

HL Paper 3

Type Ia supernovae typically have a peak luminosity of around $5 \times 10^5 L_s$, where L_s is the luminosity of the Sun (3.8×10^{26} W). A type Ia supernova is observed with an apparent peak brightness of 1.6×10^{-6} W m⁻².

- a. Describe the formation of a type Ia supernova. [2]
- b.i. Show that the distance to the supernova is approximately 3.1×10^{18} m. [2]
- b.ii. State **one** assumption made in your calculation. [1]

Markscheme

- a. a white dwarf accretes mass «from a binary partner»

when the mass becomes more than the Chandrasekhar limit ($1.4M_s$) «then a supernova explosion takes place»

[2 marks]

$$b.i. d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{5 \times 10^5 \times 3.8 \times 10^{26}}{4\pi \times 1.6 \times 10^{-6}}}$$

$$d = 3.07 \times 10^{18} \text{ «m»}$$

At least 3 sig fig required for MP2.

[2 marks]

- b.ii. type Ia supernova can be used as standard candles

there is no dust absorbing light between Earth and supernova

their supernova is a typical type Ia

[1 mark]

Examiners report

a. [N/A]

b.i. [N/A]

b.ii. [N/A]

- a.i. Derive, using the concept of the cosmological origin of redshift, the relation [2]

$$T \propto \frac{1}{R}$$

between the temperature T of the cosmic microwave background (CMB) radiation and the cosmic scale factor R .

- a.ii. The present temperature of the CMB is 2.8 K. This radiation was emitted when the universe was smaller by a factor of 1100. Estimate the temperature of the CMB at the time of its emission. [2]
- b. State how the anisotropies in the CMB distribution are interpreted. [1]

Markscheme

a.i. the cosmological origin of redshift implies that the wavelength is proportional to the scale factor: $\lambda \propto R$

combining this with Wien's law $\lambda \propto \frac{1}{T}$

OR

use of $kT \propto \frac{hc}{\lambda}$

«gives the result»

Evidence of correct algebra is needed as relationship $T = \frac{k}{R}$ is given.

[2 marks]

a.ii. use of $T \propto \frac{1}{R}$

= 2.8 x 1100 x 3080 ≈ 3100 «K»

[2 marks]

b. CMB anisotropies are related to fluctuations in density which are the cause for the formation of structures/nebulae/stars/galaxies

OWTTE

[1 mark]

Examiners report

a.i. [N/A]

a.ii. [N/A]

b. [N/A]

a. Explain the evidence that indicates the location of dark matter in galaxies. [3]

b. Outline why a hypothesis of dark energy has been developed. [3]

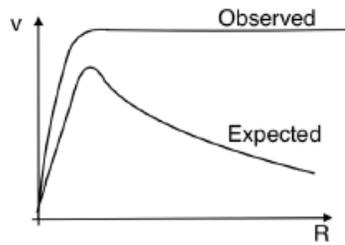
Markscheme

a. «rotational» velocity of stars are expected to decrease as distance from centre of galaxy increases

the observed velocity of outer stars is constant/greater than predicted

implying large mass on the edge «which is dark matter»

1st and 2nd marking points can be awarded from an annotated sketch with similar shape as the one below



[3 marks]

b. data from type 1a supernovae shows universe expanding at an accelerated rate

gravity was expected to slow down the expansion of the universe

OR

this did not fit the hypotheses at that time

dark energy counteracts/opposes gravity

OR

dark energy causes the acceleration

OWTTE

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

a. The Sun is a second generation star. Outline, with reference to the Jeans criterion (M_J), how the Sun is likely to have been formed. [4]

b. Suggest how fluctuations in the cosmic microwave background (CMB) radiation are linked to the observation that galaxies collide. [3]

c. Show that the critical density of the universe is [3]

$$\frac{3H^2}{8\pi G}$$

where H is the Hubble parameter and G is the gravitational constant.

Markscheme

a. interstellar gas/dust «from earlier supernova»

gravitational attraction between particles

if the mass is greater than the Jean's mass/ M_J , the interstellar gas coalesces

as gas collapses temperature increases leading to nuclear fusion

MP3 can be expressed in terms of potential and kinetic energy

- b. fluctuations in CMB due to differences in temperature/mass/density

during the inflationary period/epoch/early universe

leading to the formation of galaxies/stars/structures

gravitational interaction between galaxies can lead to collision

[Max 3 Marks]

- c. **ALTERNATIVE 1**

kinetic energy of galaxy $\frac{1}{2}mv^2 = \frac{1}{2}mH^2r^2$ «uses Hubble's law»

potential energy = $\frac{GMm}{r} = G\frac{4}{3}\pi r^3\rho\frac{m}{r}$ «introduces density»

KE=PE to get expression for critical ρ

ALTERNATIVE 2

escape velocity of distant galaxy $v = \sqrt{\frac{2GM}{r}}$

where $H_0r = \sqrt{\frac{2GM}{r}}$

substitutes $M = \frac{4}{3}\pi r^3\rho$ to get result

Examiners report

- a. [N/A]
b. [N/A]
c. [N/A]

-
- a. Outline, with reference to the Jeans criterion, why a cold dense gas cloud is more likely to form new stars than a hot diffuse gas cloud. [2]
- b. Explain how neutron capture can produce elements with an atomic number greater than iron. [2]

Markscheme

- a. «For a star to form»: magnitude of PE of gas cloud > KE of gas cloud

OR

Mass of cloud > Jean's mass

OR

Jean's criterion is the critical mass

hence a hot diffuse cloud could have KE which is too large/PE too small

OR

hence a cold dense cloud will have low KE/high PE

OR

a cold dense cloud is more likely to exceed Jeans mass

OR

a hot diffuse cloud is less likely to exceed the Jeans mass

Accept $E_p + E_k < 0$ **[2 marks]**

b. Neutron capture creates heavier isotopes / heavier nuclei / more unstable nucleus

 β^- decay of heavy elements/iron increases atomic number «by 1»

OWTTE

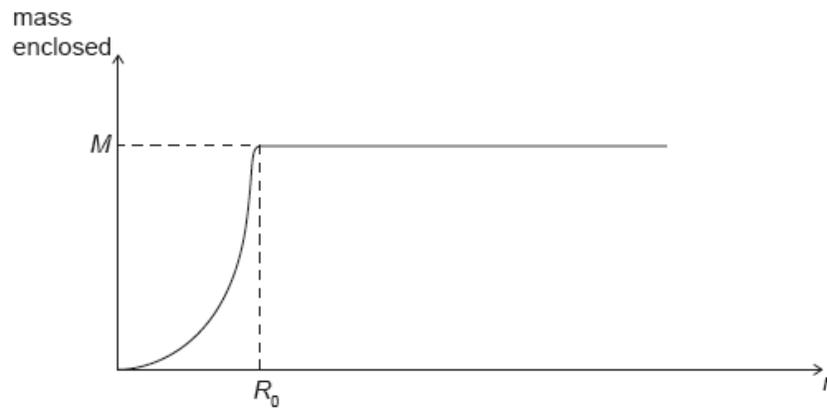
[2 marks]

Examiners report

a. [N/A]

b. [N/A]

A galaxy can be modelled as a sphere of radius R_0 . The distance of a star from the centre of the galaxy is r .



For this model the graph is a simplified representation of the variation with r of the mass of **visible matter** enclosed inside r .

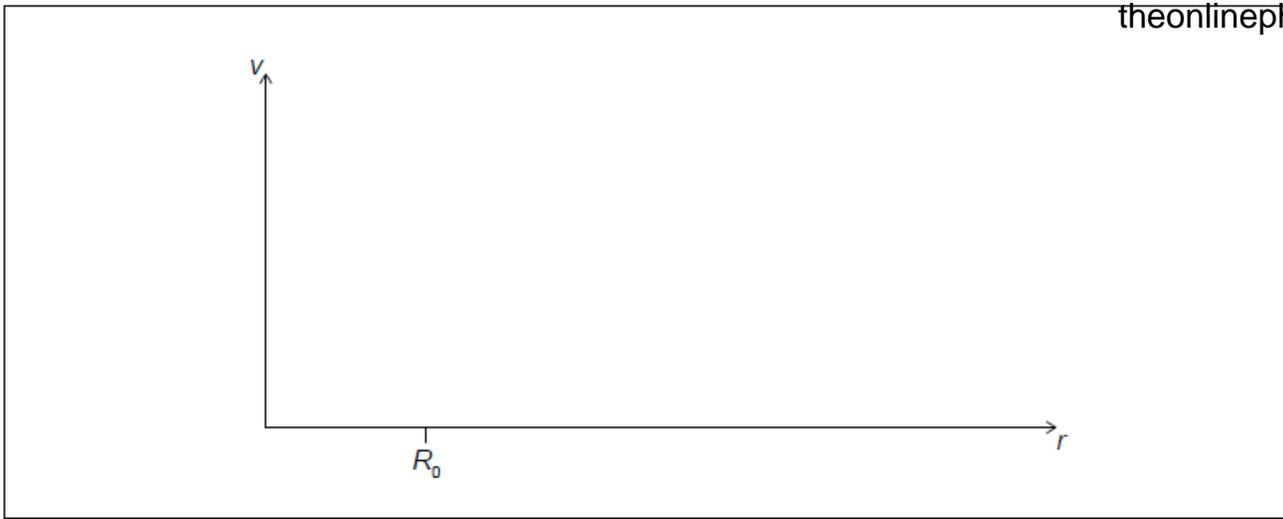
a. The mass of visible matter in the galaxy is M .

[1]

Show that for stars where $r > R_0$ the velocity of orbit is $v = \sqrt{\frac{GM}{r}}$.

b. Draw on the axes the observed variation with r of the orbital speed v of stars in a galaxy.

[2]



- c. Explain, using the equation in (a) and the graphs, why the presence of visible matter alone cannot account for the velocity of stars when $r > R_0$. [2]

Markscheme

- a. $\frac{mv^2}{r} = \frac{GMm}{r^2}$ and correct rearranging

[1 mark]

- b. linear / rising until R_0

then «almost» constant

[2 marks]

- c. for v to stay constant for r greater than R_0 , M has to be proportional to r

but this contradicts the information from the M - r graph

OR

if M is constant for r greater than R_0 , then we would expect $v \propto r^{-\frac{1}{2}}$

but this contradicts the information from the v - r graph

[2 marks]

Examiners report

- a. [N/A]
 b. [N/A]
 c. [N/A]

This question is about Hubble's law.

The spectrum of hydrogen from a source in the laboratory has a spectral line at wavelength 656 nm. The same line, viewed from Earth, in the spectrum of a distant galaxy has wavelength 682 nm.

a. Suggest why the two wavelengths are different.

b. Determine the distance to this galaxy from Earth using a Hubble constant of $74 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

[2]

Markscheme

a. the galaxy is moving away from the Earth and so the wavelength is Doppler/Red shifted;

or

the universe is expanding and so the space between galaxies is stretched/increases and this means that the wavelength of the received light will also be stretched/increased;

Do not accept answers such as "the galaxy is red-shifted".

$$b. v = \left(\frac{\Delta\lambda}{\lambda} c = \frac{682-656}{656} \times 3 \times 10^8 = 3.96 \times 10^{-2} \times 3 \times 10^8 = \right) 1.2 \times 10^4 \text{ (km s}^{-1}\text{)};$$

$$d = \left(\frac{v}{H_0} = \frac{1.2 \times 10^4}{74} = \right) 160 \text{ (Mpc)}; \text{ (allow } 5 \times 10^{24} \text{ (m))}$$

Allow ECF from first marking point for [1 max].

For example use of 682 in denominator also giving 160/155 (Mpc).

Award [0] for second marking point if 160 pc, 160 kpc and 160 Gpc are given. These are power of ten errors, not unit errors.

Examiners report

a. In (a) there were far too many vague responses that just stated 'galaxies are red-shifted'.

b. Many correct answers were seen in (b), but there were also many power of ten errors where km s^{-1} were not used in the calculation of distance.

Another common mistake was to use the observed frequency in the denominator.

This question is about the Hubble constant.

A recent estimate for the value of the Hubble constant is $70 \text{ km}^{-1} \text{ Mpc}^{-1}$.

a. Estimate, in seconds, the age of the universe.

[2]

b. The wavelength of the lines in the absorption spectrum of hydrogen is 656.3 nm when measured on Earth. Analysis of light from a distant galaxy [2]

shows that the same line has a wavelength of 725.6 nm. Determine the recessional velocity of the distant galaxy.

Markscheme

$$a. T = \frac{1}{H} \left(= \frac{1}{70 \text{ km s}^{-1} \text{ Mpc}^{-1}} \right);$$

$$T = \left(\frac{1}{70 \text{ 000 s}^{-1}} \times 10^6 \times 9.46 \times 10^{15} \times 3.26 = \right) 4.4 \times 10^{17} \text{ s};$$

$$b. \left(\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \right)$$

$$\frac{\Delta\lambda}{\lambda} = \frac{725.6-656.3}{656.3} = 0.106;$$

$$v = \left(\frac{\Delta\lambda}{\lambda} \times c = 0.106 \times 3 \times 10^8 = \right) 3.17 \times 10^7 \text{ m s}^{-1};$$

Examiners report

- a. The majority of candidates answered both questions well.
 - b. The majority of candidates answered both questions well.
-

This question is about cosmic microwave background (CMB) radiation.

