

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Attempts $\vec{AB} = \vec{OB} - \vec{OA}$ or similar	M1	1.1b
	$\vec{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $ \vec{AB} = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$ \vec{AB} = 5\sqrt{5}$	A1ft	1.1b
		(2)	
(4 marks)			
Notes (a) M1: Attempts subtraction but may omit brackets A1: cao (allow column vector notation) (b) M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a) A1ft: $\vec{AB} = 5\sqrt{5}$ ft from their answer to (a) <i>Note that the correct answer implies M1A1 in each part of this question</i>			

Q2.

Question	Scheme	Marks	AOs
(a)	Attempts $\vec{AB} = \vec{OB} - \vec{OA}$ or similar	M1	1.1b
	$\vec{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $ \vec{AB} = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$ \vec{AB} = 3\sqrt{10}$	A1ft	1.1b
		(2)	
(4 marks)			
Notes			
(a) M1: Attempts subtraction either way around. This may be implied by one correct component $\vec{AB} = \pm 9\mathbf{i} \pm 3\mathbf{j}$ There must be some attempt to write in vector form. A1: cao (allow column vector notation but not the coordinate) Correct notation should be used. Accept $-9\mathbf{i} + 3\mathbf{j}$ or $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} -9\mathbf{i} \\ 3\mathbf{j} \end{pmatrix}$ (b) M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a) Note that $\vec{AB} = \sqrt{(9)^2 + (3)^2}$ is also correct. Condone missing brackets in the expression $\vec{AB} = \sqrt{-9^2 + (3)^2}$ Also allow a restart usually accompanied by a diagram. A1ft: $\vec{AB} = 3\sqrt{10}$ ft from their answer to (a) as long as it has both an $\mathbf{i}$ and $\mathbf{j}$ component. It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0 <i>Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question</i>			

Q3.

Question	Scheme	Marks	AOs
(a)	Angle $ACB = 33^\circ$	B1	1.1b
	Attempts $\{AB^2 = 8.2^2 + 15.6^2 - 2 \times 8.2 \times 15.6 \cos 33^\circ\}$	M1	1.1b
	Distance = awrt 9.8 {km}	A1	1.1b
		(3)	
(b)	- Explains that the road is not likely to be straight {and therefore the distance will be greater}. - Explains that there are likely to be objects in the way {that they must go around and therefore the distance travelled will be greater}. - The {bases of the} masts are not likely to lie in the same {horizontal} plane {and so the distance will be greater}.	B1	3.2b
		(1)	
(4 marks)			

Notes:

(a)

B1: 33 seen anywhere but allow $72 - 39$. May be indicated on a diagram (including incorrectly) or on the given Figure 1 and it might be named incorrectly.

M1: Uses the given model and attempts to use the cosine rule to find the distance or distance²
Award for $8.2^2 + 15.6^2 - 2 \times 8.2 \times 15.6 \cos \dots$ where $\dots$ must be a value.

A1: awrt 9.8 {km} isw

(a) **Alternative**

B1: $\{AB = \pm \left(\frac{15.6 \cos 51 - 8.2 \cos 18}{15.6 \sin 51 - 8.2 \sin 18} \right) \text{ or } \pm \left(\frac{15.6 \sin 39 - 8.2 \sin 72}{15.6 \cos 39 - 8.2 \cos 72} \right) \text{ o.e.}$

May be implied by calculation that leads to $\left(\frac{\text{awrt } \pm 2.0}{\text{awrt } \pm 9.6} \right)$ e.g. $\left(\frac{9.8}{12.1} \right) - \left(\frac{7.8}{2.5} \right)$

Note: they may find components separately and condone, e.g., $\left(\frac{\text{awrt } \pm 9.6}{\text{awrt } \pm 2.0} \right)$

M1: Attempts to find $\overline{AB}$ (as above) and uses Pythagoras to find distance or distance²

A1: awrt 9.8 {km} isw

(b)

B1: A valid reason based on the assumptions, i.e., the plane is not really horizontal or the journey not being in a straight line.

Do not accept answers referencing the accuracy of the answer to part (a) being to 1d.p. or the accuracy of the values given in the question, but ignore if there is a separate, valid reason.

Some examples:

“Because it is unlikely the bearings are exact” – B0 see above.

“Because they may not walk in a straight line because they could take another longer or shorter route as their route could be more curved” – B0 – incorrect comment about there being a shorter route.

