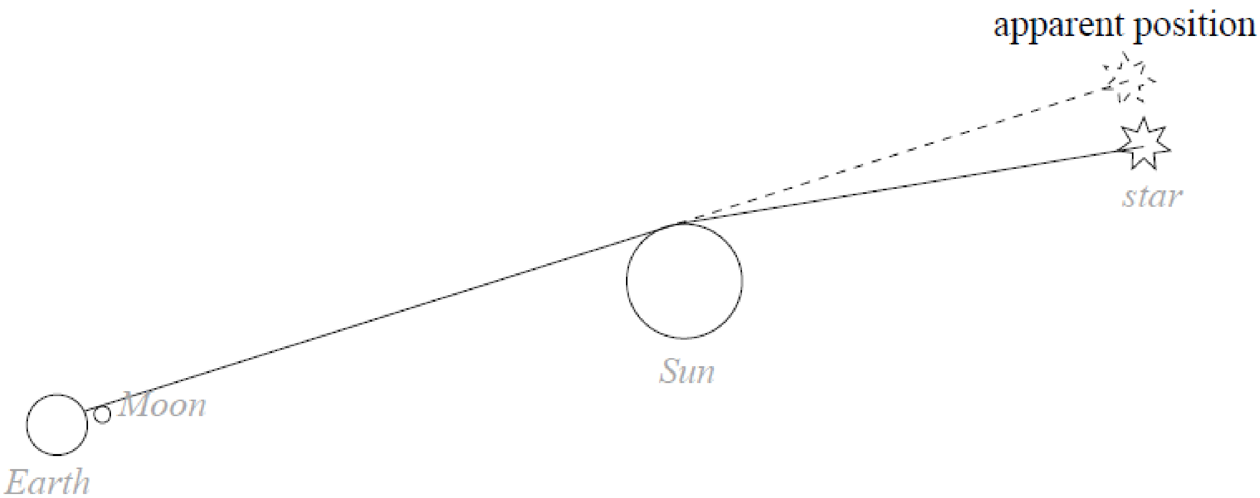
b. balloon moves to the right;

   rocket frame is equivalent to a rocket at rest in a gravitational field directed to the left;

   helium balloons rise in gravitational fields;

c. (i) so that the star could be seen during the day;

   (ii)



   ray from star curving past the Sun towards the Earth and that ray extended backwards along a straight line to a position higher/lower than real position of star;
   Allow ray path to travel above or below the Sun.

   (iii) the position of the star when light from the star reaches the Earth without going past the Sun;

   (iv) the angle of deflection will be greater;
   a greater mass will cause a greater curvature of spacetime;

Examiners report

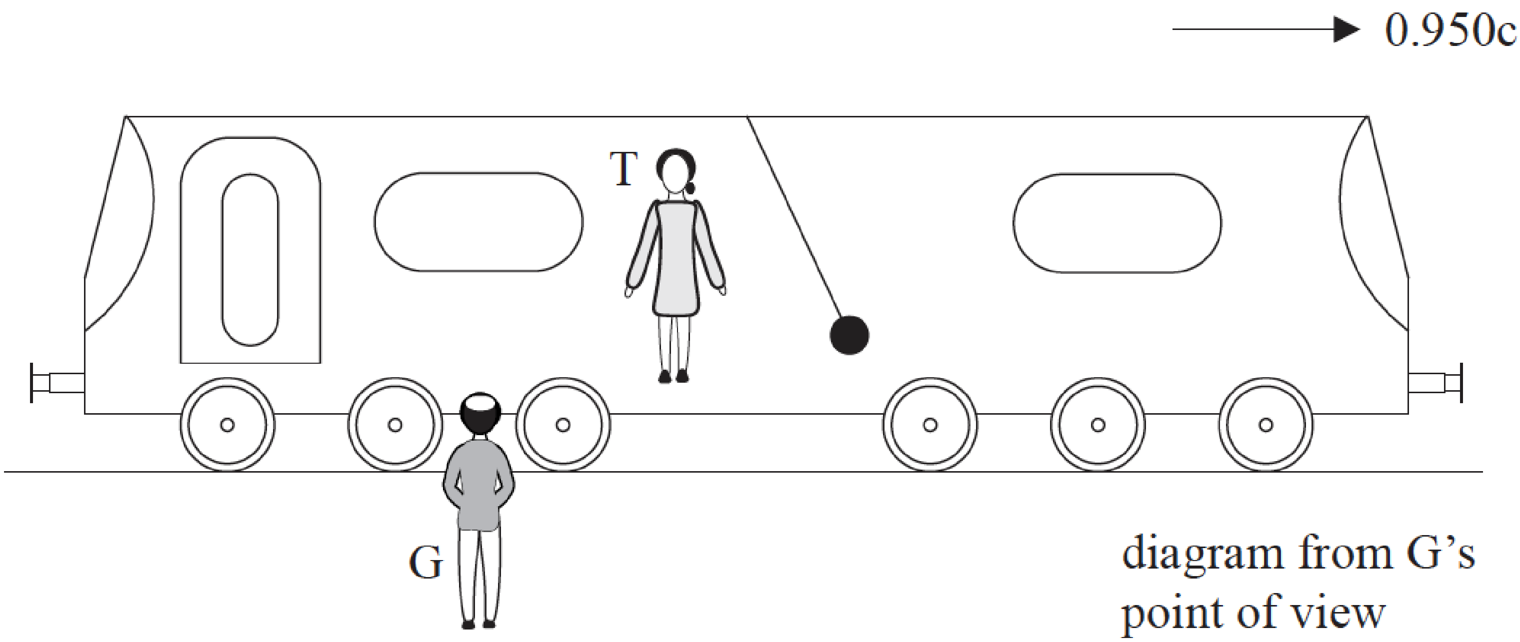
a. The equivalence principle was usually stated, but not always in unambiguous terms.

b. In (b) very few candidates could work out what would happen. Many identified that this was equivalent to a gravitational field, but did not state that the direction was to the left. Few mentioned that helium was less dense than air and so would move right, in the same direction as the acceleration of the spaceship. Admittedly it is counter-intuitive.

c. (c)(i) and (ii) were an easy two marks. But in (c)(iii) far too many candidates just stated that the position of the star should also be measured at night, without realizing that at night the star would be still be 'close' to the sun and so not in the night sky. It was expected that they would mention measuring the star's position (at night!) when it is not 'close' to the sun - such as in six months time. More usually they referred, incorrectly, to measuring the distance to the star or measuring the mass of the sun. (c)(iv) was done well.

This question is about relativistic kinematics.

In a thought experiment, a train is moving at a speed of 0.950c relative to the ground. A pendulum attached to the ceiling of the train is set into oscillation.



An observer T on the train and an observer G on the ground measure the period of oscillation of the pendulum.

Observer G sees a second train moving towards the first train (*i.e.* towards the left) at a speed of 0.950c with respect to the ground. Calculate the relative speed of the trains.