A line in the hydrogen spectrum is measured in the laboratory to have a wavelength of 656 nm. The same line from a distant galaxy is measured to have a wavelength of 730 nm. Assuming that the Hubble constant H_0 is $69.3 \text{ km s}^{-1}\text{Mpc}^{-1}$,

- b.i. calculate the distance of this galaxy from Earth. [2]
- b.ii. discuss why different measurements of the Hubble constant do not agree with each other. [1]

Markscheme

b.i. $\left(\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \right) v = \left(\frac{3.00 \times 10^8 \times 74}{656} = \right) 3.38 \times 10^7 \text{ (ms}^{-1}\text{)};$

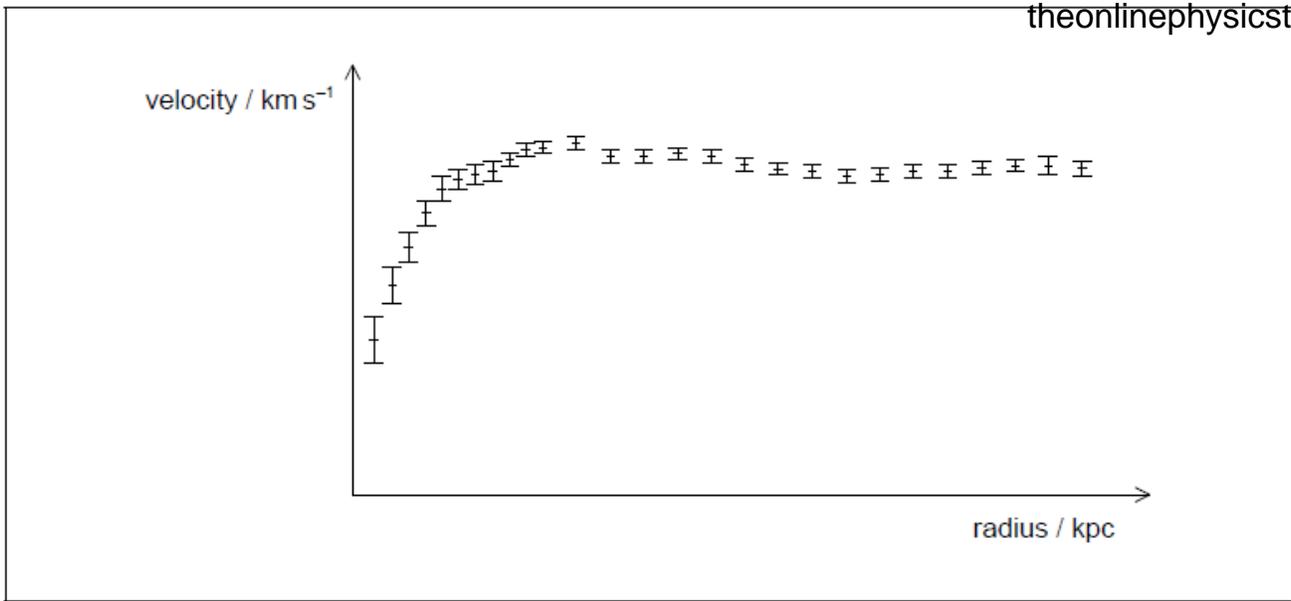
$$d = \frac{v}{H_0} = \frac{3.38 \times 10^4}{69.3} = 488 \text{ Mpc};$$

- b.ii. measurements from distant galaxies have large uncertainties;

Examiners report

- b.i. Well done by candidates, weaker candidates did not write their ideas clearly enough in (a)(ii). Part (b) was also quite well done, but only better candidates mentioned uncertainty in measurement of distances to galaxies.
 - b.ii. Well done by candidates, weaker candidates did not write their ideas clearly enough in (a)(ii). Part (b) was also quite well done, but only better candidates mentioned uncertainty in measurement of distances to galaxies.
-

The graph shows the observed orbital velocities of stars in a galaxy against their distance from the centre of the galaxy. The core of the galaxy has a radius of 4.0 kpc.



- a. Calculate the rotation velocity of stars 4.0 kpc from the centre of the galaxy. The average density of the galaxy is $5.0 \times 10^{-21} \text{ kg m}^{-3}$. [2]
- b. Explain why the rotation curves are evidence for the existence of dark matter. [2]

Markscheme

a. $v = \sqrt{\frac{4\pi G \rho}{3} r} = \sqrt{\frac{4}{3} \times \pi \times 6.67 \times 10^{-11} \times 5.0 \times 10^{-21} \times (4000 \times 3.1 \times 10^{16})}$

v is about 146000 «m s⁻¹» **or** 146 «km s⁻¹»
 Accept answer in the range of 140000 to 160000 «m s⁻¹».

- b. rotation curves/velocity of stars were expected to decrease outside core of galaxy
 flat curve suggests existence of matter/mass that cannot be seen – now called dark matter

Examiners report

- a. [N/A]
- b. [N/A]

- a. Describe how some white dwarf stars become type Ia supernovae. [3]
- b. Hence, explain why a type Ia supernova is used as a standard candle. [2]
- c. Explain how the observation of type Ia supernovae led to the hypothesis that dark energy exists. [3]

Markscheme

- a. white dwarf must have companion «in binary system»
 white dwarf gains material «from companion»

- b. a standard candle represents a «stellar object» with a known luminosity

this supernova occurs at an certain/known/exact mass so luminosity/energy released is also known

OWTTE

MP1 for indication of known luminosity, MP2 for any relevant supportive argument.

- c. distant supernovae were dimmer/further away than expected

hence universe is accelerating

dark energy «is a hypothesis to» explain this

Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

- a. Outline, with reference to star formation, what is meant by the Jeans criterion. [2]

- b. In the proton–proton cycle, four hydrogen nuclei fuse to produce one nucleus of helium releasing a total of 4.3×10^{-12} J of energy. The Sun will spend 10^{10} years on the main sequence. It may be assumed that during this time the Sun maintains a constant luminosity of 3.8×10^{26} W. [2]

Show that the total mass of hydrogen that is converted into helium while the Sun is on the main sequence is 2×10^{29} kg.

- c. Massive stars that have left the main sequence have a layered structure with different chemical elements in different layers. Discuss this structure by reference to the nuclear reactions taking place in such stars. [2]

Markscheme

- a. a star will form out of a cloud of gas

when the gravitational potential energy of the cloud exceeds the total random kinetic energy of the particles of the cloud

OR

the mass exceeds a critical mass for a particular **radius and temperature**

[2 marks]

- b. number of reactions is $\frac{10^{10} \times 365 \times 24 \times 3600 \times 3.8 \times 10^{26}}{4.3 \times 10^{-12}} = 2.79 \times 10^{55}$

H mass used is $2.79 \times 10^{55} \times 4 \times 1.67 \times 10^{-27} = 1.86 \times 10^{29}$ «kg»

[2 marks]

- c. nuclear fusion reactions produce ever heavier elements depending on the mass of the star / temperature of the core

the elements / nuclear reactions arrange themselves in layers, heaviest at the core lightest in the envelope

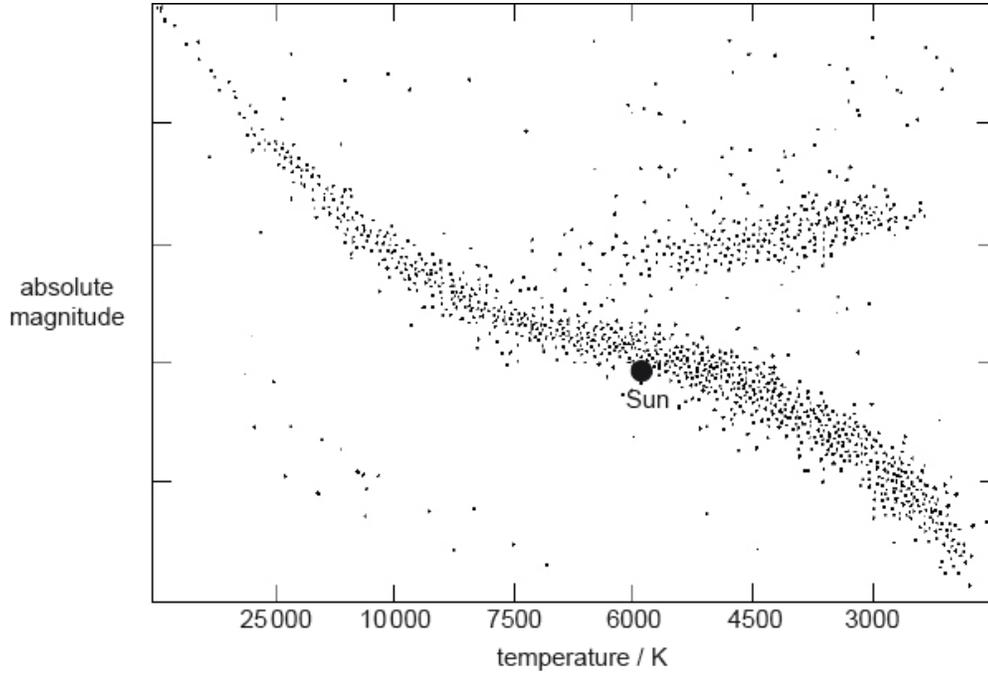
[2 marks]

Examiners report

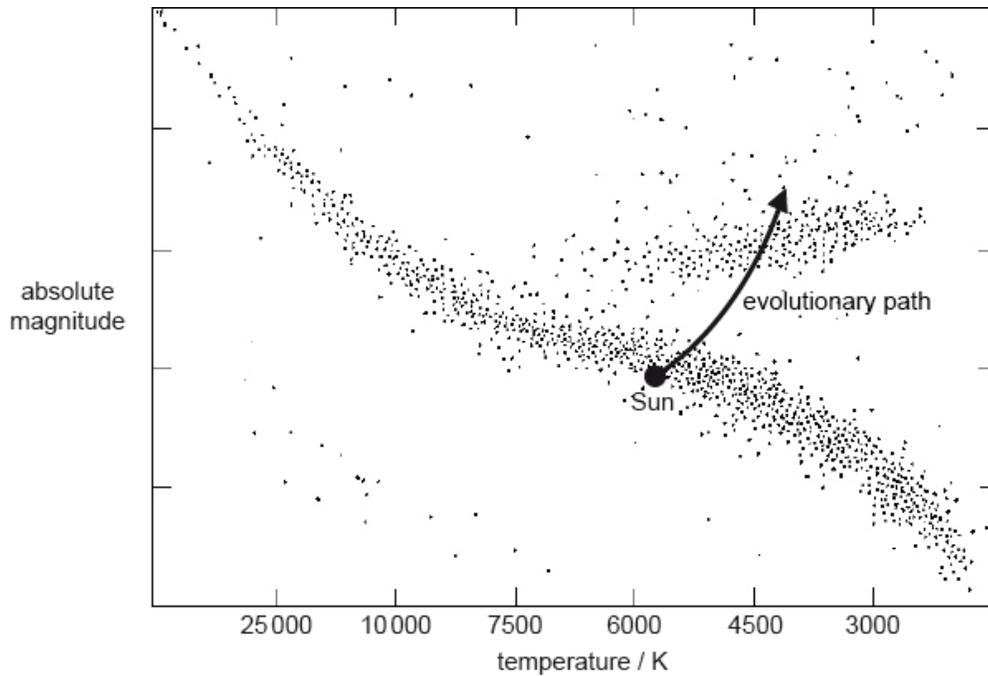
- b. [N/A]
- c. [N/A]

This question is about the Hertzsprung–Russell (HR) diagram and the Sun.

A Hertzsprung–Russell (HR) diagram is shown.



The Sun will remain on the main sequence of the HR diagram for about another five billion years. After this time it will become a red giant, following the evolutionary path shown in the diagram.



e.i. Outline why the Sun will leave the main sequence, and describe the nuclear processes that occur as it becomes a red giant. [4]

e.ii. Describe the two physical changes that the Sun will undergo as it enters the red giant stage. [2]

Markscheme

e.i.insufficient hydrogen (to continue fusion);

star collapses (under gravity);

temperature increases;

initiated fusion of helium, (energy released causes) rapid expansion of star;

e.ii.rapid expansion / increase of size;

decrease in temperature / cooler stars appear red in colour / increase of luminosity;

Examiners report

e.i.Well-prepared candidates (both HL and SL) only had a problem with the part related to the use of a non-linear temperature scale. Average prepared candidates displayed difficulty in the experimental measurement of the temperature of the distant star and also with details of nuclear processes occurring in the Sun during transformation to a red giant.

e.ii.Well-prepared candidates (both HL and SL) only had a problem with the part related to the use of a non-linear temperature scale. Average prepared candidates displayed difficulty in the experimental measurement of the temperature of the distant star and also with details of nuclear processes occurring in the Sun during transformation to a red giant.

This question is about the characteristics of the stars Procyon A and Procyon B.

The star Betelgeuse is about five times the mass of Regulus. One possible outcome of the final stage of the evolution of Betelgeuse is for it to become a black hole. State the

- i. The luminosity of the main sequence star Regulus is $150 L_S$. Assuming that, in the mass–luminosity relationship, $n = 3.5$ show that the mass of Regulus is $4.2 M_S$ where M_S is the mass of the Sun. [2]
- j. (i) other possible outcome of the final stage of the evolution of Betelgeuse. [2]
- (ii) reason why the final stage in (j)(i) is stable.

Markscheme

i. $\frac{150}{1} = \left(\frac{M_R}{1}\right)^{3.5}$ or $150 = M_R^{3.5}$;

evidence of algebraic manipulation e.g. $M_R = [150]^{\frac{1}{3.5}}$;

$= 4.2 M_S$

To award [2] there must be evidence of algebraic manipulation shown.

- j. (i) neutron star;

Examiners report

- i. Some candidates struggle with the manipulation of logs.
- j. A significantly large number of candidates in (j) recognised that Betelgeuse might become a neutron star and that neutron degeneracy pressure would account for its final stability.

This question is about Hubble's law.

- a. State Hubble's law. [1]
- b. Measured values of the Hubble constant can vary between $40 \text{ kms}^{-1} \text{ Mpc}^{-1}$ and $90 \text{ kms}^{-1} \text{ Mpc}^{-1}$. State the reason for this wide variation in values. [1]
- c. The blue line in the spectrum of atomic hydrogen as measured in the laboratory is 490 nm. The same line in the spectrum of light from a galaxy has a wavelength of 500 nm. [3]

Determine the distance of the galaxy from Earth. You may assume that the Hubble constant = $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Markscheme

- a. the recessional speed of galaxies is proportional to their distance from Earth/us/each other / $v = H_0 d$ (with terms defined);
- b. there is a large uncertainty in the measurement of galactic distances / it is difficult to accurately determine galactic distances;

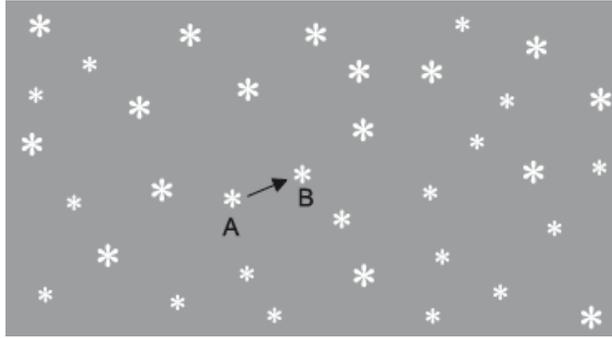
c. $v = c \frac{\Delta\lambda}{\lambda}$;
 $= \left(3 \times 10^8 \times \frac{10}{490} \right) 6.1 \times 10^3 \text{ kms}^{-1}$;
 $d = \frac{v}{H} \left(= \frac{6100}{70} \right) = 87 \text{ Mpc}$;

Examiners report

- a.
- b.
- c.

This question is about determining the distance to a nearby star.

Two photographs of the night sky are taken, one six months after the other. When the photographs are compared, one star appears to have moved from position A to position B, relative to the other stars.



Discuss whether Hubble's Law can be used to determine reliably the distance from Earth to this star.