“Because they won’t travel in one direction due to the roads” – B1 BOD

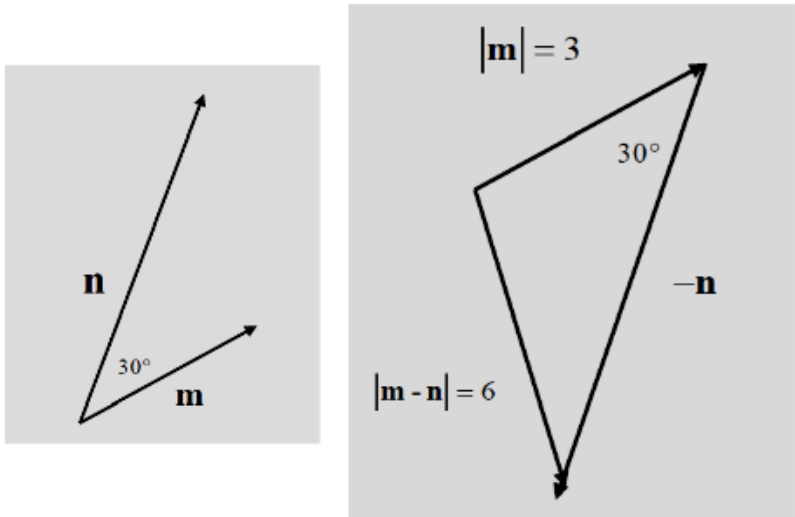
“Impossible and unrealistic to walk in a straight line” – B1

Q4.

Question	Scheme	Marks	AOs
(a)	Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}$, $\overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i} - 10\mathbf{j}) = -2 \times (12\mathbf{i} + 5\mathbf{j})$	M1	1.1b
	Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point O .	A1	2.4
		(2)	
(b)	Attempts distance $AB = \sqrt{(12+24)^2 + (10+5)^2}$	M1	1.1b
	Attempts speed = $\frac{\sqrt{(12+24)^2 + (10+5)^2}}{4}$	dM1	3.1a
	Speed = 9.75 ms^{-1}	A1	3.2a
		(3)	
(5 marks)			
Alt(a)	Attempts to find the equation of the line which passes through A and B E.g. $y - 5 = \frac{5+10}{12+24}(x-12)$ ($y = \frac{5}{12}x$)	M1	1.1b
	Shows that when $x = 0$, $y = 0$ and concludes the stone passes through the point O .	A1	2.4

Notes	
(a)	M1: Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}$, $\overrightarrow{OB}$ or $\overrightarrow{AB}$ either way around. E.g. States that $(-24\mathbf{i} - 10\mathbf{j}) = -2 \times (12\mathbf{i} + 5\mathbf{j})$ Alternatively, allow an attempt finding the gradient using any two of AO, OB or AB Alternatively attempts to find the equation of the line through A and B proceeding as far as $y = \dots x$ Condone sign slips. A1: States that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ or as AO is parallel to OB (and the stone is travelling in a straight line) the stone passes through the point O. Alternatively, shows that the point $(0,0)$ is on the line and concludes (the stone) passes through the point O.
(b)	M1: Attempts to find the distance AB using a correct method. Condone slips but expect to see an attempt at $\sqrt{a^2 + b^2}$ where a or b is correct dM1: Dependent upon the previous mark. Look for an attempt at $\frac{\text{distance } AB}{4}$ A1: 9.75 ms^{-1} Requires units

Q5.

Question	Scheme	Marks	AOs
(i)	Explains that a and b lie in the same direction oe	B1	2.4
		(1)	
(ii)		M1	1.1b
	Attempts $\frac{\sin 30^\circ}{6} = \frac{\sin \theta}{3}$	M1	3.1a
	$\theta = \text{awrt } 14.5^\circ$	A1	1.1b
	Angle between vector m and vector m - n is awrt 135.5°	A1	3.2a
		(4)	
(5 marks)			

Notes

(i)

B1: Accept any valid response E.g The lines are collinear. Condone "They are parallel"

Mark positively. ISW after a correct answer

Do not accept "the length of line a + b is the same as the length of line a + the length of line b

Do not accept $|a|$ and $|b|$ are parallel.

(ii)

M1: A triangle showing 3, 6 and 30° in the correct positions.

Look for 6' opposite 30° with another side of 3.

Condone the triangle not being obtuse angled and not being to scale.

Do not condone negative lengths in the triangle. This would automatically be M0

M1: Correct sine rule statement with the sides and angles in the correct positions.

If a triangle is drawn then the angles and sides must be in the correct positions.

This is not dependent so allow recovery from negative lengths in the triangle.

If the candidate has not drawn a diagram then correct sine rule would be M1 M1

Do not accept calculations where the sides have a negative length. Eg $\frac{\sin 30^\circ}{6} = \frac{\sin \theta}{-3}$ is M0

A1: $\theta = \arcsin 14.5^\circ$

A1: CSO $\arcsin 135.5^\circ$

Q6.