Markscheme

$u' = \frac{-0.950\text{c} - 0.950\text{c}}{1 - (-0.950)(0.950\text{c})}$;

$u' = -0.999\text{c}$;

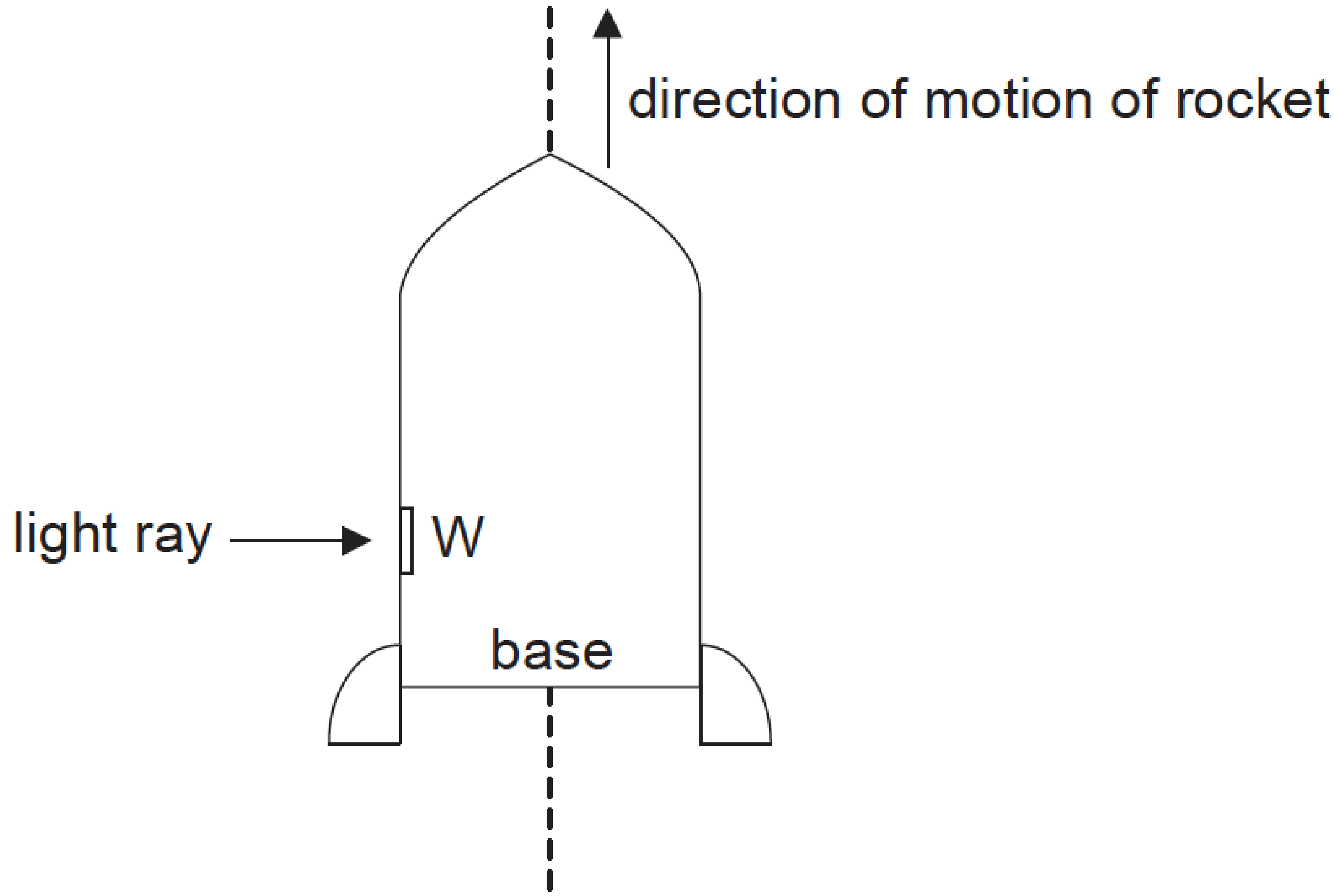
Accept calculation with left as positive to give +0.999c.

Examiners report

[N/A]

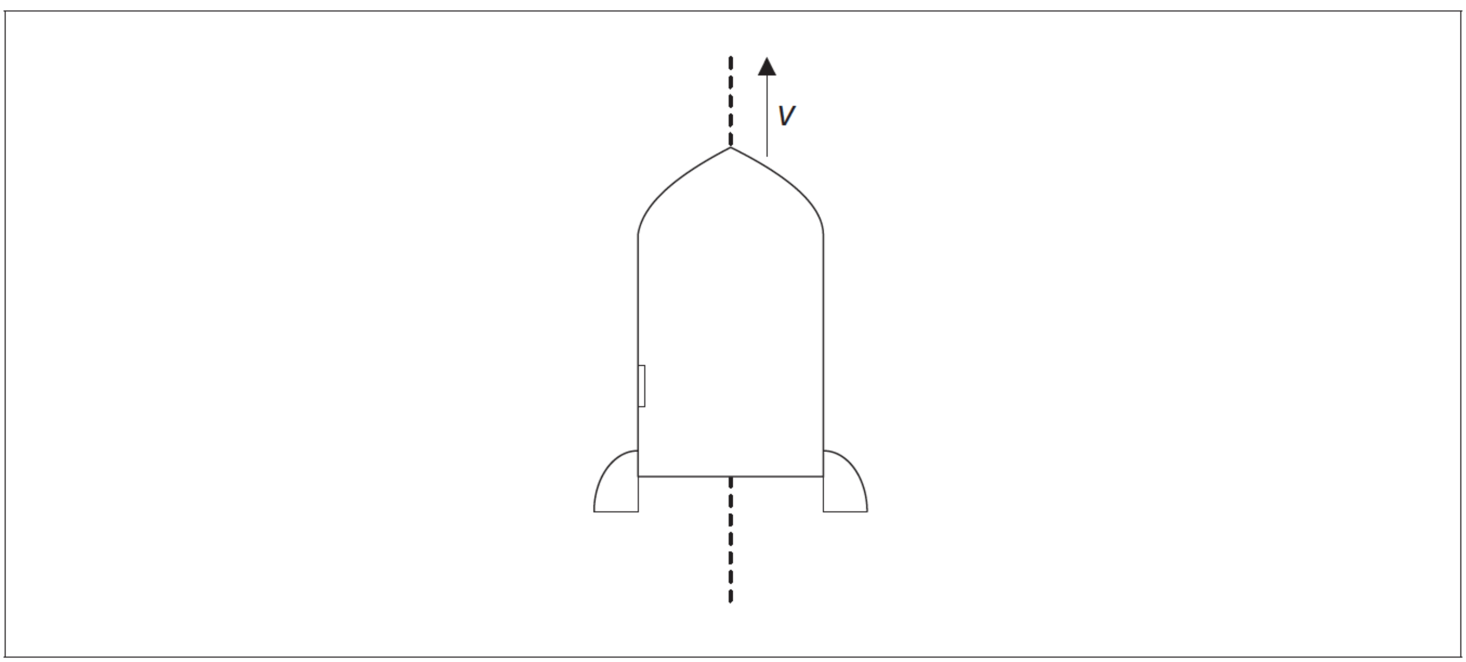
This question is about general relativity.

A rocket is in outer space far from all masses. It moves along the dotted line according to an inertial observer located outside the rocket.