Markscheme

this star is less than 1000 pc away/in our galaxy;

Hubble's law is for galaxies (not local stars) / red-shift will be too small to measure / uncertainty in Hubble constant high for such measurement;

Examiners report

Well discriminating question, better candidates realized that the star is closer to Earth and drew the diagram. Many candidates made a mistake to present diameter and the angle, giving half of the proper values. The relationships were generally well explained. In the alternative pair of quantities many candidates stated only the quantity for distance, not for the angle. The HL question related to Hubble's law was properly answered only by better candidates. The SL question was poorly answered with most confusing stellar and spectroscopic parallax.

This question is about the Big Bang model and red-shift.

- c. Many galaxies are a great distance from Earth. Explain, with reference to Hubble's law, how the measurement of the red-shift of light from such galaxies enables their distance from Earth to be determined. [3]
- d. State **one** problem associated with using Hubble's law to determine the distance of a galaxy a great distance from earth. [1]

Markscheme

- c. the amount of red-shift enables the recession speed of a galaxy to be determined;

Hubble's law states that the recession speed is proportional to its distance from Earth/ $v = H_0 d$ with terms defined;

if the constant of proportionality/ H_0 is known then d can be determined;

- d. it is difficult to determine an accurate value of the Hubble constant / difficult to measure the red-shift / Hubble constant had different values in the past;

Examiners report

- c. Too many candidates lost marks by not defining the symbols in the Hubble Law.
- d. Part (d) was usually answered correctly.

This question is about Hubble's law.

- a. A galaxy a distance d away emits light of wavelength λ . Show that the shift in wavelength $\Delta\lambda$, as measured on Earth, is given by [1]

$$\Delta\lambda = \frac{H_0 d \lambda}{c}$$

where H_0 is the Hubble constant.

- b. Light of wavelength 620 nm is emitted from a distant galaxy. The shift in wavelength measured on Earth is 35 nm. Determine the distance to the galaxy using a Hubble constant of $68 \text{ km s}^{-1} \text{ Mpc}^{-1}$. [1]

Markscheme

- a. combining $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$ and $v=H_0d$;

Answer given, check working.

- b. $d = \frac{c\Delta\lambda}{\lambda H_0} = \left(\frac{3 \times 10^8 \times 35}{620 \times 68} \right)$;

(mark is for rearrangement)

$$d=250(\text{Mpc}) \text{ or } 7.7 \times 10^{24} \text{ (m)};$$

Allow only first marking point if incorrect value for λ is used.

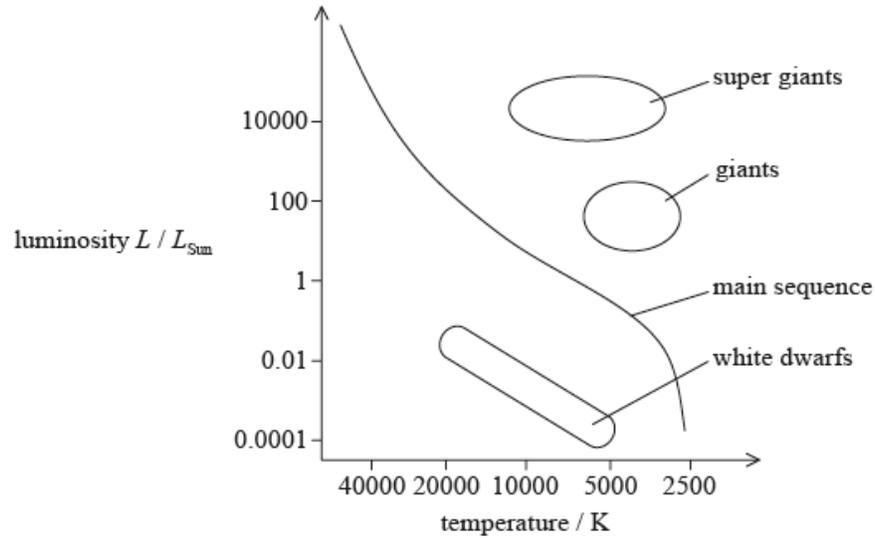
Award [2] for a bald correct answer.

Examiners report

- a. In part (a) there were almost no incorrect answers.
- b. In part (b) far too many candidates lost 1 mark because they used the wrong power of ten for velocity in Hubble's constant.

This question is about the Hertzsprung–Russell (HR) diagram and stellar evolution.

The star Phi-1 Orionis is a large star on the main sequence with a mass of approximately 18 solar masses.



- a. Calculate the luminosity of Phi-1 Orionis in terms of the luminosity of the Sun. Assume that $n = 3.5$ in the mass–luminosity relation. [2]
- b. The Sun is expected to have a lifespan of around 10^{10} years. With reference to the equilibrium between radiation pressure and gravitational pressure, discuss why Phi-1 Orionis will use up its hydrogen at a faster rate than the Sun. [2]
- c.i. Using the HR diagram on page 6, draw the evolutionary path of Phi-1 Orionis as it leaves the main sequence. [1]
- c.ii. Outline, with reference to the Oppenheimer–Volkoff limit, the fate of Phi-1 Orionis. [1]

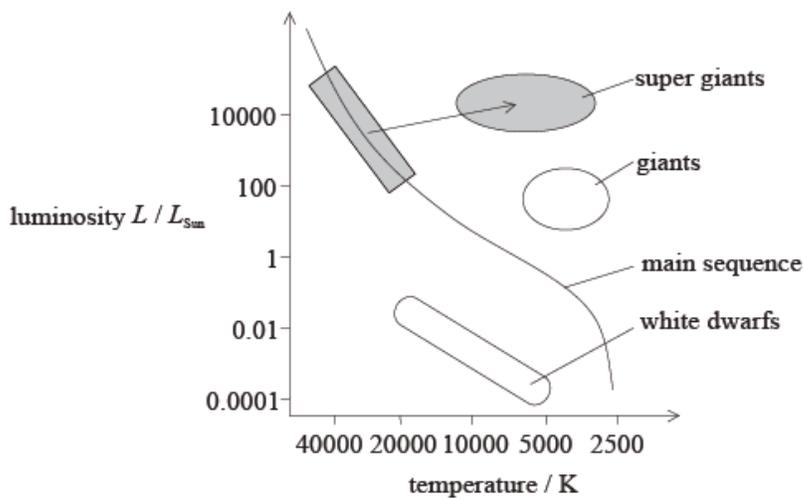
Markscheme

a. $\frac{L}{L_{\odot}} = \left(\left[\frac{m}{m_{\odot}} \right]^{3.5} \right) \left[\frac{18m_{\odot}}{m_{\odot}} \right]^{3.5};$

$L = 25000 L_{\odot};$

Answer must include L_{\odot} in correct place.

- b. Phi-1 Orionis has a larger mass so it has a larger gravitational pressure;
to remain in equilibrium it requires (an equal) radiation pressure which is provided by burning (hydrogen) at a faster rate;
- c.i. line drawn starting from the top-left of the main sequence towards (red) super giants; } (allow anywhere within the grey shaded regions)



c.ii.(mass of star is more than 15 solar masses so can be predicted, that) after supernova explosion it will be more than 3 solar masses/Oppenheimer-Volkoff limit and become a black hole;

or

if the mass after supernova explosion will be less than then Oppenheimer-Volkoff limit, it will become a neutron star;

Examiners report

- a. This question was not easy. The majority of candidates calculated the luminosity of the star and used the balance between radiation pressure and gravitational pressure. Stronger candidates identified the evolution of the star on the HR diagram and showed the ability to distinguish between a black hole and neutron star.
- b. This question was not easy. The majority of candidates calculated the luminosity of the star and used the balance between radiation pressure and gravitational pressure. Stronger candidates identified the evolution of the star on the HR diagram and showed the ability to distinguish between a black hole and neutron star.
- c.i. This question was not easy. The majority of candidates calculated the luminosity of the star and used the balance between radiation pressure and gravitational pressure. Stronger candidates identified the evolution of the star on the HR diagram and showed the ability to distinguish between a black hole and neutron star.
- c.ii. This question was not easy. The majority of candidates calculated the luminosity of the star and used the balance between radiation pressure and gravitational pressure. Stronger candidates identified the evolution of the star on the HR diagram and showed the ability to distinguish between a black hole and neutron star.

This question is about stellar distances.

- b. The star Sirius A is 3 pc from Earth. The apparent brightness of Sirius A is $1.2 \times 10^{-7} \text{ W m}^{-2}$. Determine the luminosity of Sirius A.

[2]

- c. The luminosity of the Sun is 3.8×10^{26} W. Determine the mass of Sirius A relative to the mass of the Sun. (Assume that $n=3.6$ in the mass-luminosity relation.) [2]

Markscheme

b. $L (= 4\pi bd^2) = 4 \times \pi \times 1.2 \times 10^{-7} \times [8.1 \times 10^{16}]^2$;

$9.9 \times 10^{27} \text{ (W)}$;

Allow $1.3 \times 10^{28} \text{ (W)}$ if candidates use 3 (pc) from (a).

c. $\frac{M_{\text{Sirius}}}{M_{\text{Sun}}} \left(= \left[\frac{L_{\text{Sirius}}}{L_{\text{Sun}}} \right]^{\frac{1}{3.5}} \right) = \left[\frac{9.9 \times 10^{27}}{3.8 \times 10^{26}} \right]^{\frac{1}{3.5}}$;

$M_{\text{Sirius}} = 2.5 M_{\text{Sun}}$;

Allow ECF from (b).

Examiners report

- b. Some candidates had difficulty in manipulating a logarithmic equation. (b) discriminated well. Many candidates used the equation from the data booklet value in non-SI unit and forgot to convert pc to meters. This was not a surprise to the examining team. Quite a high number forgot to square the distance.
- c. In (c), many candidates did not present their working in logical manner, especially those who did not understand mass-luminosity relations and incorrectly used the formula from data booklet.

This question is about stellar evolution.

- a. The mass of a main sequence star is two solar masses. Estimate, in terms of the solar luminosity, the range of possible values for the luminosity of this star. [2]
- b. The star in (a) will eventually leave the main sequence. [3]
- State
- (i) the condition that must be satisfied for this star to eventually become a white dwarf.
- (ii) the source of the energy that the white dwarf star radiates into space.
- (iii) **one** likely element, other than hydrogen and helium, that may be found in a white dwarf.
- c. Explain why a white dwarf maintains a constant radius. [2]

Markscheme

a. $\frac{L}{L_{\odot}} = 2^n$ with n between 3 and 4;

so $8L_{\odot} < L < 16L_{\odot}$;

Award [2] for a bald correct answer.

b. (i) the core/remnant mass must be less than the Chandrasekhar limit/1.4 solar masses; } (must see core or remnant or similar term)

(ii) residual/thermal/internal energy of the star / OWTTE;

Do not allow fusion.

(iii) C/O/Ne/Mg; (accept no others)

c. gravitational attraction/pressure is balanced by;

electron (degeneracy) pressure/repulsion / pressure/force due to Pauli exclusion principle;

Award the first marking point independently of the second.

Examiners report

a. In part (a) most candidates correctly referred to the mass–luminosity equation and used it to determine the luminosity range for the star.

b. Part (b)(i) was answered well by many, but there were also many who did not refer to the remnant or core mass being below the Chandrasekhar limit. In (b)

(ii) there were far too many candidates who referred to fusion continuing in a white dwarf. In part (b)(iii) carbon or oxygen were almost always correctly stated.

c. In (c) it was expected that electron degeneracy pressure would be mentioned, many did so but fusion radiation pressure was also incorrectly mentioned.

This question is about the mass–luminosity relation.

Star X is 1.5×10^5 more luminous than the Sun and has a mass 30 times that of the Sun.

a. Identify whether star X is on the main sequence. Assume that $n = 3.5$ in the mass–luminosity relation. [2]

b. (i) State the evolution of star X. [3]

(ii) Explain the eventual fate of star X.

Markscheme

a. luminosity= $30^{3.5} \times 148\,000$ (times the luminosity of the Sun) **or**

mass= $(1.5 \times 10^5)^{3.5} = 30$ (times the mass of the Sun);

(this is close to the quoted luminosity/mass and) so X must be on the main sequence;

b. (i) red (super)giant goes supernova with core remaining;

- (ii) Oppenheimer–Volkoff/mass of remnant will determine final fate;
(to give) neutron star/black hole;

Examiners report

- a. [N/A]
b. [N/A]
-

- a. State the Jeans criterion for star formation. [2]
- b. Describe **three** differences between type Ia and type II supernovae. [3]

Markscheme

- a. a gas cloud will collapse to form a star
if «the magnitude of» the gravitational potential energy of the particles is greater than the kinetic energy of the particles **OR** mass of the cloud is greater than the Jeans mass
- b. Ia have consistent maxima in their light curves but II vary
Ia has a strong ionized Sill line but II has hydrogen lines in their spectra
Ia was a white dwarf but II are massive stars
Ia form from binary systems but II are the result of core collapse of a star
Ia can be used as standard candles but II are not

Examiners report

- a. [N/A]
b. [N/A]
-

This question is about Hubble's law and the age of the universe.

- a. (i) State Hubble's law. [2]
(ii) State why Hubble's law cannot be used to determine the distance from Earth to nearby galaxies, such as Andromeda.
- b. (i) Show that $\frac{1}{H_0}$ is an estimate of the age of the universe, where H_0 is the Hubble constant. [3]
(ii) Assuming $H_0 = 80 \text{ km s}^{-1}\text{Mpc}^{-1}$, estimate the age of the universe in seconds.

Markscheme

- a. (i) recessional speed of a galaxy is directly proportional to distance from Earth/ $v=H_0d$ with symbols defined;

(ii) local velocity of Andromeda relative to Earth greater than (recessional) speed due to expansion of universe / OWTTE;

b. (i) relative speed between two points in universe separated by distance d is $v = \frac{d}{T}$ where T is the age of the universe;

$$v = \frac{d}{T} = H_0 d \text{ therefore } T = \frac{1}{H_0};$$

$$\text{(ii) } T = \frac{1}{80} \times \frac{10^6 \times 3.26 \times 9.46 \times 10^{15}}{1000} = 4 \times 10^{17} \text{ (s)};$$

Do not deduct unit mark if seconds not given, as question asks for answer in seconds.

Examiners report

a. [N/A]

b. [N/A]

This question is about some of the properties of the star Aldebaran and also about galactic distances.

c. Betelgeuse in the constellation of Orion is a red supergiant star.

[5]

(i) Compare the fate of Aldebaran to that of Betelgeuse.

(ii) Outline, with reference to the Chandrasekhar limit, the circumstances under which the final state of Betelgeuse could be the same as the final state of Aldebaran.

d. Distances to galaxies may be determined by using Cepheid variable stars.

[5]

By considering the nature and properties of Cepheid variable stars, explain how such stars are used to determine galactic distances.