Question Number	Scheme	Marks	AOs
(a)	Attempts a scale factor $\frac{10}{4}, \frac{4}{10}$ o.e.	M1	3.1a
	$p = -15$	A1	1.1b
		(2)	
(b)	Attempts either $ \mathbf{a} + \mathbf{c} = \left \begin{pmatrix} 4+q \\ -3 \end{pmatrix} \right = \sqrt{(4+q)^2 + 9}$ or $ \mathbf{d} = \sqrt{30^2 + 6^2} \quad \{ = \sqrt{936} = 6\sqrt{26} \}$	M1	1.1b
	$2 \mathbf{a} + \mathbf{c} = \mathbf{d} \Rightarrow 4((4+q)^2 + 9) = 30^2 + 6^2 \quad \{ = 936 \}$	dM1	1.1b
	$(4+q)^2 = 225 \Rightarrow 4+q = \pm 15 \Rightarrow q = \dots$ or $q^2 + 8q - 209 = 0 \Rightarrow q = \dots$	ddM1	2.1
	$\Rightarrow q = 11, -19$	A1	1.1b
		(4)	
(6 marks)			
Notes:			

(a)

M1: Uses the fact that the vectors are parallel and attempts a scale factor.

May be implied by e.g. $\begin{pmatrix} 4 \\ -6 \end{pmatrix} = \lambda \begin{pmatrix} 10 \\ p \end{pmatrix} \Rightarrow \lambda = \dots$ or $4 = 10\lambda \Rightarrow \lambda = \dots$

A1: $p = -15$ Sight of this with no incorrect working scores both marks including e.g. $\begin{pmatrix} 10 \\ -15 \end{pmatrix}$

Note $p = -15\mathbf{j}$ scores M1A0.

(b)

M1: Attempts a correct method of finding the modulus or modulus² for either vector $\mathbf{a} + \mathbf{c}$ or $\mathbf{d}$.

If using $\mathbf{a} + \mathbf{c}$ then they must clearly be adding the vectors.

Condone incorrect labelling, e.g., $|\mathbf{a} + \mathbf{c}| = (4+q)^2 + 9$ as it is not a “show that”.

dM1: Uses the information to set up a quadratic equation in q with square roots removed.

Dependent on the previous method mark.

When finding $|\mathbf{a} + \mathbf{c}|$ they must clearly be adding the vectors.

Correct algebraic work is required but condone poor squaring e.g. $(4+q)^2 = 16 + q^2$ and

numerical slips such as failure to square the 2 i.e. $2((4+q)^2 + 9) = 30^2 + 6^2$ or omission of the

2 i.e. $(4+q)^2 + 9 = 30^2 + 6^2$

Similarly, condone failure to multiply both components of $\mathbf{a} + \mathbf{c}$ by 2 leading to e.g.

$$\sqrt{(8+2q)^2 + 3^2} = \sqrt{30^2 + 6^2}$$

Use of $|\mathbf{a} + \mathbf{c}| \rightarrow |\mathbf{a}| + |\mathbf{c}|$ is dM0, as is poor square rooting e.g. $\sqrt{q^2 + 8q + 25} = \sqrt{q^2 + 8q} + 5$

ddM1: Full and complete attempt to find at least one value for q , condoning failure to square the 2 or similar.

Dependent on both previous method marks.

If they expand the brackets, then for this mark they must achieve a 3TQ and go on to solve it using the usual rules. They may also use a calculator, in which case values will need checking.

Note that the correct 3TQ is $4q^2 + 32q - 836 = 0$ or $q^2 + 8q - 209 = 0$

Note also that if they fail to square the 2 then they are likely to achieve $2q^2 + 16q - 886 = 0$ or $q^2 + 8q - 443 = 0$ which leads to $q = -4 \pm 3\sqrt{51}$

A1: cao Both $q = 11, -19$ and no others. Must be simplified. A0 if either value is rejected.

Note: You may see correct answers coming from incorrect work,

$$\text{e.g. } 4((4+q)^2 - 9) = 30^2 - 6^2 \text{ which will score M0dM0ddM0A0}$$

Alt(b): You may see perfectly valid responses such as

$$2 \begin{vmatrix} 4+q \\ -3 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix} \Rightarrow \begin{vmatrix} 8+2q \\ -6 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix} \Rightarrow \pm(8+2q) = 30 \Rightarrow q = 11, -19$$

M1: Sets up the modulus equation $\begin{vmatrix} 8+2q \\ -6 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix}$ which may be implied by either $|8+2q| = 30$

or both $8+2q = 30$ and $-8-2q = 30$

Condone a slip on the i component e.g. $\begin{vmatrix} 8+q \\ -6 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix}$

dM1: Deduces **either** linear equation in q following the award of the previous mark.

ddM1: For solving **both** linear equations in q

A1: cao Both $q = 11, -19$ and no others. Must be simplified. A0 if either value is rejected.