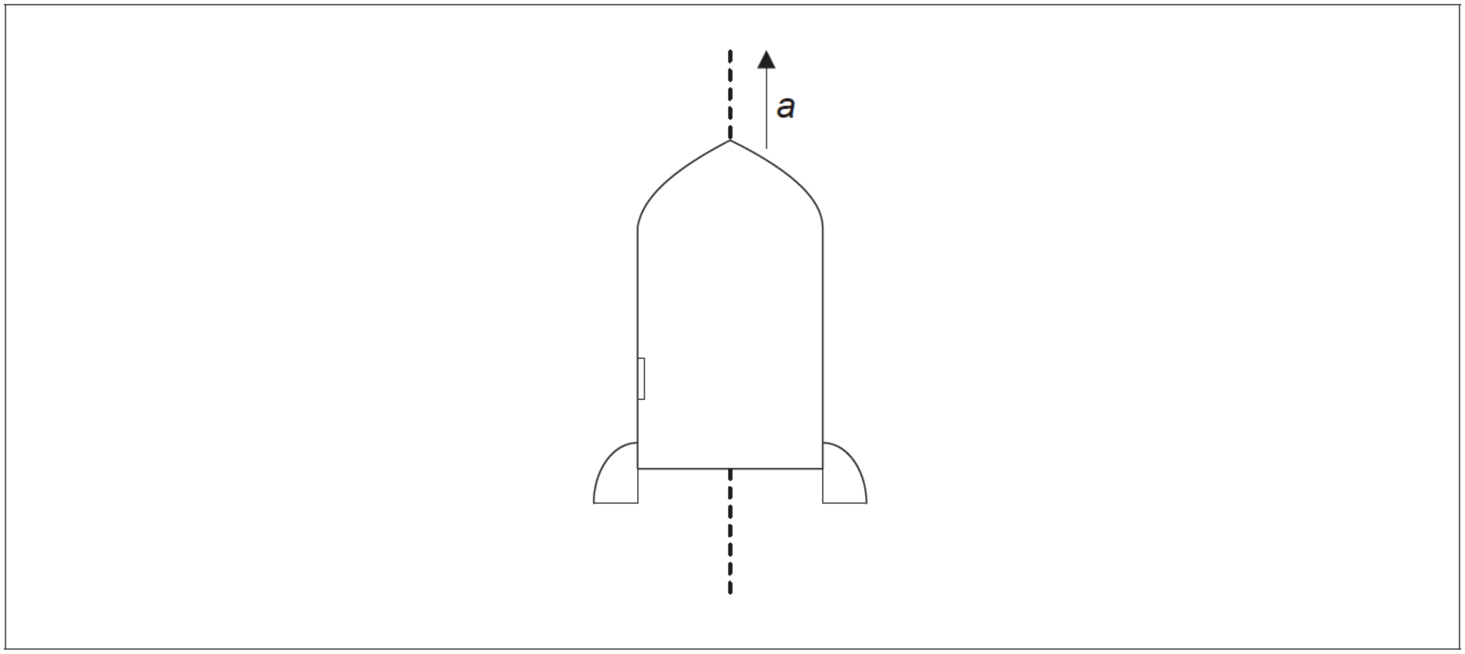


a. A ray of light is moving at right angles to the direction of the rocket according to the same inertial observer. The ray of light enters the rocket through a window W. Draw the path of the light ray according to an observer at rest inside the rocket, [2]

(i) when the rocket is moving at constant speed *v*.



(ii) when the rocket is moving at constant positive acceleration *a*.



b. The acceleration of the rocket in (a)(ii) is $12 ms^{-2}$. A gamma ray is emitted from the base of the rocket. The frequency at the base is f_{base}= 3.4×10^{18}Hz. A detector in the rocket is at a distance of 25m above the base. The frequency measured by the detector is $f_{detector}$. Determine the frequency shift $f_{detector} - f_{base}$. [3]

Markscheme

a. (i) any straight line with negative slope;

(ii) a downward curve (projectile like);

b. $\Delta f = \left(f_{\text{\\detector}} - f_{base} = \frac{fgh}{c^2} = \right) \frac{3.4\times10^{18}\times12\times25}{9.0\times10^{16}}$;

$=1.1\times10^4$ (Hz);

negative sign/red-shifted;

*Award **[3]** for a bald correct answer of -1.1×10^4 (Hz) .*

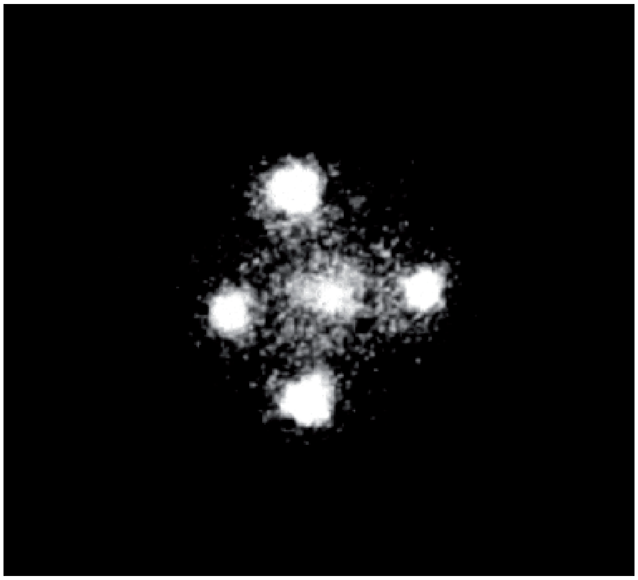
Examiners report

a. In part (a) most candidates drew a projectile path for (ii) but thought that light would travel horizontally for the rocket observer in (i).

b. Part (b) was an easy substitution into the gravitational frequency shift formula. Many forgot to square the speed of light or failed to give a negative value for Δf.