Markscheme

c. (i) *Aldebaran*:

it forms a planetary nebula which then becomes a white dwarf;

Betelgeuse:

it forms a supernova which then becomes a neutron star/black hole/pulsar;

To award **[2]** both phases are required in both responses.

Award **[1 max]** if intermediate stages (planetary nebula, supernova) are omitted.

(ii) reference to 1.4 solar mass (Chandrasekhar limit for white dwarfs);

if Betelgeuse blows away sufficient mass (in the supernova stage);

and is left with a core mass below the Chandrasekhar limit;

the core can form a white dwarf

d. the (outer layers of the star) undergo a (periodic) expansion and contraction;

which produces a (periodic) variation in its luminosity/apparent brightness;

the (average) luminosity depends on the period of variation;

by measuring the period, the luminosity can be found;

by then measuring its apparent brightness, its distance from Earth can be found;

Examiners report

c.

d.

This question is about Hubble's law.

a. The fractional change in the wavelength λ of light from the galaxy Hydra is $\frac{\Delta\lambda}{\lambda}=0.204$. The distance to Hydra is 820 Mpc. [2]

Estimate in $\text{km s}^{-1} \text{Mpc}^{-1}$ a value for the Hubble constant.

b. An estimate of the age of the universe is $\frac{1}{H}$ where H is the Hubble constant. Suggest why $\frac{1}{H}$ overestimates the age of the universe. [2]

Markscheme

a. $\frac{\Delta\lambda}{\lambda} = 0.204 = \frac{v}{c} \Rightarrow v = 6.12 \times 10^4 \text{kms}^{-1}$;

so $H = \frac{v}{d} = \frac{6.12 \times 10^4}{820} = 74.6 \text{kms}^{-1} \text{Mpc}^{-1}$;

Award [2] for a bald correct answer.

Award [1 max] if power of ten error in first marking point is carried forward.

b. present value of expansion rate is used for estimate;

but in the past the expansion rate was greater;

Examiners report

a. [N/A]

b. [N/A]

This question is about the structure of the universe.

a. (i) State, in terms of the arrangement of galaxies, the present large-scale distribution of mass in the universe. [2]

(ii) State how the separation of distant galaxies is changing with time.

b. State and explain the observational evidence for your answer to (a)(ii). [3]

Markscheme

a. (i) galaxies arranged in clusters (that are themselves arranged in superclusters);

(ii) galaxies/clusters/superclusters move further apart / distance between galaxies/ clusters/superclusters increases;

b. increase in wavelength / red-shift is observed in light from distant galaxies;

the red-shift increases with distance;

therefore (the metric of) space is expanding (with time) / the separation between galaxies is increasing;

following the Big Bang;

Galaxies are moving away from us or from Earth is not enough for the third mark.

Do not award mark for background radiation.

Examiners report

a. Candidates generally understood the distribution of galaxies in the universe and could clearly explain red-shift.

b. Candidates generally understood the distribution of galaxies in the universe and could clearly explain red-shift.

This question is about the life history of stars.

a. Outline, with reference to pressure, how a star on the main sequence maintains its stability. [3]

b. A star with a mass equal to that of the Sun moves off the main sequence. Outline the main processes of nucleosynthesis that occur in the core of this star before and after this change. [2]

c. Compare the fate of the star in (b) with that of a star of much greater mass. [3]

Markscheme

a. balance of two forces/pressures;

(balance) between radiation/pressure and gravitational force/pressure;

(radiation pressure is when) photons/radiation exert outwards force on nuclei/ particles;

(gravitational pressure is when) gravitational force between particles/layers of the star acts inwards;

b. whilst on the main sequence hydrogen fusion/burning to give helium;

after leaving the main sequence helium fusion/burning to give carbon;

c. star in (b) forms red giant, heavier star forms (red) supergiants; } (*do not allow "giant"*)

star in (b) forms planetary nebula, heavier star goes supernova;

star in (b) forms white dwarf, heavier star forms neutron star/black hole;

Examiners report

- a. (a) There was evidence of superficial learning from the syllabus. Only a few of the best candidates wrote details of radiation and/or gravitational pressure, in response to the “outline” command term.
- b. There was evidence in (b) [HL only] that some candidates do not read the question carefully. Better candidates clearly outlined the processes before and after moving off the main sequence. Only a few demonstrated a good understanding of the term nucleosynthesis and answered this question clearly. These candidates referred to hydrogen to helium while in the main sequence and helium to carbon after leaving the main sequence.
- c. In (c) [HL], quite a high number of candidates outlined only the fate of a star with much greater mass and did not compare this with the fate of a star with mass equal to mass of the Sun. Candidates who understood that comparison is required, often omitted planetary nebulae. The main issue here was the superficial reading of the questions. Many responded with memorized tracts of stellar evolution and did not answer the question.

- b. Beta Centauri is a star in the southern skies with a parallax angle of 8.32×10^{-3} arc-seconds. Calculate, in metres, the distance of this star from Earth. [2]
- c. Outline why astrophysicists use non-SI units for the measurement of astronomical distance. [1]

Markscheme

b. $d = \frac{1}{8.32 \times 10^{-3}}$ **OR** 120«pc»

$$120 \times 3.26 \times 9.46 \times 10^{15} = 3.70 \times 10^{18} \text{m}$$

Answer must be in metres, watch for POT.

- c. distances are so big/large **OR** to avoid using large powers of 10 **OR** they are based on convenient definitions

Examiners report

- b. [N/A]
c. [N/A]

This question is about the Hubble constant.

- a. Outline the measurements that must be taken in order to determine a value for the Hubble constant. [3]
- b. One estimate of the Hubble constant is $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Cygnus A is a radio galaxy at a distance of $6.0 \times 10^8 \text{ ly}$ from Earth. Calculate, in km s^{-1} , the recessional speed of Cygnus A relative to the Earth. [2]

Markscheme

- a. red shift used to measure recessional speed of galaxies;
 named measurement to yield distance to galaxies (e.g. Cepheid variable, Supernova);
 repeat for many galaxies/clusters of galaxies;
 Hubble constant is gradient of speed–distance graph; *{(any symbols used must be defined)}*

To award **[3]** reference must be made to galaxies in at least one of the marking points.

b. $v = 60 \times \frac{6.0 \times 10^8}{3.26 \times 10^6}$;
 $= 1.1 \times 10^4 \text{ km s}^{-1}$;

Examiners report

- a. [N/A]
 b. [N/A]

This question is about Hubble's law.

- a. State Hubble's law. [1]
- b. The wavelength of a line in the spectrum of atomic hydrogen, as measured in the laboratory, is 656 nm. The same line in the spectrum of light from a distant galaxy is measured to be 790 nm. The galaxy is 940 Mpc from Earth. [6]
- (i) Show that the recessional speed of the galaxy is $6.13 \times 10^4 \text{ km s}^{-1}$.
- (ii) Determine, using your answer to (b)(i), a value for the Hubble constant.
- (iii) Show, using your answer to (b)(ii), that the age of the universe is of the order of 10^{17} s. (1 pc = 3.1×10^{13} km)

Markscheme

- a. the recessional speed of galaxies from Earth is proportional to their distance from Earth;

or

$v = Hd$; *(with symbols defined)*

b. (i) $v = c \frac{\Delta\lambda}{\lambda}$;

$= 3.0 \times 10^5 \times \frac{134}{656}$;

$= 6.13 \times 10^4 \text{ kms}^{-1}$

(ii) $H = \frac{v}{d}$;

$= \left(\frac{6.13 \times 10^4}{940} \right) 65.1 \text{ kms}^{-1} \text{ Mpc}^{-1}$;

(iii) $T = \frac{1}{H}$;

$= \frac{3.1 \times 10^{19}}{65.1} = 4.76 \times 10^{17} \text{ s}$;

$\approx 10^{17} \text{ s}$

Examiners report

- a. [N/A]
 - b. [N/A]
-

This question is about Hubble's law.

- a. State [2]
 - (i) Hubble's law.
 - (ii) the significance of the reciprocal of the Hubble constant.
- b. The wavelength of a certain line in the hydrogen spectrum is measured to be 434 nm in the laboratory. The same line in the hydrogen spectrum [2]
of the galaxy 3C-273 is measured on Earth to be 504 nm.

Determine the distance of 3C-273 from Earth using a Hubble constant of $72 \text{ kms}^{-1} \text{ Mpc}^{-1}$.

Markscheme

- a. (i) galaxies move away from each other/from Earth with a speed that is proportional to their separation/distance from Earth;
Accept answer in terms of equation with symbols defined.

(ii) it gives age of the universe/time since the Big Bang;
- b. $\frac{v}{c} = \left(\frac{\Delta\lambda}{\lambda_0} = \frac{504-434}{434} = \right) 0.161$;
 $d = \left(\frac{v}{H} = \frac{0.161 \times 3 \times 10^5}{72} = \right) 672 \text{ Mpc} \approx 670 \text{ Mpc}$;
Alward [1] for 578(Mpc) if 504 used in the denominator.

Examiners report

- a. [N/A]
 - b. [N/A]
-

This question is about stellar evolution.

Achernar may evolve to become a neutron star.

- a. Achernar is a main sequence star with a mass that is eight times the mass of the Sun. Deduce that Achernar has a greater temperature than the [2]
Sun.
- b. Outline why Achernar will spend less time on the main sequence than the Sun. [2]
- c. (i) State the condition relating to mass that must be satisfied for Achernar to become a neutron star. [3]

- (ii) Some neutron stars rotate about their axes and have strong magnetic fields. State how these stars may be detected.

Markscheme

- a. from the mass–luminosity relation, Achernar has a higher luminosity; { *(reference to mass–luminosity is essential)*

so it is above/to the left of the Sun on the main sequence / temperature increases with luminosity in the main sequence;

Ignore irrelevant statements that $L = \sigma AT^4$.

Allow second marking point even if only mass is discussed.

- b. Achernar has greater luminosity/temperature and so fuses hydrogen at a (disproportionately) higher rate than the Sun / *OWTTE*;

so it will run out of hydrogen/move to red giant region in less time than the Sun / *OWTTE*;

- c. (i) the remnant mass/the mass of its core/the mass after the supernova stage; { *(do not allow “mass” bald)*

must be between the Chandrasekhar and Oppenheimer limits / $1.4M_{\square} < M_{\text{core}} < 3M_{\square}$; { *(allow answer in words or numerical values)*

Allow $2.5 M_{\square}$ to $3 M_{\square}$ as O–V limit.

- (ii) detection of EM radiation from pulsars/stars that pulsate / stars whose intensity varies rapidly / *OWTTE*;

Accept answers that refer to any regions of the electromagnetic spectrum.

Examiners report

- a. In (a) most candidates correctly referred to the mass-luminosity equation, but then asserted that luminosity was proportional to temperature without consideration of a star’s surface area.

- b. (b) was answered well by most.

- c. In (c)(i) the Chandrasekhar and Oppenheimer-Volkoff limits (or their values) were both expected but not often provided. Also it was common to refer to a star’s ‘mass’ rather than ‘remnant mass’ or ‘mass of the core’. Many candidates realised that a pulsar was being described in (c)(ii).

This question is about the evolution of stars.

- a. State what is meant by the

[2]

(i) Chandrasekhar limit.

(ii) Oppenheimer–Volkoff limit.

- b. Suggest how your answers in (a) can be used to predict the fate of a main sequence star.

[3]

Markscheme

- a. (i) sets upper limit on mass of white dwarf;

(ii) sets upper limit on mass of neutron star;

b. (if in the supernova phase) the mass blown leaves behind a mass of $1.4M_{\text{Sun}}$ / less than the Chandrasekhar limit;

the star will evolve to a white dwarf;

mass greater than about $1.4M_{\text{Sun}}$, but less than the O–V limit, will evolve (because of the O–V limit) into a neutron star;

Examiners report

a. [N/A]

b. [N/A]

This question is about the main sequence star Khad (Phi Orionis).

The luminosity of Khad is $2.0 \times 10^4 L_S$, where L_S is the luminosity of the Sun.

a. Assuming that the exponent n in the mass–luminosity relation is 3.5, show that the mass of Khad is about 17 solar masses. [2]

b. Outline the likely evolution of the star Khad after it leaves the main sequence. [3]

Markscheme

a. $\frac{L_K}{L_S} = \left[\frac{m_K}{m_S} \right]^{3.5}$;

$$m = \left[\frac{L_K}{L_S} \times m_S^{3.5} \right]^{\frac{1}{3.5}} = [2.0 \times 10^4]^{\frac{1}{3.5}} m_S = 16.9 m_S$$

hence $m_K \approx 17 m_S$

b. Khad will become a red supergiant/superrred/superrred giant;

a supernova will take place;

the core/remnant will form a neutron star or black hole;

Examiners report

a. [N/A]

b. [N/A]

This question is about Hubble's law.

a. State Hubble's law. [1]

b. Light from the galaxy M31 received on Earth shows a blue-shift corresponding to a fractional wavelength shift $\frac{\Delta\lambda}{\lambda}$ of 0.001. [5]

(i) Calculate the velocity of M31 relative to Earth.

(ii) The distance to M31 from Earth is 0.77 Mpc. Estimate, using the answer to (b)(i), a value of the Hubble constant.

(iii) Comment on your answer to (b)(ii).

Markscheme

a. recessional speed of (distant) galaxies is proportional to their separation / $v = Hd$;

Symbols must be defined for the equation.

b. (i) $\left(\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow\right) v = 3 \times 10^5 \text{ (ms}^{-1}\text{) or } 3 \times 10^2 \text{ (kms}^{-1}\text{)}$;

towards Earth;

(ii) 390 (kms⁻¹Mpc⁻¹); (allow other units: 390000(ms⁻¹Mpc⁻¹), 0.39(ms⁻¹ pc⁻¹))

(iii) the value is wrong/cannot be used/cannot be relied upon / we cannot use this galaxy to calculate H ;
Hubble's law only applies to galaxies moving away / more distant galaxies;

Examiners report

a. The statement of Hubble's law is a frequent question and was generally well answered. However too many candidates still mention planets or stars rather than galaxies or omit to mention recessional velocity.

b. In (b)(i) the value of the velocity of M31 was almost always correct, but 'towards Earth' was usually missing. Hubble's constant calculations were done well in (b)(ii), with a variety of units used. It was expected that candidates would refer to the fact that M31 is not very distant and does not recede as a reason for the value in (b)(ii) being invalid. Few did. Instead reference was often made to the reason for the general uncertainty in the value of H - probably because the latter has been a more usual question in recent past papers. In teaching this topic it is obvious that a range of values for H can be found in textbooks. These can easily become out of date, but the value obtained in (b)(ii) was about 5 times that currently accepted. Candidates are obviously expected to have some idea of this value.

This question is about red-shift.