Question	Scheme	Marks	AOs
(a)	$\left(\left \overrightarrow{OB} \right = \right) \sqrt{(\dots)^2 + (\pm 2)^2} \text{ or } \left(\left \overrightarrow{OB} \right = \right) \sqrt{(\pm 6)^2 + (\dots)^2}$ or $\left(\left \overrightarrow{OB} \right ^2 = \right) (\dots)^2 + (\pm 2)^2 \text{ or } \left(\left \overrightarrow{OB} \right ^2 = \right) (\pm 6)^2 + (\dots)^2$	M1	1.1b
	$\left(\left \overrightarrow{OB} \right = \right) \sqrt{(\pm 2)^2 + (\pm 6)^2}$	dM1	1.1b
	$\sqrt{40} \text{ or } 2\sqrt{10}$	A1	1.1b
		(3)	
(b)	$\left \overrightarrow{OA} \right = \sqrt{73} \text{ or } \left \overrightarrow{AB} \right = \sqrt{29}$	B1	1.1b
	$40 = 73 + 29 - 2\sqrt{73}\sqrt{29} \cos OAB \Rightarrow \cos OAB = \frac{73 + 29 - 40}{2\sqrt{73}\sqrt{29}}$ $\Rightarrow OAB = \cos^{-1} \left(\frac{73 + 29 - 40}{2\sqrt{73}\sqrt{29}} \right)$ or $\cos OAB = \frac{73 + 29 - 40}{2\sqrt{73}\sqrt{29}} \Rightarrow OAB = \dots$	M1	3.1a
	$OAB = 47.6^\circ$	A1	1.1b
		(3)	
(6 marks)			

Notes:

Note that marks in (a) can be scored in (b) as long as they are not contradictory.
 Note that they are not asked for $\overrightarrow{OB}$ in (a) but $\left| \overrightarrow{OB} \right|$. As such, all they need are the magnitudes of the components of $\overrightarrow{OB}$ to find $\left| \overrightarrow{OB} \right|$ so you can ignore if $\overrightarrow{OB}$ is correct or not in both parts and full marks can be awarded even if there are sign errors in their $\overrightarrow{OB}$ if they write it as a vector.

(a)

M1: Attempts $\left| \overrightarrow{OB} \right|$ or $\left| \overrightarrow{OB} \right|^2$ with one component correct and the other component non-zero.

Allow $\sqrt{(\pm 2)^2 + (\dots)^2}$ or $\sqrt{(\pm 6)^2 + (\dots)^2}$ or $(\dots)^2 + (\pm 2)^2$ or $(\pm 6)^2 + (\dots)^2$

and condone e.g. -2^2 or -6^2

But it must clearly not be an attempt at e.g. $\left| \overrightarrow{AB} \right|$ e.g. $\sqrt{5^2 + 2^2}$

dM1: Complete and correct method for $\left| \overrightarrow{OB} \right|$ i.e. $\left| \overrightarrow{OB} \right| = \sqrt{(\pm 2)^2 + (\pm 6)^2}$

A1: $\sqrt{40}$ or $2\sqrt{10}$ only but isw if they then use decimals.

Beware in (b) that assuming OAB is right angled can give answers that look approximately correct e.g. $\sin OAB = \frac{OB}{AB} = \frac{\sqrt{40}}{\sqrt{73}} \Rightarrow OAB = \sin^{-1} \frac{\sqrt{40}}{\sqrt{73}} = 47.75\ldots$ but is an incorrect method.

In (b) mark the method that is most successful.

(b) Way 1: Cosine rule

B1: Finds either of $|\overrightarrow{OA}| = \sqrt{73}$ or $|\overrightarrow{AB}| = \sqrt{29}$ allow for sight of these values even if not associated with a vector. They may be seen on a diagram or embedded in an attempt at the cosine rule. May be implied by decimal values (see diagram)

M1: A complete and correct method for finding angle OAB with their OA , OB and AB .

Correct attempt at the cosine rule leading to a value for angle OAB using arccos.

Following the correct use of the cosine rule, if a value for angle OAB is just written down or there is no evidence of arccos, you may need to check.