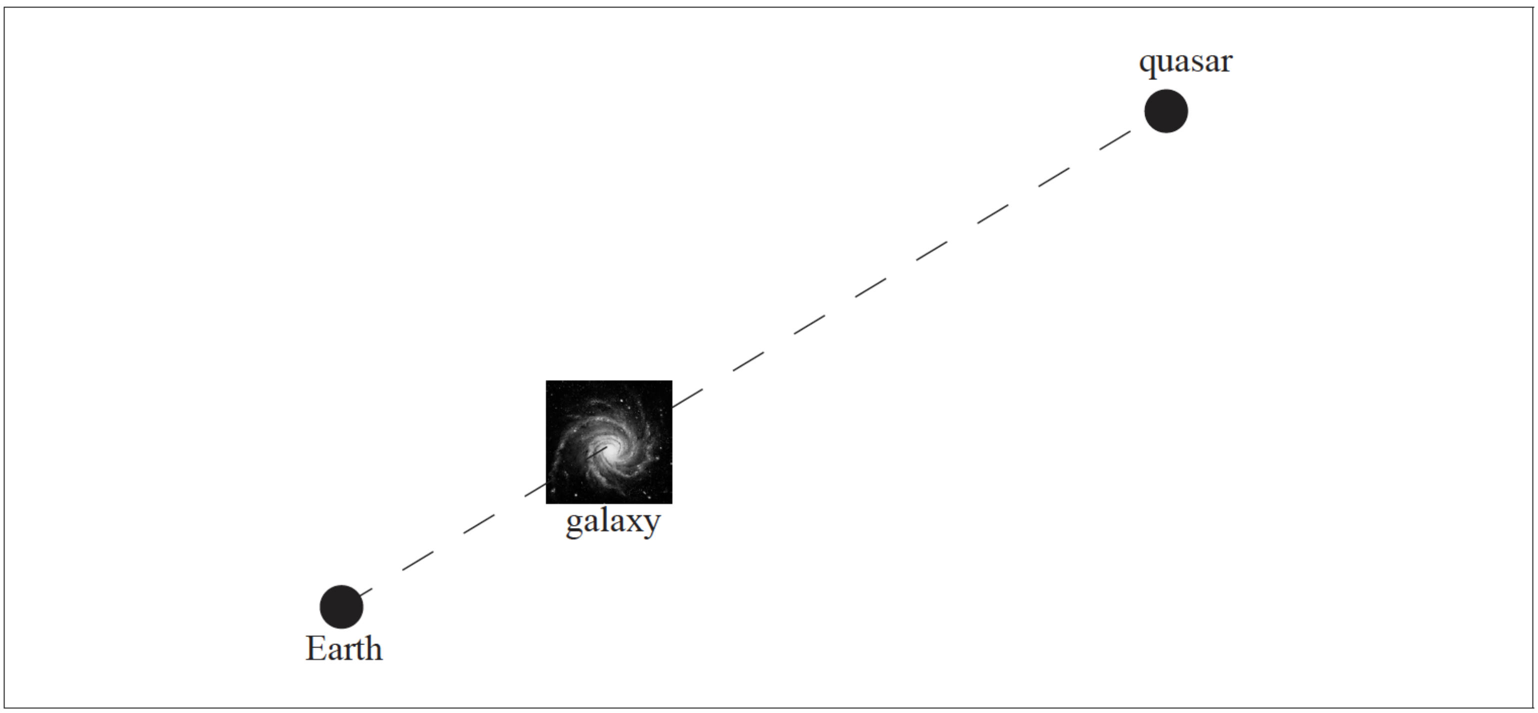
This question is about evidence that supports general relativity.

The astronomical photograph taken from Earth shows four separate images of a single distant quasar that appear to surround a galaxy. The galaxy is closer to Earth than the quasar.



[Source: www.hubblesite.org/newscenter/archive/releases/1990/20/image/a/]

Outline how one image of the quasar is formed. You may draw on the diagram below that shows the arrangement of the Earth, the galaxy and the quasar to support your answer.



[Source: adapted from www.sciencephoto.com/media/332699/enlarge]

Markscheme

mention of gravitational lensing;

galaxy has a very large mass/gravitational field;

this field/mass bends the direction of light emitted by the quasar;

spacetime is distorted by this field/mass;

Award ***[1 max]*** *for a diagram showing only a curve joining the quasar and the Earth.*
Award ***[3]*** *for an annotated diagram.*

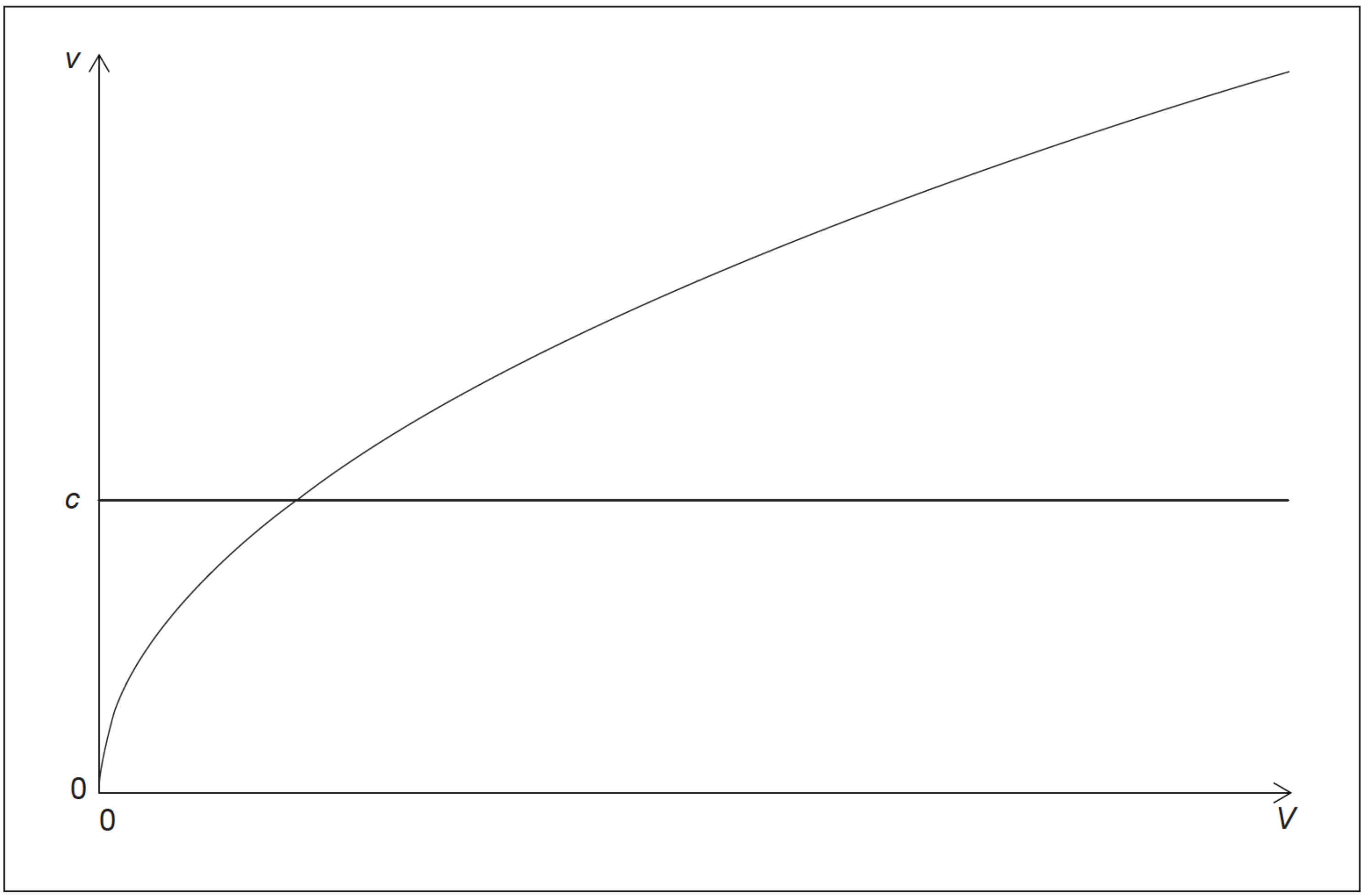
Examiners report

Many well prepared candidates realized that the light is bent. However, “outline” requires a brief account or summary, so a more detailed answer was required here. Information such as the galaxy has very large mass should be mentioned, at least implicitly.

This question is about relativistic energies.

a. Calculate the speed of an electron when its total energy is equal to five times its rest mass energy. [3]

b. The electron is accelerated from rest through a potential difference V. The graph shows how the speed v of the electron after acceleration varies with V assuming that Newtonian mechanics applies. [2]



On the graph, sketch a line to show the variation with V of v according to relativistic mechanics.