The wavelengths of radio signals from galaxy A are found to be red-shifted from the wavelengths that would be observed from sources at rest relative to Earth.

The fractional change in wavelength of the radio signals from galaxy A is 9.4×10^{-3} .

Calculate, in km s^{-1} , the average velocity of galaxy A relative to Earth.

Markscheme

$v = 0.0094c$;

2800 (km s^{-1});

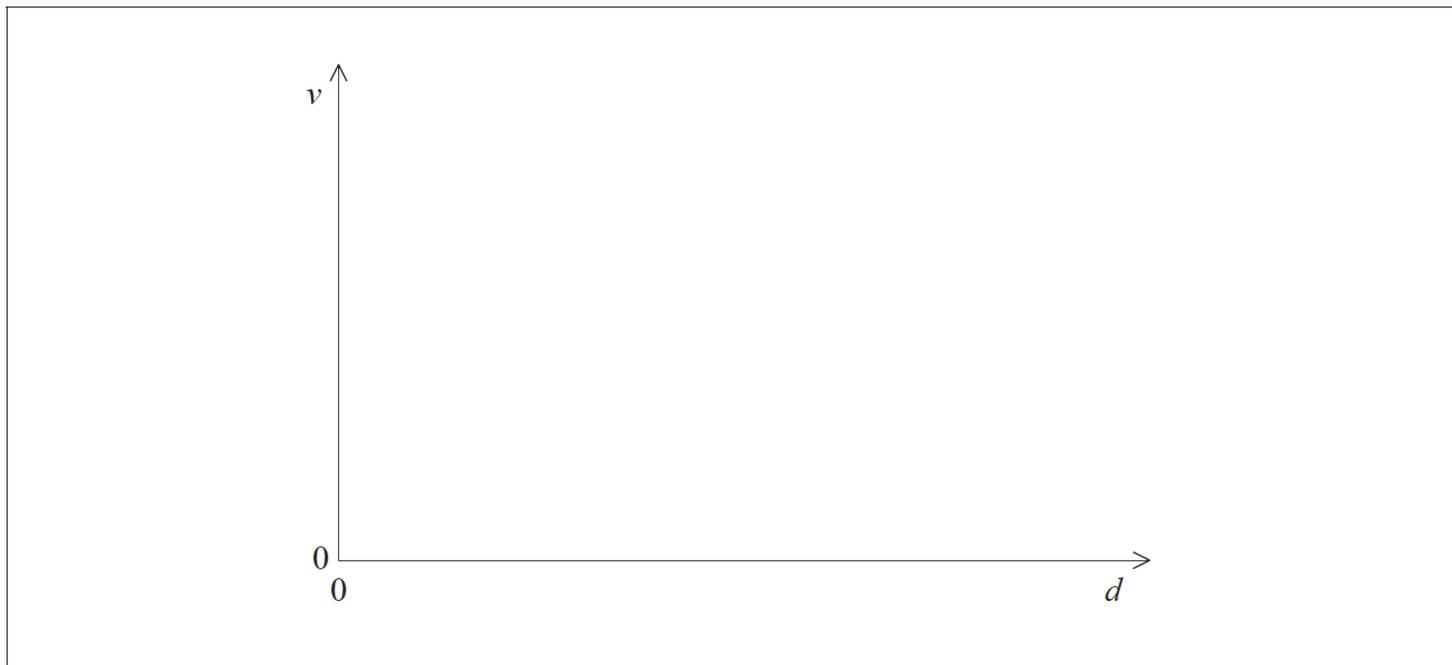
Examiners report

was very well done.

This question is about red-shift.

a. (i) On the axes, sketch a graph to show how the recessional speed v of a galaxy varies with distance d from the Earth.

[3]



(ii) Outline how the graph in (a)(i) can be used to determine the age of the universe.

b. Astronomers use the factor z to report the red-shift of an object relative to Earth where

[2]

$$z = \frac{\text{shift in wavelength detected by Earth observer}}{\text{wavelength of light emitted by object}}$$

Quasar 3C273 is thought to be the closest quasar to Earth and has $z=0.18$. Assuming that the Hubble constant is $70 \text{ kms}^{-1} \text{ Mpc}^{-1}$, determine the distance of this object from Earth.

Markscheme

a. (i) straight line that passes through the origin (or would do so if extrapolated);

(ii) gradient is H_0 /Hubble's constant;
age of universe is $\frac{1}{H_0}$ **or** age is $\frac{1}{\text{gradient}}$;

b. $v=(0.18 \times c)=5.4 \times 10^7(\text{ms}^{-1})$ **or** $5.4 \times 10^4(\text{kms}^{-1})$;

$$d = \frac{v}{70} = 770 \text{ (Mpc)} \text{ **or** } 2.4 \times 10^{25}(\text{m});$$

Award **[1 max]** if v is correctly calculated but left in ms^{-1} , giving an answer of 770 000 (Mpc).

Award **[1 max]** ECF for an incorrect value of v used correctly in kms^{-1} .

Award **[0]** for an incorrect value of v and failure to convert to kms^{-1} .

Award **[2]** for a bald correct answer.

Examiners report

a. [N/A]

b. [N/A]

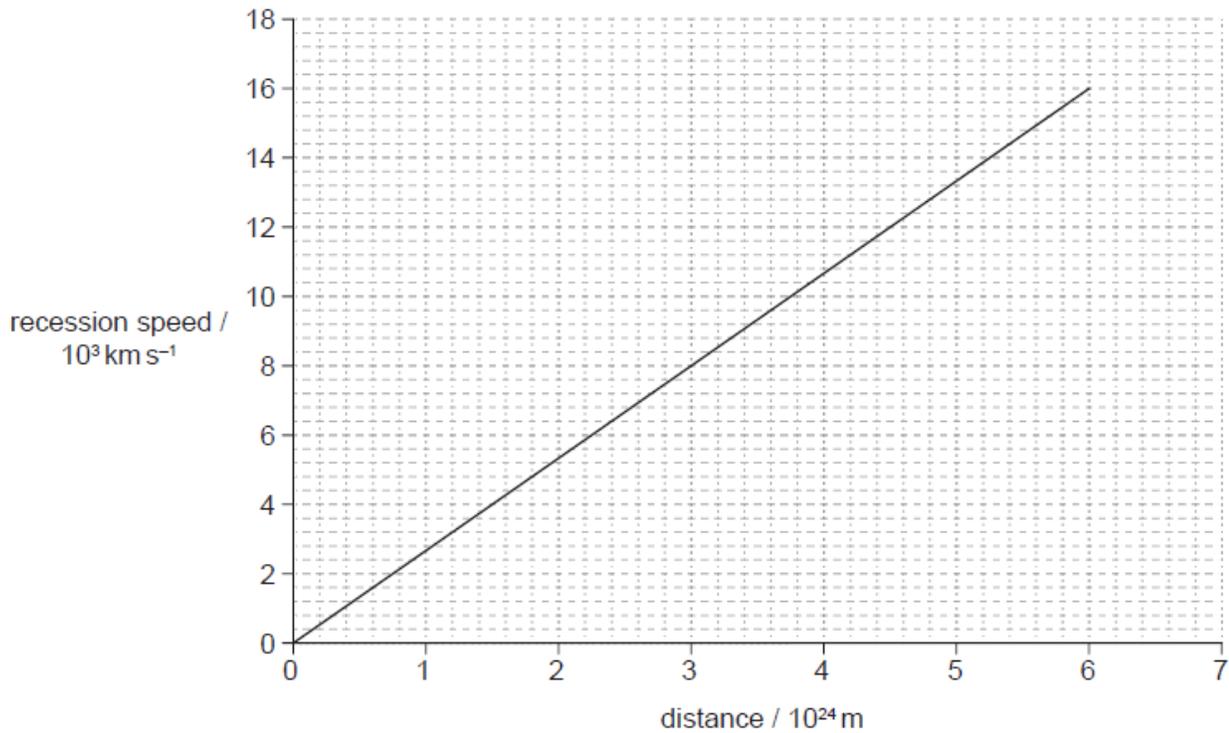
This question is about Hubble's law.

a. The light from distant galaxies is red-shifted. Explain how this red-shift arises.

[3]

b. The graph shows the variation of recession speed with distance from Earth for some galactic clusters.

[4]



(i) Calculate, in s^{-1} , the Hubble constant..

(ii) Estimate, in s, the age of the universe.

(iii) State the assumption that you made in your estimate in (b)(ii).

Markscheme

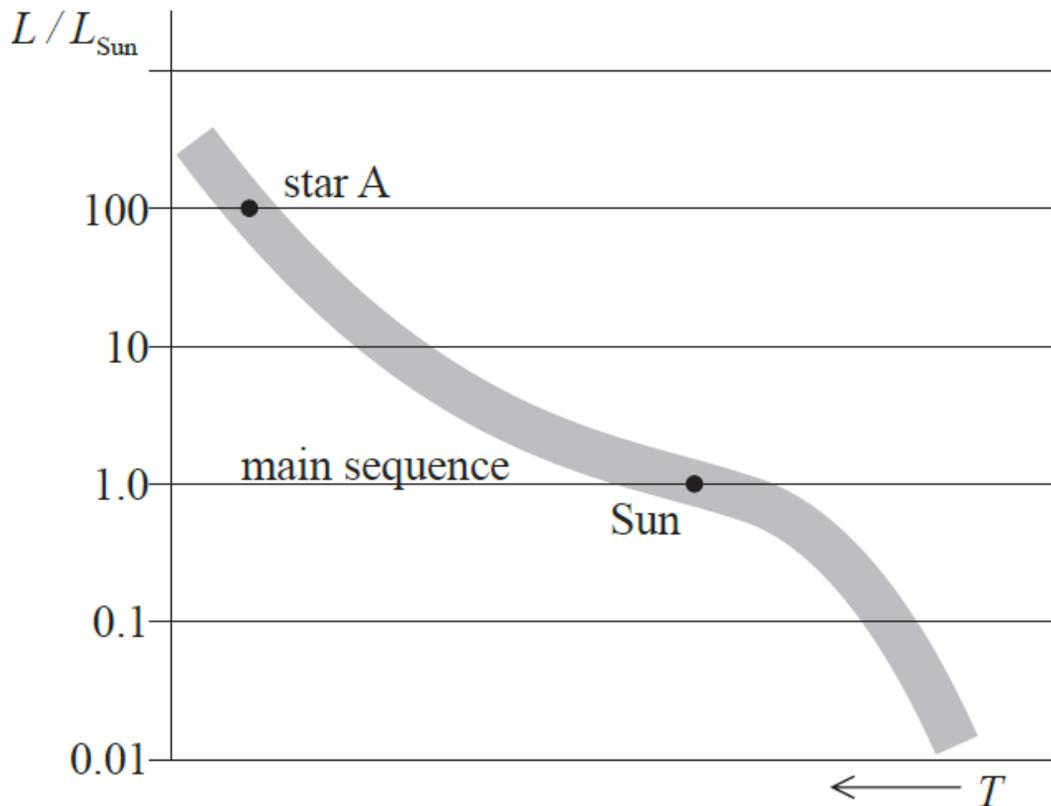
- a. (fabric of the) universe is expanding;
 so the wavelength of the light increases;
 during the time it has traveled from emitter to detector;
- b. (i) correct substitution into $H_0 = \frac{v}{d}$;
 $2.7 \times 10^{-18} \text{ (s}^{-1}\text{)}$;
 (ii) $\frac{1}{2.7 \times 10^{-18}} = 3.8 \times 10^{17} \text{ (s)} / 3.7 \times 10^{17}$;
 Allow ECF from (b)(i).
 (iii) universe has always had a constant rate of expansion;

Examiners report

- a. [N/A]
 b. [N/A]

This question is about stellar evolution.

- a. The Hertzsprung–Russell (HR) diagram shows the Sun, a star A and the main sequence. [2]



Using the mass–luminosity relation $L \propto M^{3.5}$, determine the ratio of the mass of star A to the mass of the Sun.

- b. Star A will leave the main sequence and will evolve to become a neutron star. State the [2]
 (i) change in star A that marks its departure from the main sequence.

(ii) range of mass of a neutron star.

Markscheme

a. $\frac{L_A}{L_S} = 100 = \left[\frac{M_A}{M_S} \right]^{3.5}$;
 $\left(\frac{M_A}{M_S} = 100^{1/3.5} = 3.7 \right) \approx 4$

Award marks only if a ratio is calculated.

b. (i) depletion of hydrogen in the core / fusion moves to outer layers;

(ii) $1.4M_\odot < M < 3M_\odot$;

Allow between $2M_\odot$ and $3M_\odot$ as the upper bound OV limit.

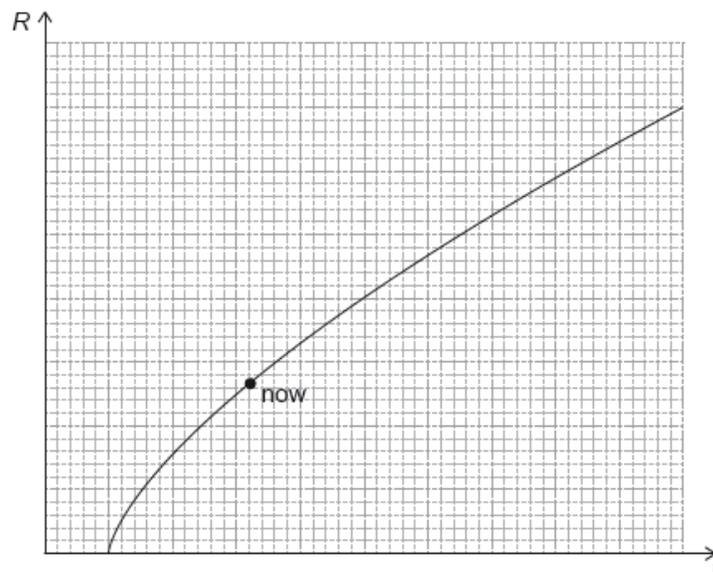
Examiners report

a. (stellar mass ratio) was well answered, although sometimes the working was poorly presented. Hydrogen depletion was usually not specifically mentioned.

b. (b)(i) as the cause for a star to leave the main sequence. Many gave unnecessary information about the subsequent path to a neutron star. For (b)(ii) the upper or lower limits for the mass of a neutron star were known, but rarely both. The range 1.4Ms to 2.5Ms or 3Ms was allowed. Often the names of the limits were given, but not the values.

Recent evidence from the Planck observatory suggests that the matter density of the universe is $\rho_m = 0.32 \rho_c$, where $\rho_c \approx 10^{-26} \text{ kg m}^{-3}$ is the critical density.

a. The graph shows the variation with time t of the cosmic scale factor R in the flat model of the universe in which dark energy is ignored. [1]



On the axes above draw a graph to show the variation of R with time, when dark energy is present.

b.i. The density of the observable matter in the universe is only $0.05 \rho_c$. Suggest how the remaining $0.27 \rho_c$ is accounted for.