Following the correct use of the cosine rule, sufficient evidence could be e.g.

$$\cos OAB = k \Rightarrow OAB = \cos^{-1} k = \dots$$

A1: awrt 47.6° Condone omission of degrees symbol.

(b) Way 2: Right angled triangles

B1: Finds any of $\tan^{-1}\left(\frac{8}{3}\right) = 69.4^\circ$, $\tan^{-1}\left(\frac{2}{5}\right) = 21.8^\circ$, $\tan^{-1}\left(\frac{3}{8}\right) = 20.6^\circ$, $\tan^{-1}\left(\frac{5}{2}\right) = 68.2^\circ$

May be implied.

M1: A complete and correct method for finding angle OAB .

e.g. attempts $\tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{8}\right)$ or $\tan^{-1}\left(\frac{8}{3}\right) - \tan^{-1}\left(\frac{2}{5}\right)$ or $90^\circ - \tan^{-1}\left(\frac{3}{8}\right) - \tan^{-1}\left(\frac{2}{5}\right)$

leading to a value for angle OAB .

A1: awrt 47.6° Condone omission of degrees symbol.

(b) Way 3: Scalar product

B1: Finds $\overrightarrow{AO} \cdot \overrightarrow{AB} = \begin{pmatrix} -3 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 2 \end{pmatrix} = 15 + 16 = 31$

Allow $\pm \overrightarrow{AO} \cdot \pm \overrightarrow{AB} = \pm 31$

M1: A complete and correct method for finding angle OAB with their OA , OB and AB .

$$\text{e.g. } 31 = |\overrightarrow{AO}| |\overrightarrow{AB}| \cos OAB = \sqrt{3^2 + 8^2} \sqrt{5^2 + 2^2} \cos OAB \Rightarrow \cos OAB = \frac{31}{\sqrt{73} \sqrt{29}} \Rightarrow OAB = \dots$$

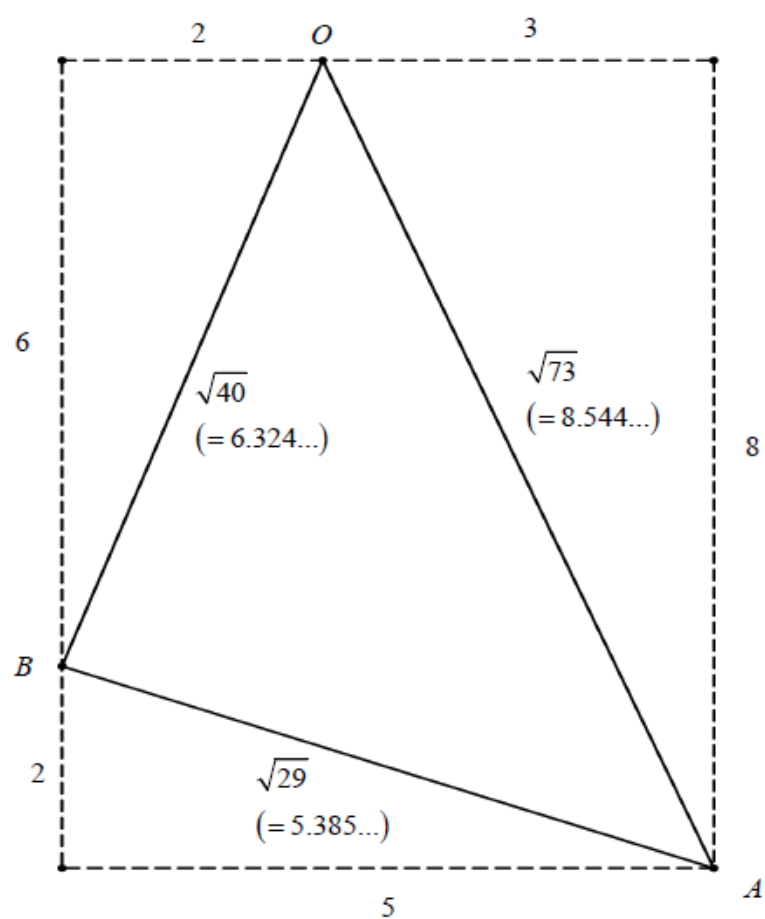
$$\text{If e.g. } \overrightarrow{OA} \cdot \overrightarrow{AB} \text{ is attempted then they need to find e.g. } OAB = 180^\circ - \cos^{-1} \frac{-31}{\sqrt{73} \sqrt{29}}$$

Following the correct use of the scalar product, if a value for angle OAB is just written down or there is no evidence of arccos, you may need to check.

A1: awrt 47.6° Condone omission of degrees symbol.

There may be other methods for finding angle OAB .