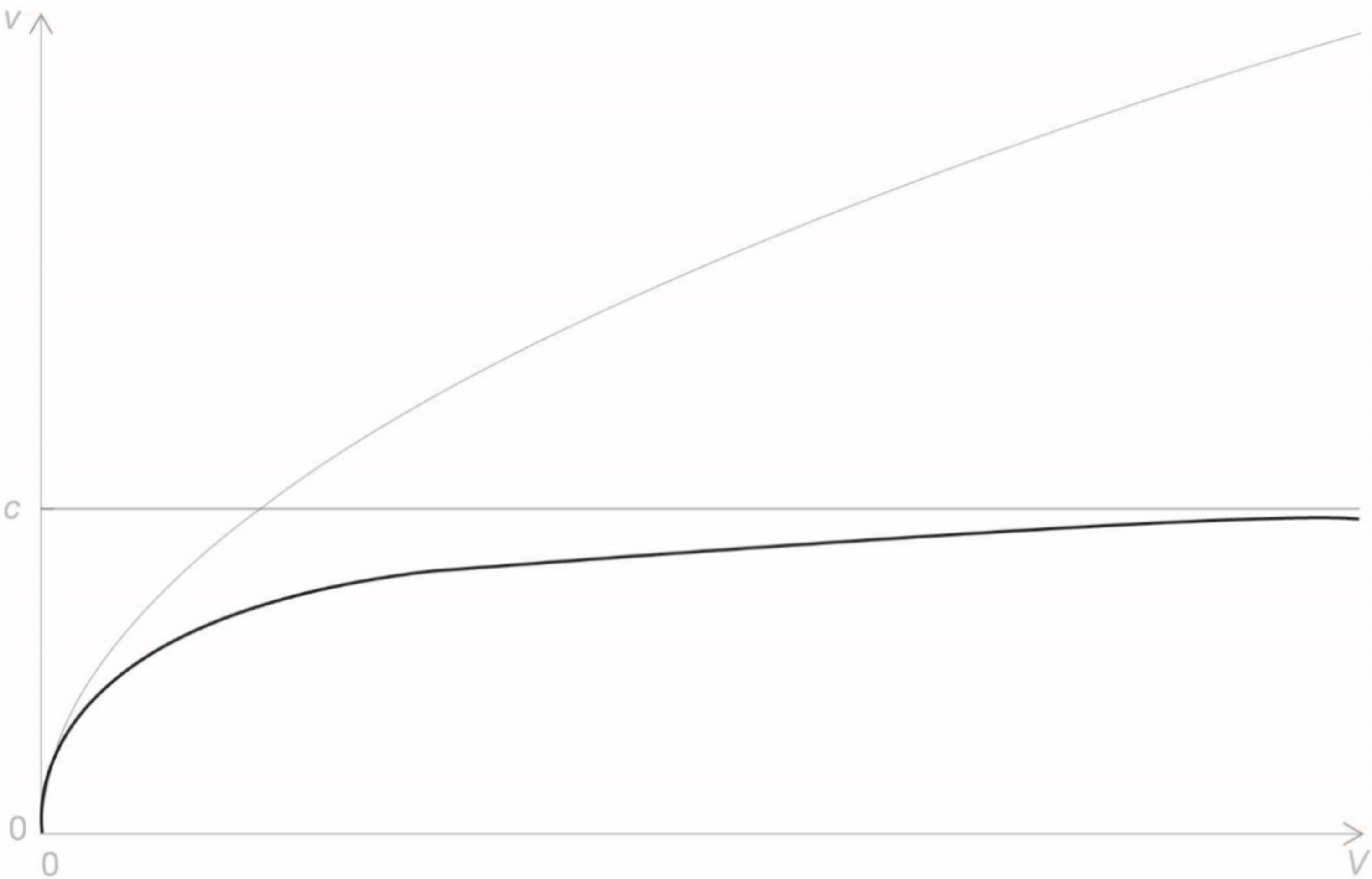
Markscheme

a. $E = \gamma mc^2 = 5mc^2$ **or** $\gamma = 5$;

so $\frac{v^2}{c^2} = 1 - 0.04$;

$v = 0.98c$;

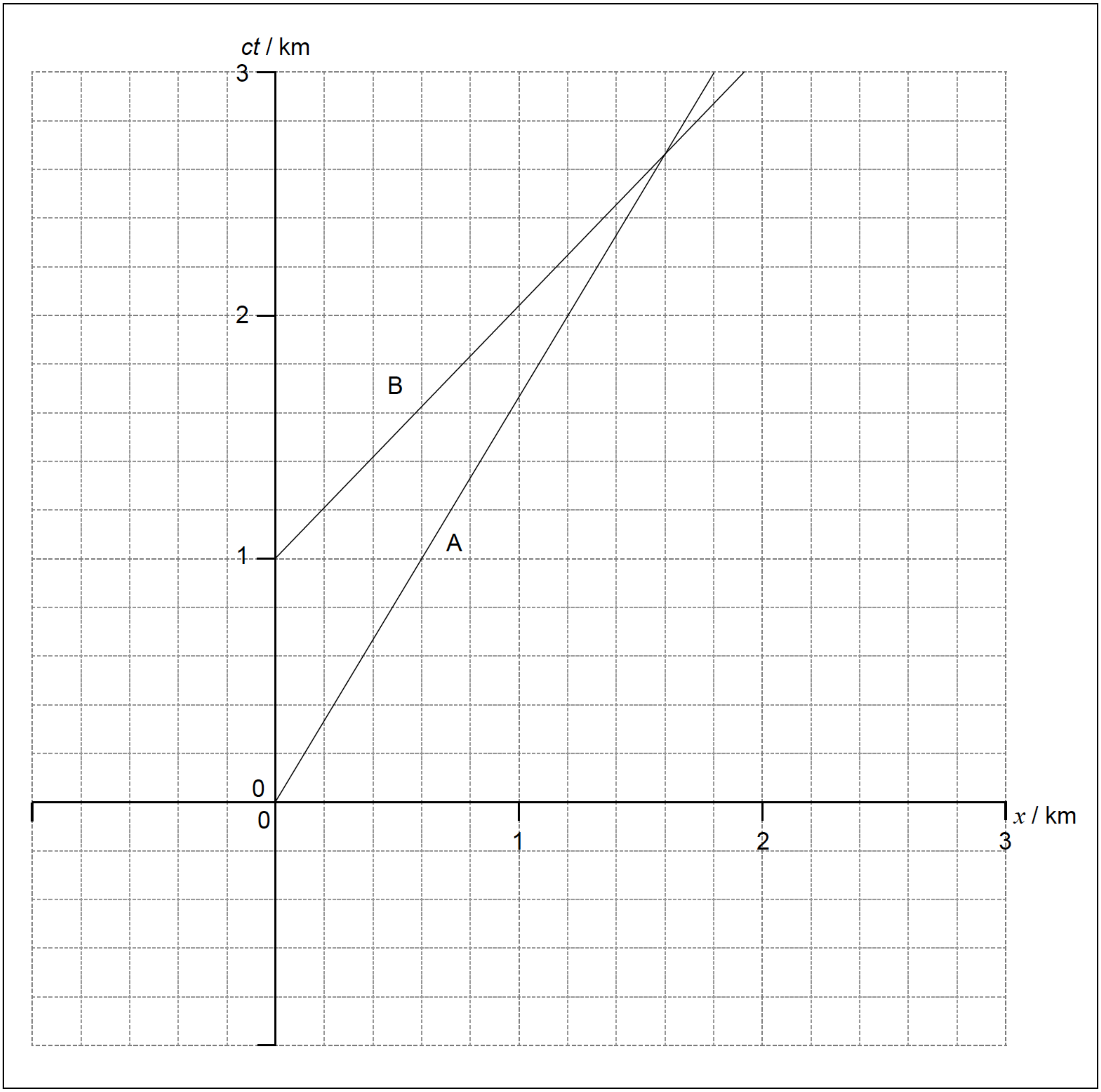
b. close to Newton for small v and below Newton for large V; asymptotic to c;



Examiners report

a. [N/A]
b. [N/A]

An observer on Earth watches two rockets, A and B. The spacetime diagram shows part of the motion of A and B in the reference frame of the Earth observer. A and B are travelling in the same direction.