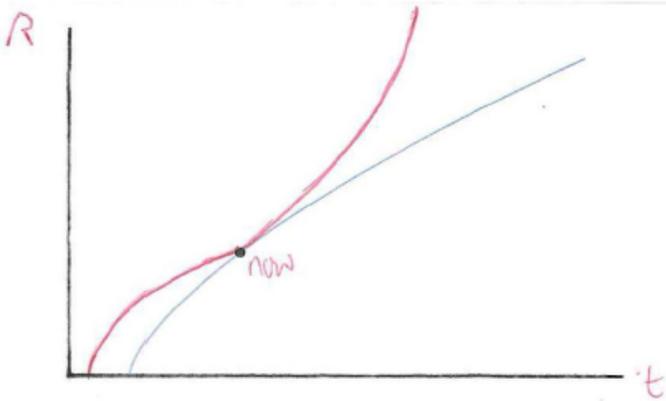
[1]

b.ii. The density of dark energy is $\rho_\Lambda c^2$ where $\rho_\Lambda = \rho_c - \rho_m$. Calculate the amount of dark energy in 1 m^3 of space.

[2]

Markscheme

a. curve starting earlier, touching at now and going off to infinity



[1 mark]

b.i. there is dark matter that does not radiate / cannot be observed

Unexplained mention of "dark matter" is not sufficient for the mark.

[1 mark]

b.ii. $\rho_\Lambda = 0.68\rho_c = 0.68 \times 10^{-26} \text{ «kgm}^{-3}\text{»}$

energy in 1 m^3 is therefore $0.68 \times 10^{-26} \times 9 \times 10^{16} \approx 6 \times 10^{-10} \text{ «J»}$

[2 marks]

Examiners report

a. [N/A]

b.i. [N/A]

b.ii. [N/A]

a. Describe what is meant by dark matter.

[2]

b. The distribution of mass in a spherical system is such that the density ρ varies with distance r from the centre as

[1]

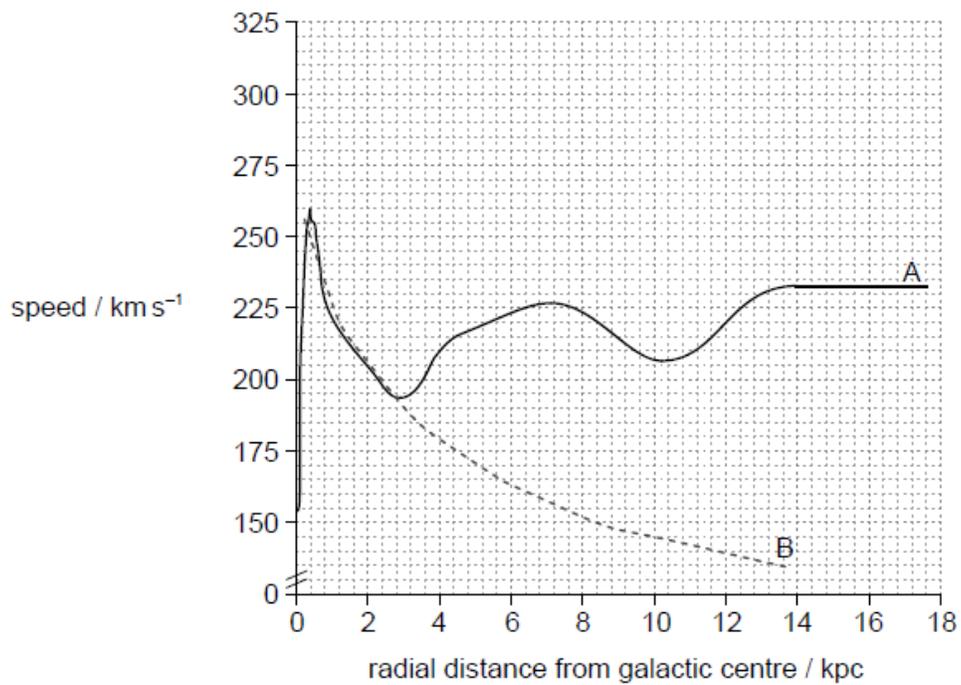
$$\rho = \frac{k}{r^2}$$

where k is a constant.

Show that the rotation curve of this system is described by

$$v = \text{constant.}$$

c. Curve A shows the actual rotation curve of a nearby galaxy. Curve B shows the predicted rotation curve based on the visible stars in the galaxy. [2]



Explain how curve A provides evidence for dark matter.

Markscheme

a. dark matter is invisible/cannot be seen directly

OR

does not interact with EM force/radiate light/reflect light

interacts with gravitational force

OR

accounts for galactic rotation curves

OR

accounts for some of the “missing” mass/energy of galaxies/the universe

OWTTE

[6 marks]

b. «from data booklet formula» $v = \sqrt{\frac{4\pi G\rho}{3}} r$ substitute to get $v = \sqrt{\frac{4\pi Gk}{3}}$

Substitution of ρ must be seen.

[1 mark]

c. curve A shows that the outer regions of the galaxy are rotating faster than predicted

this suggests that there is more mass in the outer regions that is not visible

OR

more mass in the form of dark matter

Examiners report

- a. [N/A]
 b. [N/A]
 c. [N/A]

This question is about stellar evolution.

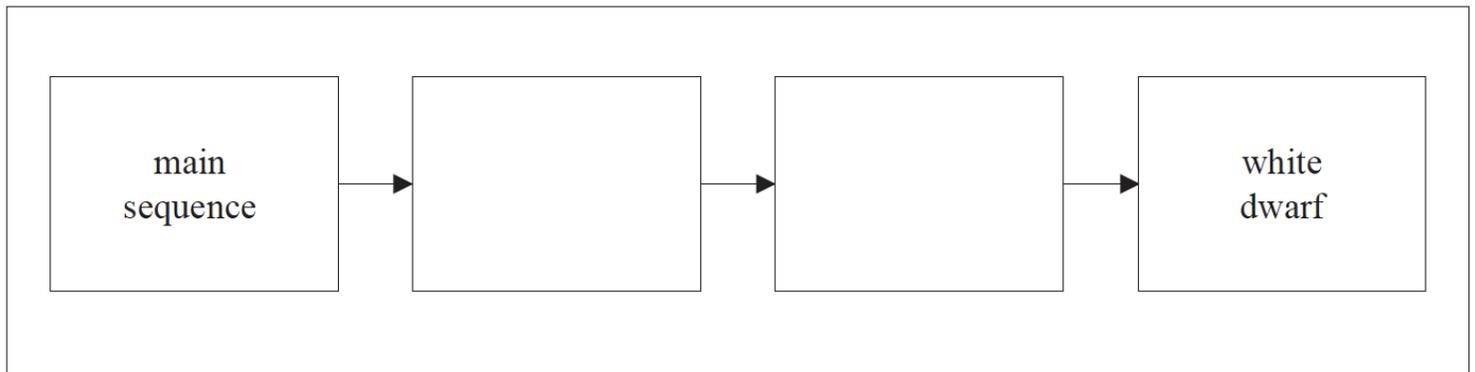
- a. A main sequence star has a mass of $2.2M_{\odot}$ where $M_{\odot} = 1$ solar mass. The lifetime of a star on the main sequence is proportional to $\frac{M}{L}$ where [3]

M is the mass and L is the luminosity of the star.

Using the mass–luminosity relation $L \propto M^{3.5}$ show that the

- (i) luminosity of the star is $16L_{\odot}$ where $L_{\odot} = 1$ solar luminosity.
 (ii) lifetime of this star on the main sequence will be approximately $\frac{1}{7}$ of the lifetime of the Sun.

- b. The star in (a) will evolve to become a white dwarf. The diagram represents the stages in the evolution of the star. [3]



- (i) On the diagram, label the **two** intermediate stages.
 (ii) State what may be deduced about the mass of this star when it is in the white dwarf stage.

Markscheme

- a. (i) $L/\text{luminosity} = 2.2^{3.5} L_{\odot}$;

$$(L \approx 16 L_{\odot})$$

Some explanatory algebra required, not just $2.2^{3.5}$.

$$\text{(ii) (since } T \propto \frac{M}{L}) \frac{T}{T_{\odot}} = \left[\frac{M_{\odot}}{M} \right]^{2.5} \text{ or } \frac{T}{T_{\odot}} = \left[\frac{M_{\odot}}{M} \right]^{-2.5};$$

$$\frac{T}{T_{\odot}} = \left(\frac{1}{2.2^{2.5}} \right) \frac{1}{7.2};$$

(the lifetime will be approximately 7 times less than that of the Sun)

or

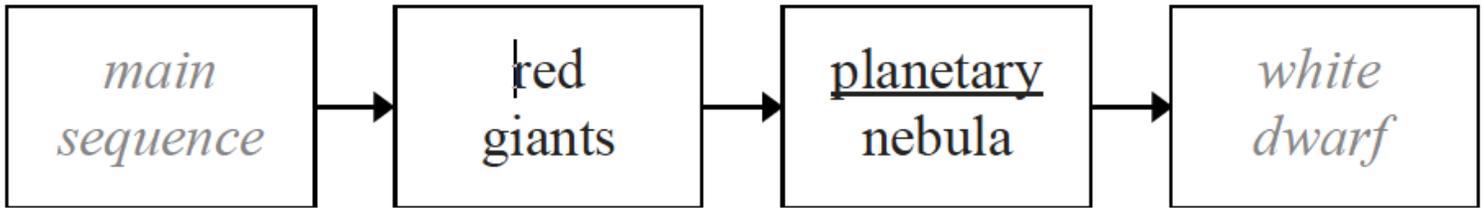
$$\frac{T}{T_{\odot}} = \frac{M}{L} \times \frac{L_{\odot}}{M_{\odot}};$$

$$= \frac{2.2}{16} = 0.14;$$

(the lifetime will be approximately 7 times less than that of the Sun)

Be careful to check that the working is valid for the value obtained.

b. (i)



(ii) less than the Chandrasekhar limit / less than $1.4M_{\odot}$;

Examiners report

a. [N/A]

b. [N/A]

This question is about stars in the constellation Canis Minor.

d. (ii) Gomeisa has a radius four times that of the Sun. Use the data in (c) to show that the ratio

[7]

$$\frac{\text{luminosity of Gomeisa}}{\text{luminosity of Sun}}$$

is about 200.

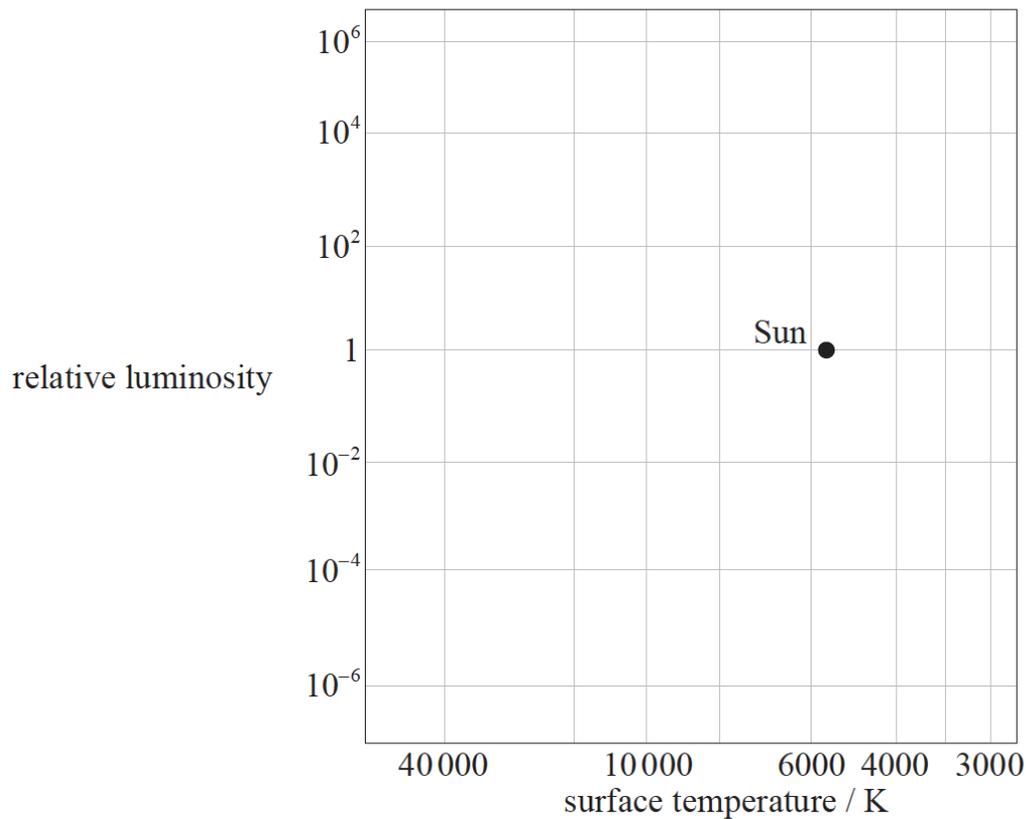
(iii) Assuming the value of n in the mass–luminosity equation to be 3.5, calculate

$$\frac{\text{mass of Gomeisa}}{\text{mass of Sun}}$$

(iv) Outline, with reference to the Chandrasekhar limit, the likely eventual fate of Gomeisa.

f.

[1]



On the HR diagram above, sketch the likely evolutionary path of Luyten's star.

Markscheme

d. (i) $2.9 = -0.7 + 51g\left(\frac{d}{10}\right)$;

$$\frac{d}{10} = 10^{\frac{3.6}{5}};$$

52(pc);

Award **[2 max]** ECF if magnitudes are reversed giving 1.9 (pc).

Award **[2 max]** if data for Luyten's star is used and no credit for the distance of 4 (pc) has already been given in (c)(i).

Award **[3]** for a bald correct answer.

(ii) $\frac{L_G}{L_S} = \left[\frac{R_G}{R_S}\right]^2 \left[\frac{T_G}{T_S}\right]^4$;

$$= 4^2 \times \left[\frac{11000}{5800}\right]^4$$

=210; (must see this answer to better than 1 significant figure)

Approximate answer of 200 is given in the question so correct steps in the working are required to award any marks.

(iii) $\frac{m_G}{m_S} = \left[\frac{L_G}{L_S}\right]^{\frac{1}{3.5}}$ / OWTTE;

allow values in the range of 4.3 to 4.6; [2]

Allow ECF from (d)(ii).

Award **[2]** for a bald correct answer.

(iv) mentions value of (Chandrasekhar limit) 1.4 solar masses;

if (remnant) mass of G is greater than the Chandrasekhar limit, it would become a neutron star; { (ignore any reference to a black hole)

if (remnant) mass of G is less than the Chandrasekhar limit, it would become a white dwarf;

Do not award both second and third marking points.