For reference:



Q8.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ} = 13\mathbf{i} - 15\mathbf{j} - (3\mathbf{i} + 5\mathbf{j})$	M1	1.1a
	$= 10\mathbf{i} - 20\mathbf{j}$	A1	1.1b
		(2)	
(b)	$ \overrightarrow{QR} = \sqrt{10^2 + (-20)^2}$	M1	2.5
	$= 10\sqrt{5}$	A1ft	1.1b
		(2)	
(c)	$\overrightarrow{PS} = \overrightarrow{PQ} + \frac{3}{5}\overrightarrow{QR} = 3\mathbf{i} + 5\mathbf{j} + \frac{3}{5}(10\mathbf{i} - 20\mathbf{j}) = \dots$ or $\overrightarrow{PS} = \overrightarrow{PR} + \frac{2}{5}\overrightarrow{RQ} = 13\mathbf{i} - 15\mathbf{j} + \frac{2}{5}(-10\mathbf{i} + 20\mathbf{j}) = \dots$	M1	3.1a
	$= 9\mathbf{i} - 7\mathbf{j}$	A1	1.1b
		(2)	
(6 marks)			

Notes
(a) M1: Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component. eg $10\mathbf{i} - 10\mathbf{j}$ on its own can score M1. A1: Correct answer. Allow $10\mathbf{i} - 20\mathbf{j}$ and $\begin{pmatrix} 10 \\ -20 \end{pmatrix}$ but not $\begin{pmatrix} 10\mathbf{i} \\ -20\mathbf{j} \end{pmatrix}$ (b) M1: Correct use of Pythagoras. Attempts to “square and add” before square rooting. The embedded values are sufficient. Follow through on their $\overrightarrow{QR}$ A1ft: $10\sqrt{5}$ following (a) of the form $\pm 10\mathbf{i} \pm 20\mathbf{j}$ (c) M1: Full attempt at finding a $\overrightarrow{PS}$. They must be attempting $\overrightarrow{PQ} \pm \frac{3}{5}\overrightarrow{QR}$ or $\overrightarrow{PS} = \overrightarrow{PR} \pm \frac{2}{5}\overrightarrow{RQ}$ but condone arithmetical slips after that. Cannot be scored for just stating eg $\overrightarrow{PQ} \pm \frac{3}{5}\overrightarrow{QR}$ Follow through on their $\overrightarrow{QR}$. Terms do not need to be collected for this mark. If no method shown it may be implied by one correct component following through on their $\overrightarrow{QR}$

A1: Correct vector as shown. Allow $9\mathbf{i} - 7\mathbf{j}$ and $\begin{pmatrix} 9 \\ -7 \end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix} 9\mathbf{i} \\ -7\mathbf{j} \end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix} 10\mathbf{i} \\ -20\mathbf{j} \end{pmatrix}$

Alt (c) (Expressing $\overrightarrow{PS}$ in terms of the given vectors) They must be attempting $\frac{2}{5}\overrightarrow{PQ} + \frac{3}{5}\overrightarrow{PR}$

M1: $(\overrightarrow{PS} = \overrightarrow{PQ} + \frac{3}{5}\overrightarrow{QR} = \overrightarrow{PQ} + \frac{3}{5}(\overrightarrow{PR} - \overrightarrow{PQ}))$
 $\Rightarrow \frac{2}{5}\overrightarrow{PQ} + \frac{3}{5}\overrightarrow{PR} = \frac{2}{5}(3\mathbf{i} + 5\mathbf{j}) + \frac{3}{5}(13\mathbf{i} - 15\mathbf{j}) = \dots$

A1: Correct vector as shown. Allow $9\mathbf{i} - 7\mathbf{j}$ and $\begin{pmatrix} 9 \\ -7 \end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix} 9\mathbf{i} \\ -7\mathbf{j} \end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix} 10\mathbf{i} \\ -20\mathbf{j} \end{pmatrix}$

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-8\mathbf{i} + 9\mathbf{j}) - (10\mathbf{i} - 3\mathbf{j})$	M1	1.1b
	$= -18\mathbf{i} + 12\mathbf{j}$	A1	1.1b
		(2)	
(b)	$ \overrightarrow{AB} = \sqrt{18^2 + 12^2} = \sqrt{468}$	M1	1.1b
	$= 6\sqrt{13}$	A1	1.1b
		(2)	
(c)	For the key step in using the fact that BCA forms a straight line in an attempt to find "p" $\overrightarrow{AB} = \lambda \overrightarrow{BC} \Rightarrow -18\mathbf{i} + 12\mathbf{j} = 6\lambda\mathbf{i} + \lambda(p-9)\mathbf{j}$ with components equated leading to a value for λ and to $p = \dots$	M1	2.1
	(i) $p = 5$	A1	1.1b
	(ii) ratio = 2: 3	B1 (A1 on EPEN)	2.2a
		(3)	
(7 marks)			

Notes:

(a) **Must be seen in (a)**

M1: Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component.