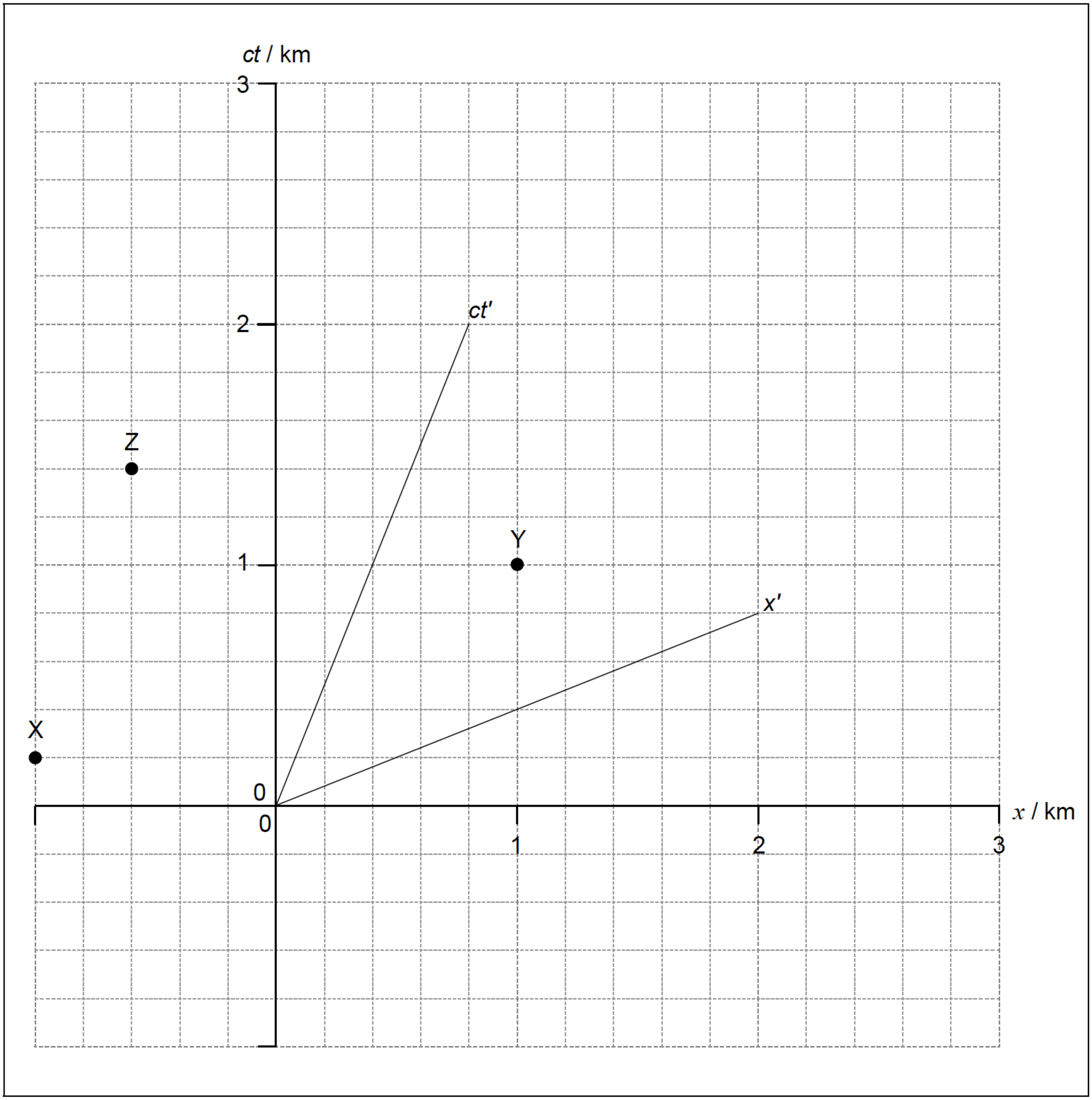
a. For the reference frame of the Earth observer, calculate the speed of rocket A in terms of the speed of light *c*. [2]

b. One rocket passes the other at event E. For the reference frame of the Earth observer, estimate [2]

(i) the space coordinate of E, in kilometres.

(ii) the time coordinate of E, in seconds.

c. Three flashing light beacons, X, Y and Z, are used to guide another rocket C. The flash events are shown on the spacetime diagram. The diagram shows the axes for the reference frames of Earth and of rocket C. The Earth observer is at the origin. [4]



Using the graph opposite, deduce the order in which

(i) the beacons **flash** in the reference frame of rocket C.

(ii) the Earth observer **sees** the beacons flash.