Award second or third marking point for the general idea, consistent with any value used for Chandrasekhar limit.

Allow ECF from (d)(iii).

For masses of G , from (d)(iii), which are over 8 solar masses, allow reference to a black hole as eventual fate.

- f. any anticlockwise path that goes above and right of the Sun and passes through/ends below and left of the Sun;

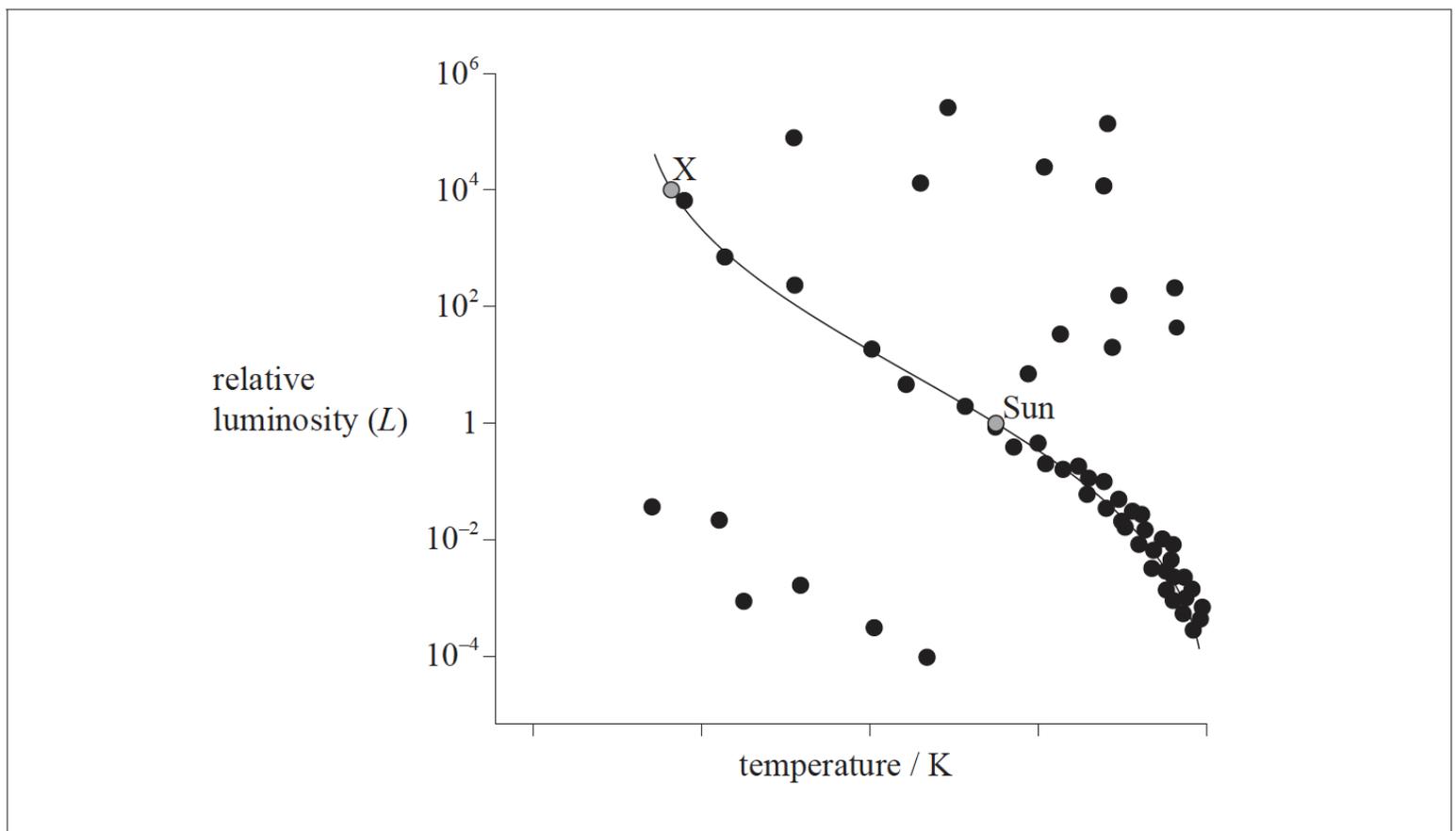
Examiners report

d. [N/A]

f. [N/A]

This question is about stellar evolution.

In the HR diagram below, the Sun and another main sequence star, X, have been marked.



- a. (i) On the diagram above, draw a line to show the evolutionary path of the Sun from its present position on the main sequence to the final stage [3] in its evolution.
- (ii) Explain, by reference to the Chandrasekhar limit, why the final stage in the evolution of the Sun is the one you indicated in (a)(i).
- b. (i) Show that the mass of star X is approximately 14 solar masses. (Assume that $n=3.5$ in the mass–luminosity relation.) [3]
- (ii) State the likely final stage of star X.

Markscheme

a. (i) any line from the sun to red giants (anywhere above and right of the sun) and then anticlockwise to white dwarfs (anywhere below and left of the sun);

(ii) after the planetary nebula stage/red giants stage;

the remnant mass/core mass of the star will be less than the Chandrasekhar limit (and so will end up as a white dwarf);

b. (i) $\frac{M_X}{M_\odot} = \left[\frac{L_X}{L_\odot} \right]^{\frac{1}{3.5}}$;

$$\frac{M_X}{M_\odot} = \left(10^{\frac{4}{3.5}} \right) 13.9;$$

Evidence of calculation required for second mark (i.e. 3 sf or more showing).

Award [2 max] for valid alternative route to this answer.

(ii) neutron star / black hole;

Examiners report

a. [N/A]

b. [N/A]

This question is about the properties of a star.

b. (i) The radius of star X is $4.5 R_S$ where R_S is the radius of the Sun. The surface temperature of the Sun is $5.7 \times 10^3 \text{K}$. [6]

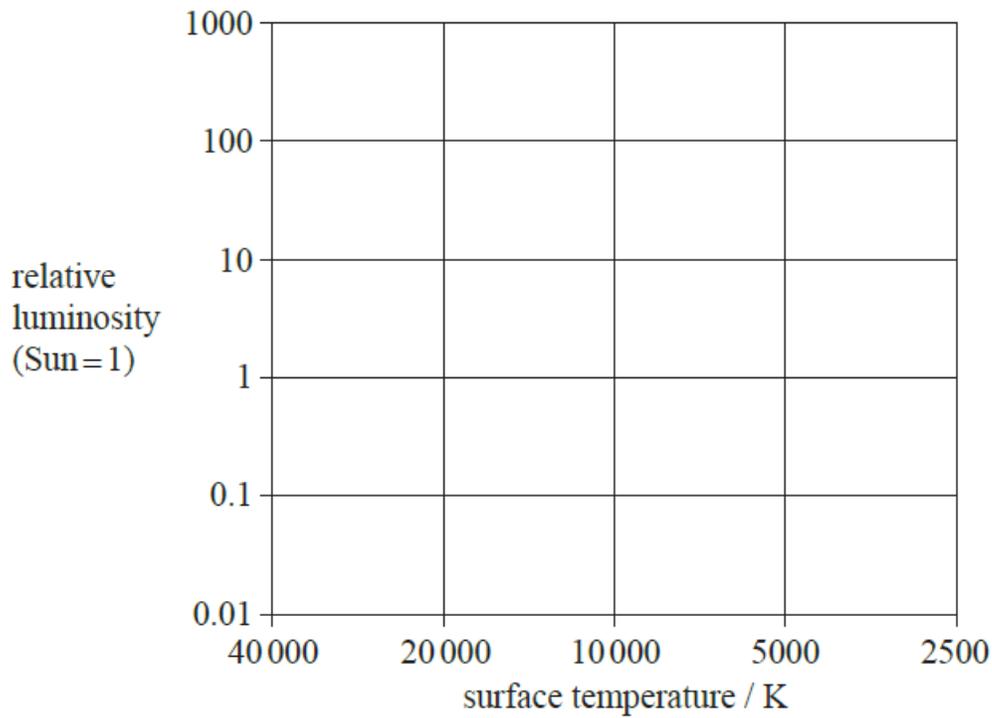
Determine the ratio $\frac{\text{luminosity of star X}}{\text{luminosity of the Sun}}$.

(ii) Calculate, assuming that the power in the mass–luminosity relationship is 3.5, the ratio $\frac{\text{mass of star X}}{\text{mass of Sun}}$.

c. On the Hertzsprung–Russell diagram, label [2]

(i) the position of star X with the letter X.

(ii) the position of the Sun with the letter S.



d. Explain, with reference to the Chandrasekhar limit, whether or not star X will become a white dwarf.

[2]

Markscheme

$$\text{b. (i) } \frac{L_X}{L_S} = \frac{\sigma r_X^2 T_X^4}{\sigma r_S^2 T_S^4};$$

$$= \frac{4.5^2 \times 9700^4}{5700^4};$$

$$= 170;$$

Accept answers that use $T=10000$ (K) to give an answer of 190.

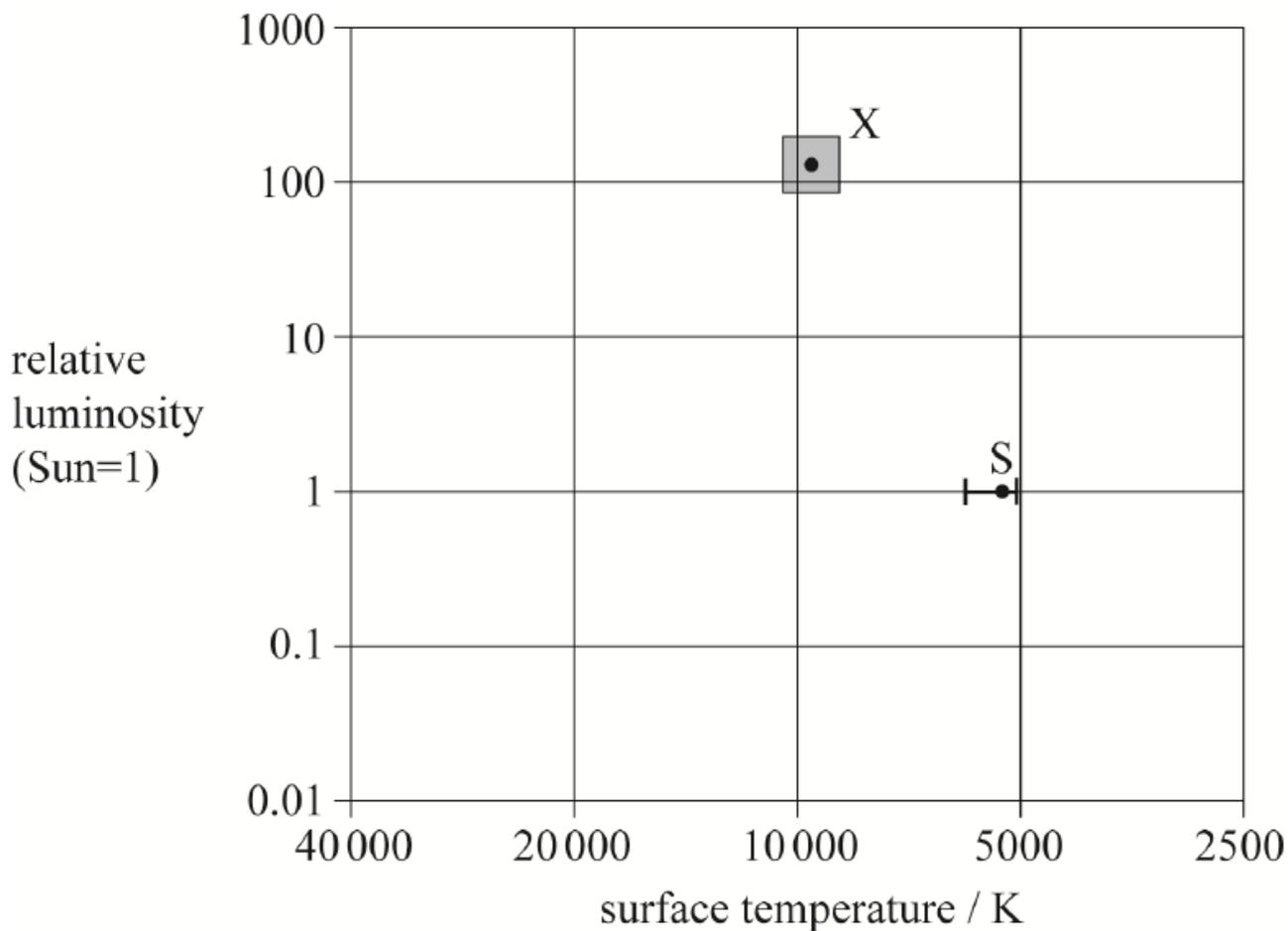
$$\text{(ii) } \frac{M_X}{M_S} = \left[\frac{L_X}{L_S} \right]^{\frac{1}{3.5}};$$

$$= [170]^{\frac{1}{3.5}};$$

$$= 4.3;$$

Award **[3]** for a bald correct answer.

c.



(i) X marked correctly within range shown;

(ii) S marked correctly within range shown;

Allow ECF from (b)(ii).

d. Chandrasekhar limit below which white dwarf stars can exist is 1.4 solar masses;

X is not going to form a white dwarf / X will not form a white dwarf if its final core/remnant mass is greater than the Chandrasekhar limit;

Allow ECF from (b)(ii).