Allow as coordinates for this mark. Condone missing brackets, e.g., $-8\mathbf{i} + 9\mathbf{j} - 10\mathbf{i} - 3\mathbf{j}$

A1: cao $-18\mathbf{i} + 12\mathbf{j}$ o.e. $\begin{pmatrix} -18 \\ 12 \end{pmatrix}$ Condone $\begin{pmatrix} -18 \\ 12 \end{pmatrix}$

Do not allow $\begin{pmatrix} -18\mathbf{i} \\ 12\mathbf{j} \end{pmatrix}$ or $(-18, 12)$ or $\begin{pmatrix} -18 \\ 12 \end{pmatrix}$ for the A1.

(b)

M1: Attempts to use Pythagoras' theorem on their vector from part (a). Allow restarts.

$|\overrightarrow{AB}| = \sqrt{18^2 + 12^2} = \sqrt{468}$ Note that -18 will commonly be squared as 18

May be implied by awrt 21.6 This will need checking if (a) is incorrect.

A1: cao $6\sqrt{13}$ May come from $\begin{pmatrix} \pm 18 \\ \pm 12 \end{pmatrix}$

(c)

M1: For the key step in using the fact that BCA forms a straight line in an attempt to find "p"

Condone sign slips. Award, for example, for $\pm \frac{p-9}{6} = \pm \frac{2}{3}$ leading to $p = \dots$

It is implied by $p = 5$ unless it comes directly from work that is clearly incorrect.

e.g., award for an attempt to use

- $\overrightarrow{AB} = \alpha \overrightarrow{AC} \Rightarrow -18\mathbf{i} + 12\mathbf{j} = -12\alpha\mathbf{i} + \alpha(p+3)\mathbf{j}$ with components equated leading to a value for α and to $p = \dots$
- gradient $BC = \text{gradient } BA = -\frac{2}{3}$ e.g., $\frac{p-9}{6} = \frac{9-3}{-8-10}$ leading to $p = \dots$
- triangles BCM and BAN are similar with lengths in a ratio 1:3. e.g., $p = 9 - \frac{1}{3} \times 12$ or $p = -3 + \frac{2}{3} \times 12$
- attempt to find the equation of line AB using both points (FYI line AB has equation $y = -\frac{2}{3}x + \frac{11}{3}$) and then sub in $x = -2$ leading to $p = \dots$
- $\frac{p+3}{12} = \frac{2}{3}$ or $\frac{p+3}{2} = 9-p$ leading to $p = \dots$

A1: $p = 5$ Correct answer implies both marks, unless it comes directly from work that is clearly incorrect.

B1: States ratio = 2: 3 or equivalent such as 1: 1.5 or 22:33
Note that 3:2 is incorrect but condone $\{\text{Area}\}AOB : \{\text{Area}\}AOC = 3: 2$
This might follow incorrect work or even incorrect p
For reference, area $AOC = 22$, area $AOB = 33$ and area $BOC = 11$

Q10.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{PQ} = (3-9)\mathbf{i} + (-5+8)\mathbf{j}$	M1	1.1a
	$= -6\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Gradient of $PQ = \frac{-5-8}{3-9} \left(= -\frac{1}{2} \right)$ and Gradient of $QR = \frac{18}{9} (= 2)$ or $ \overrightarrow{PQ} = \sqrt{(-6)^2 + 3^2} (= 3\sqrt{5})$ and $ \overrightarrow{QR} = \sqrt{9^2 + 18^2} (= 9\sqrt{5})$ and $ \overrightarrow{PR} = \sqrt{3^2 + 21^2} (= 15\sqrt{2})$	M1	3.1a
	e.g. shows that $-\frac{1}{2} \times 2 = -1$ and deduces angle $PQR = 90^\circ$ * or e.g. shows $ \overrightarrow{PQ} ^2 + \overrightarrow{QR} ^2 = \overrightarrow{PR} ^2$ and deduces angle $PQR = 90^\circ$ *	A1*	2.4
		(2)	
(c)	Attempts to find the length PQ and at least one of QR or PS using Pythagoras' Theorem correctly e.g. $ \overrightarrow{PQ} = \sqrt{(-6)^2 + 3^2}$ and either $ \overrightarrow{QR} = \sqrt{9^2 + 18^2}$ or $ \overrightarrow{PS} = \sqrt{27^2 + 54^2}$	M1	2.1
	$ \overrightarrow{PQ} = 3\sqrt{5}$ and either $ \overrightarrow{QR} = 9\sqrt{5}$ or $ \overrightarrow{PS} = 27\sqrt{5}$	A1ft	1.1b
	e.g. Area $= \frac{1}{2} \times (9\sqrt{5} + 27\sqrt{5}) \times 3\sqrt{5}$ or $\frac{1}{2} \times 4 \times 9\sqrt{5} \times 3\sqrt{5}$	dM1	3.1a
	$= 270$	A1	1.1b
		(4)	
(8 marks)			