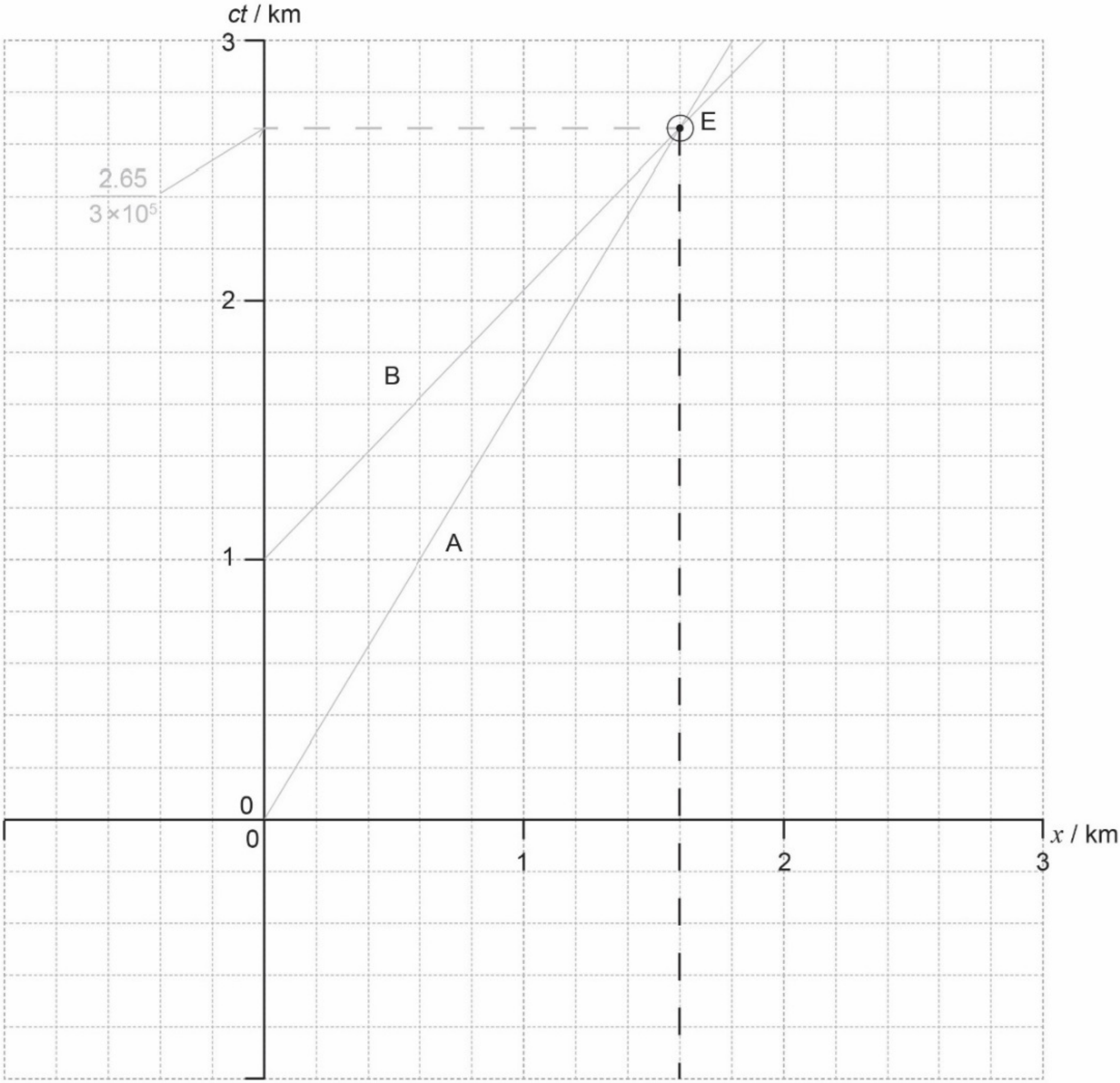
Markscheme

a. *Δct*=2.0km ***AND*** *Δx*=1.2km

$$v = \ll \frac{\Delta x}{\Delta ct} = \frac{1.2}{2.0} \gg = 0.60\mathrm{c}$$

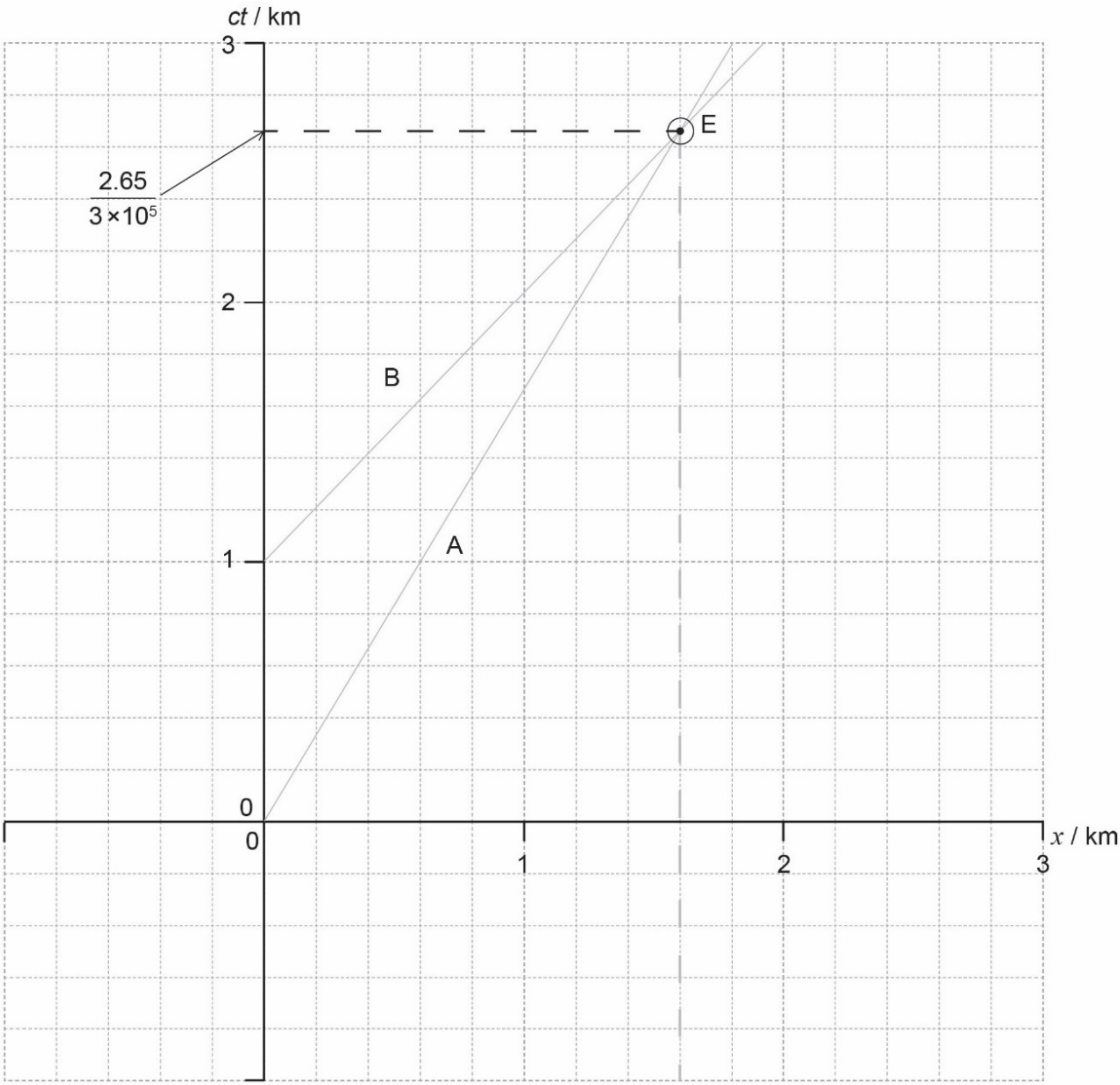
Allow any correct read-off from graph.
Accept answers from 0.57c to 0.63c.

b. (i) 1.6km



Allow ±0.1km.