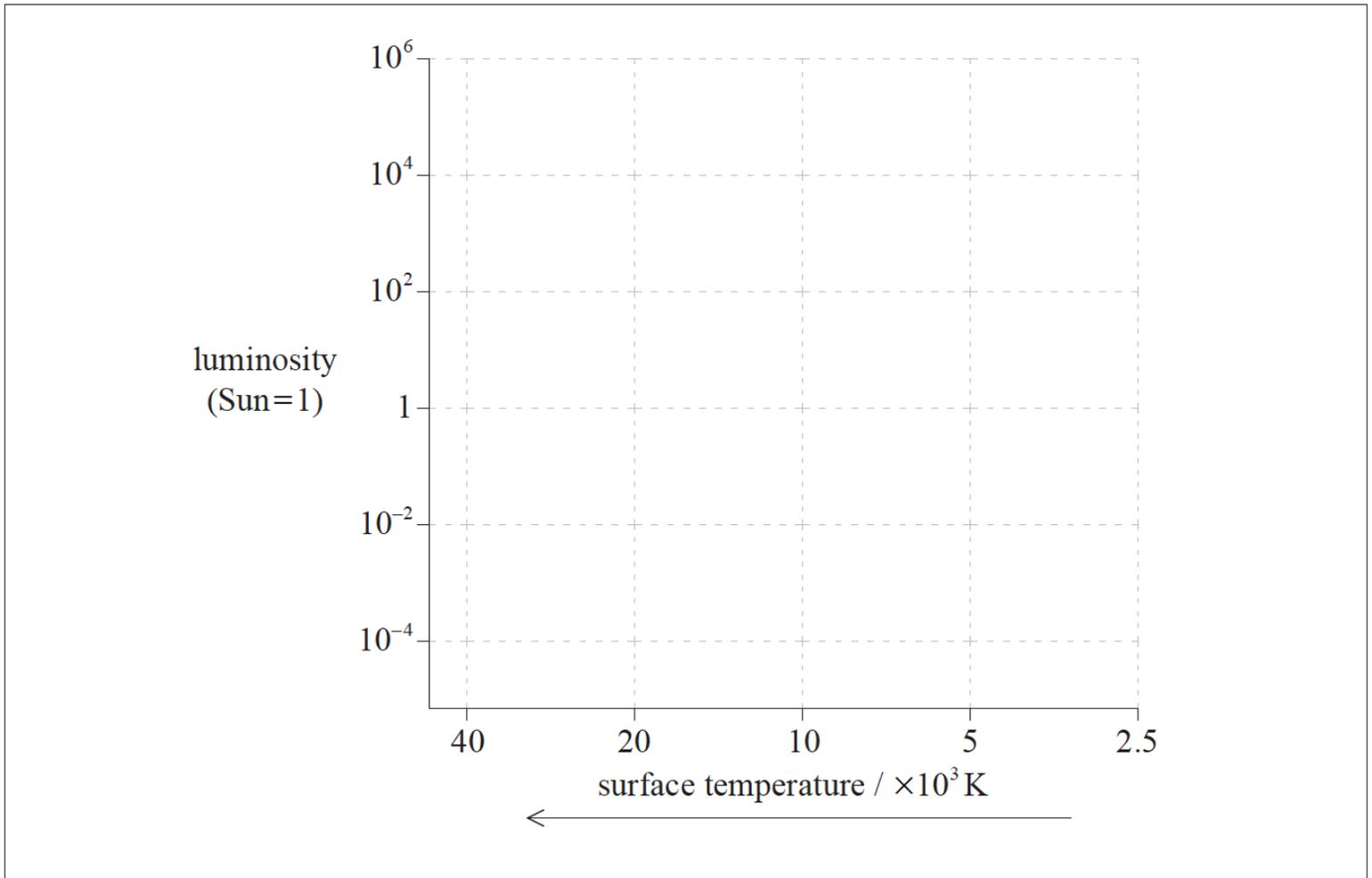
Examiners report

b. There was a lot of poor algebra and messy working. There was a much poorer understanding of ratio in this question, in particular dealing with the inverse power of 3.5.

c. was well done in general.

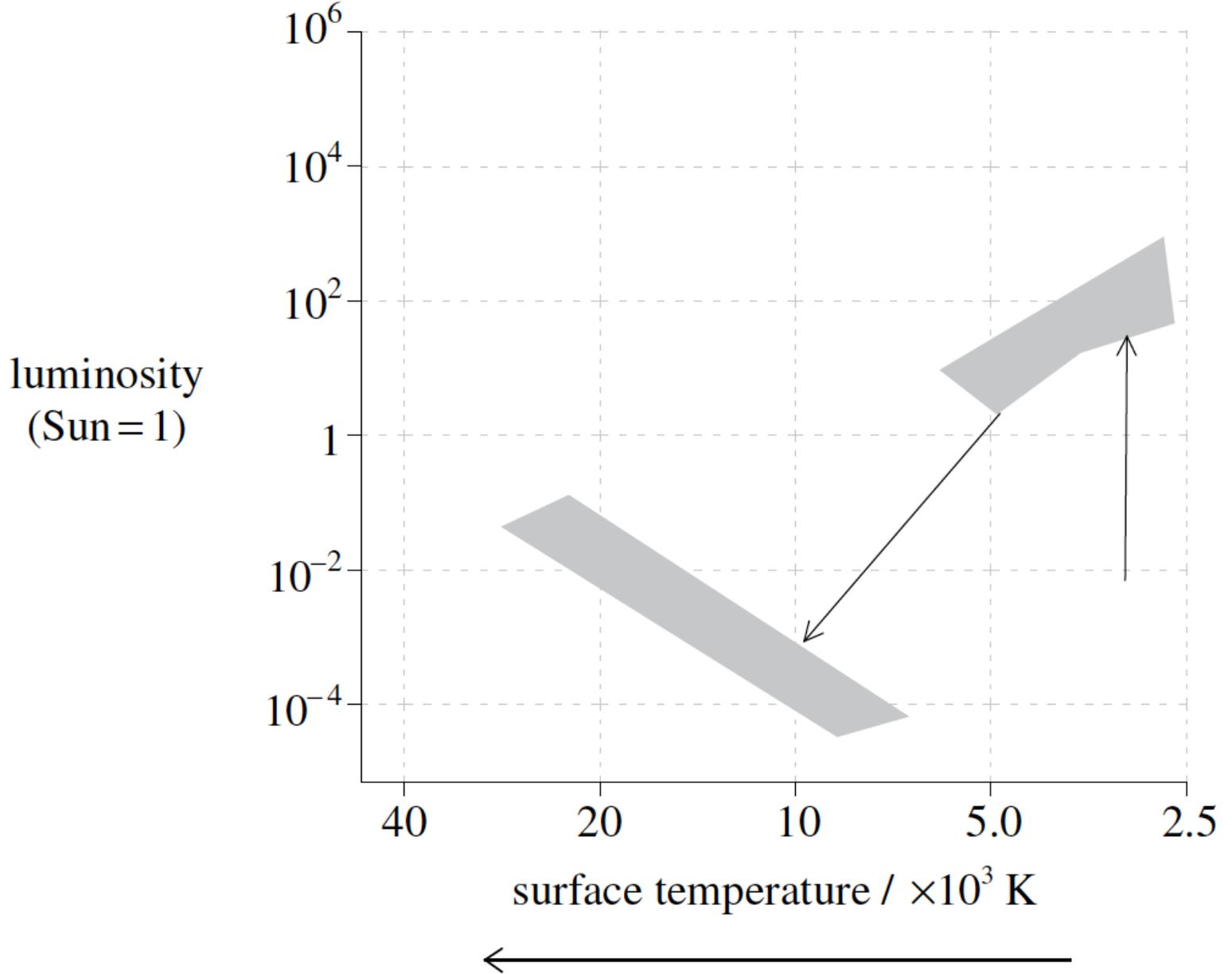
d. was well done in general, but many candidates thought that the Chandrasekhar limit was $4 M_{\odot}$. The question was slightly confusing because the mass worked out in (b)(ii) is for a main sequence star, not a remnant after a supernova. The markscheme was flexible enough to allow for candidates who took the calculated mass to be the mass of a remnant, or those who said that the star would form a white dwarf if the remnant mass was less than the Chandrasekhar limit.

This question is about stellar distances and stellar properties.



On the HR diagram on page 2, draw the evolutionary path of Barnard's star after it leaves the main sequence.

Markscheme

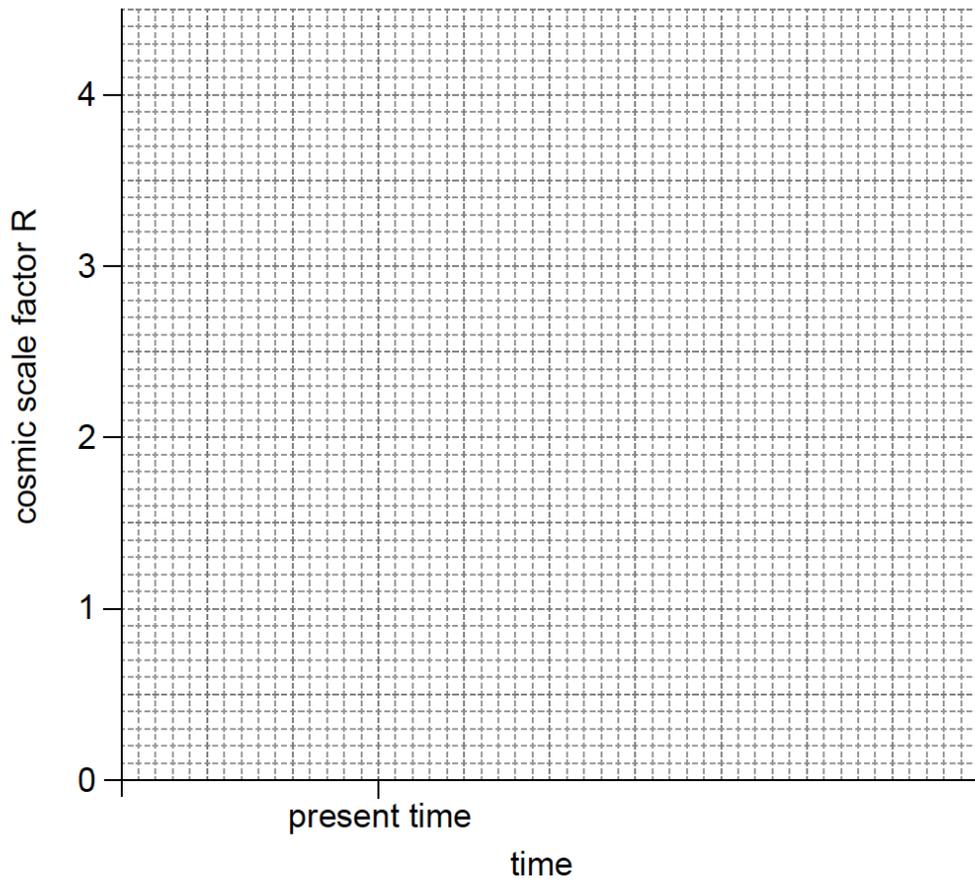


approximate starting position [$10^{-1} \rightarrow 10^{-3}$, 3000 \rightarrow 4000];
 line to region of red giants;
 line from red giants to white dwarfs;
The shaded areas are the limits within which the lines must be drawn.

Examiners report

[N/A]

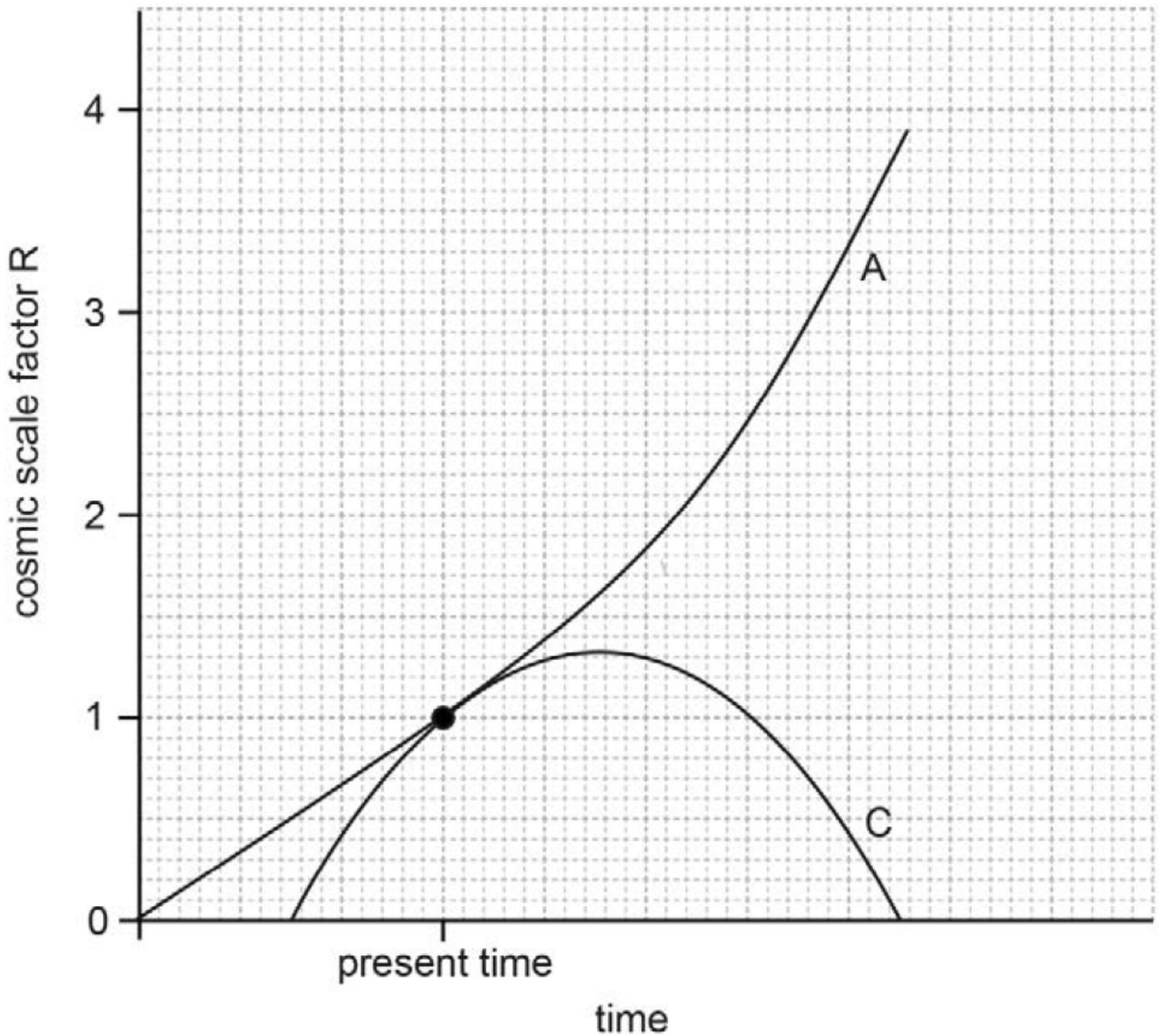
The Hot Big Bang model suggests several outcomes for the universe. There is now evidence that dark energy and dark matter exist.



- a. On the axes, sketch a graph of the variation of cosmic scale factor with time for [3]
- (i) a closed universe without dark energy. Label this curve C.
 - (ii) an accelerating universe with dark energy. Label this curve A.
- b. Explain **one** experimental observation that supports the presence of dark **matter**. [2]

Markscheme

- a. (i) curve beginning on $R=0$ before present time and ending after present time on $R=0$



(ii) curve starting earlier than C with general shape shown above coincides with curve C at present time

Judge by eye.

b. rotation speeds of galaxies is greater at the edges than expected

so the density at the edges must be greater than that supplied by luminous matter alone

Accept any other valid piece of evidence, eg gravitational lensing, which provides a good measure of galactic cluster masses.

Examiners report

a. [N/A]

b. [N/A]