Notes	
Note that work seen must be used in the relevant part. If there is a lack of labelling of parts then award the marks to the parts which leads to the highest total overall.	
(a)	
M1:	Attempts subtraction either way round (does not need to be evaluated). It cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component or sight of $\mp 6\mathbf{i} \pm 3\mathbf{j}$.
A1:	Correct answer. Allow $-6\mathbf{i} + 3\mathbf{j}$ or $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$ but do not allow $\begin{pmatrix} -6\mathbf{i} \\ 3\mathbf{j} \end{pmatrix}$ isw once a correct answer is seen.
(b)	Condone lack of labelling / poor notation for lengths/angles provided the intention is clear
M1:	Attempts to find the gradient of the line PQ and the gradient of the line QR . If they find the reciprocals of both they must be labelled e.g. $\frac{dx}{dy}$ o.e. (but not gradient or m)
	Do not allow sign slips for this mark. Alternatively they may find the lengths PQ , QR and PR or PQ^2 , QR^2 and PR^2

Be aware of Further Maths methods such as attempting the dot product

$$\begin{pmatrix} -6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 18 \end{pmatrix} = (-6 \times 9) + (3 \times 18)$$

A1*: Correct working and conclusion that angle $PQR = 90^\circ$

- Using gradients or their reciprocals they need to state or show that the product is equal to -1 o.e. or refer to the values being negative reciprocals of each other
- Using Pythagoras' Theorem they must state or show that $|PQ|^2 + |QR|^2 = |PR|^2$
- Using the cosine rule and finding angle $PQR = 90^\circ$
- Using the scalar dot product they must show that $\begin{pmatrix} -6 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 18 \end{pmatrix} = 0$

In all cases there must be some sort of minimal conclusion that angle $PQR = 90^\circ$ e.g. "hence right angle" or if they start with a preamble it is acceptable to state "hence proven", "QED" or a tick. Use of e.g. cosine rule resulting in 90° is sufficient.

(c) Condone lack of labelling / poor notation for lengths provided the intention is clear

M1: Correct use of Pythagoras' Theorem to find the length of PQ and at least one of QR or PS . Must be used or seen in (c) to score this mark. Condone working using rounded or truncated values.

A1ft: Correct length of PQ and at least one of QR or PS . Follow through on their vectors for PQ and PS but QR must be $\sqrt{405}$ or equivalent. Lengths do not need to be simplified but they must be exact. Must be used or seen in (c) to score this mark.

dM1: Correct method to find the area of the trapezium. It is dependent on the first method mark and the method to find any lengths must be correct.

This may be achieved by calculating $\frac{1}{2} \times 4 \times |QR| \times |PQ|$

Alternatively, they may find the area of a rectangle + triangle so look for:

$$\text{e.g. } |PQ| \times |QR| + \frac{1}{2} \times (|PS| - |QR|) \times |PQ| = \sqrt{45} \times 9\sqrt{5} + \frac{1}{2} \times 18\sqrt{5} \times \sqrt{45}$$

Note that there are other combinations of lengths to find the area of a rectangle and either add or subtract triangles as appropriate. Condone working using rounded values.

A1: 270

Alt (c) "Shoelace method" or other methods using position vectors

M1: Correct method to find either the position vector of R or the position vector of S . May be seen as coordinates. Check any diagram drawn.

A1: R has position vector $12i + 13j$ and S has position vector $36i + 46j$ (or equivalent). May be seen as coordinates. Check any diagram drawn.

dM1: Correct method to find the area of the trapezium via the "shoelace" method:

$$\begin{vmatrix} 9 & -8 \\ 3 & -5 \\ 12 & 13 \\ 36 & 46 \\ 9 & -8 \end{vmatrix} = \frac{1}{2} |(9 \times (-5) + 3 \times 13 + 12 \times 46 + 36 \times (-8)) - (3 \times (-8) + 12 \times (-5) + 36 \times 13 + 9 \times 46)|$$

$$= \frac{1}{2} |258 - 798|$$

A1: 270