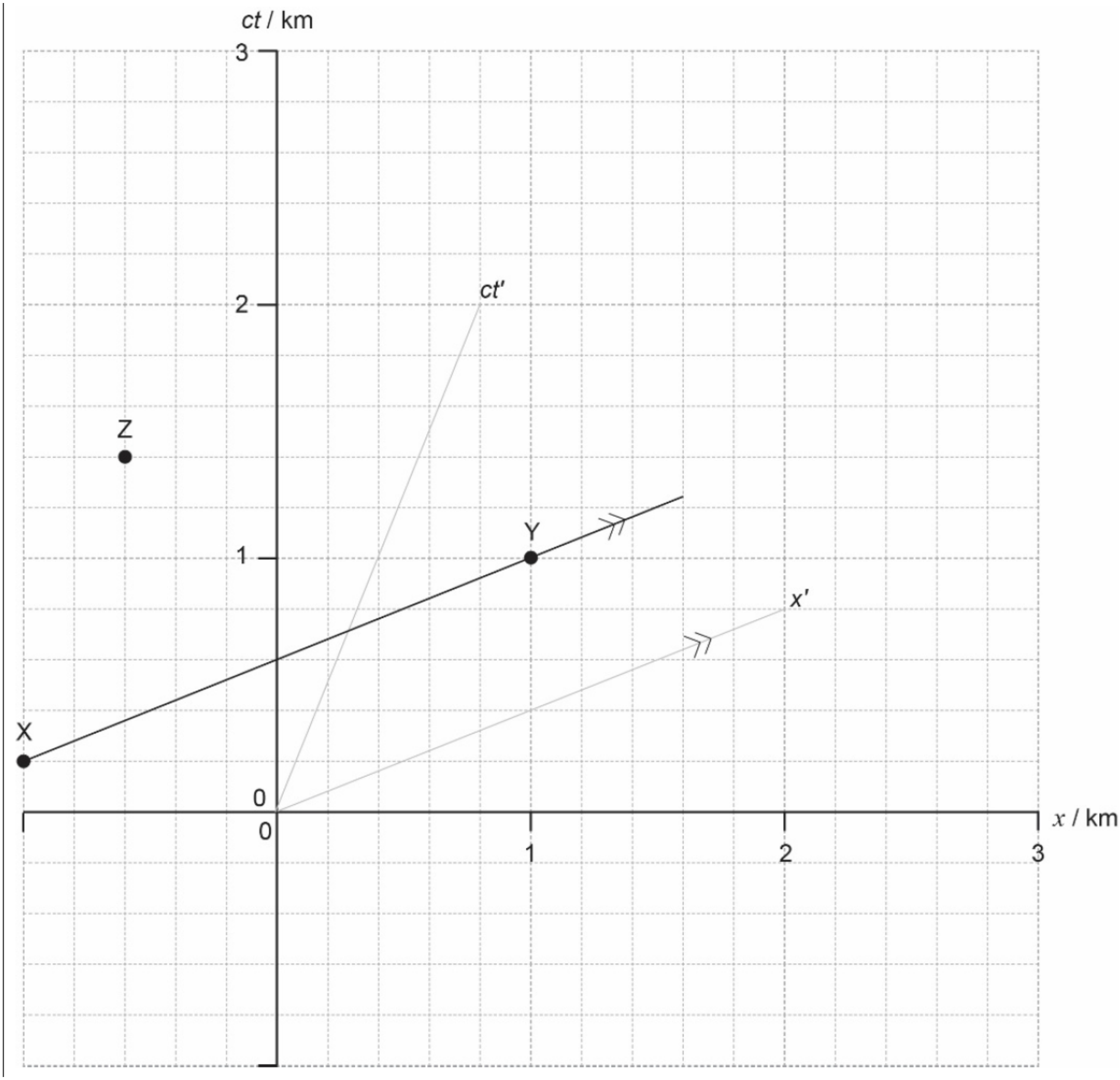
(ii) 8.8 μs



Allow ±0.5 μs.
Allow ECF, the answer can be calculated from answers to (a) and (b)(i).

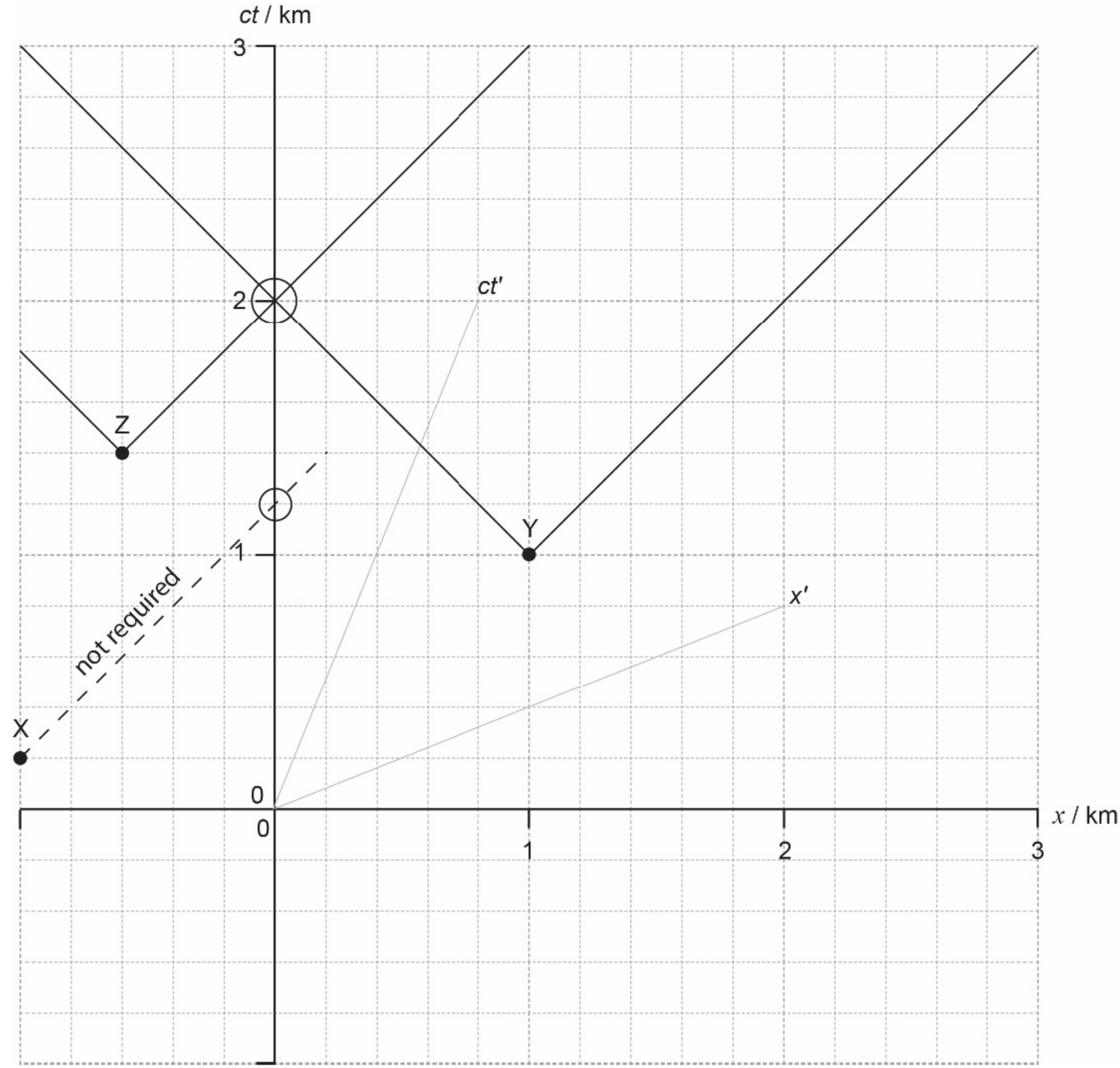
c. (i) events at same perpendicular distance from x' axis of rocket are simultaneous ***OR*** line joining X to Y is parallel to x' axis

X and Y simultaneously then Z



MP1 may be present on spacetime diagram.

(ii) constructs light cones to intersect worldline of observer
X first followed by Y and Z simultaneously



Only Y and Z light cones need to be seen.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]