Episode 525: Binding energy

Change in mass

Notice that in both these reactions the mass number and charge (proton number) are conserved. Energy, however, is only conserved if you take account of changes to the rest energy – in effect of changes to the masses – of the particles.

In Cockcroft and Walton’s experiment, the masses of the particles are:

- H: 1.0073 atomic mass units
- Li: 7.0160 atomic mass units
- He: 4.0015 atomic mass units.

An atomic mass unit, symbol u, is equal to 1.6605×10^{-27} kg.

1 Show that the mass decreases in this reaction.

$\rightarrow$

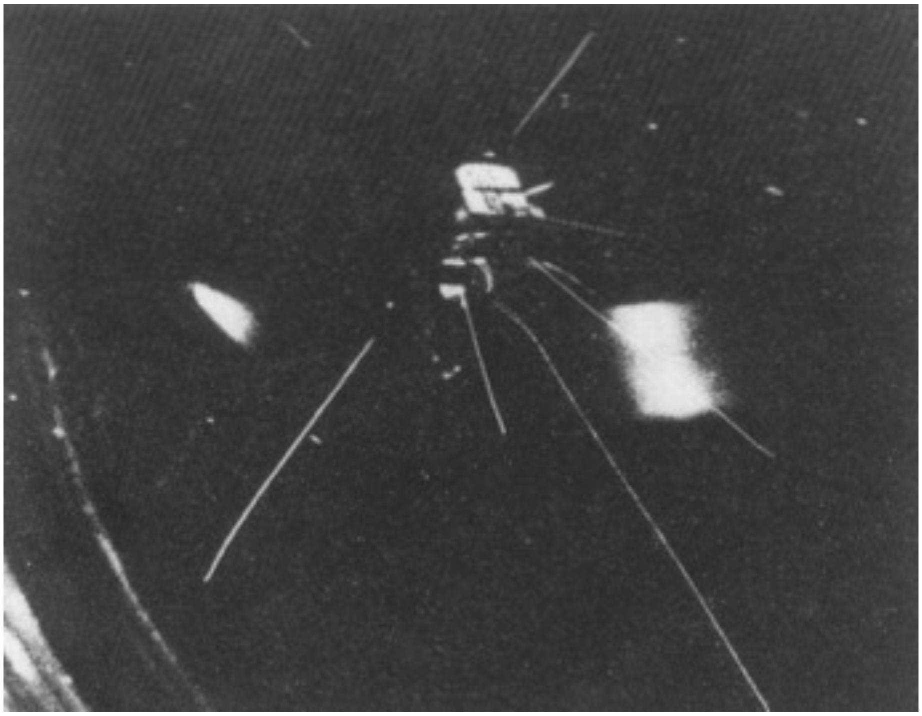
$${}^{1}_{1}\mathrm{H} + {}^{7}_{3}\mathrm{Li} \text{ ® } {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He}.$$

Calculate Δm in atomic mass units and in kilograms.

Change in energy

2 The energy of the protons was 800 000 electron volts (800 keV). The lithium was in solid form so the nuclei would only have been vibrating due to thermal energy, less than an electron volt.

The reaction was captured in this photograph:



Two pairs of alpha particles, emerging in opposite directions, can be seen in the photograph.

From the range of the tracks through the cloud chamber the energy of the alpha particles was measured to be 8.5 MeV each.

Show that the total kinetic energy of the particles increases, and calculate ΔE in MeV and in joules.

3 If the increase in kinetic energy comes from the decrease in rest energy you should expect $\Delta E = \Delta mc^2$. Calculate the ratio of the change in kinetic energy to the change in mass $\Delta E/\Delta m$ in J kg^{-1}.

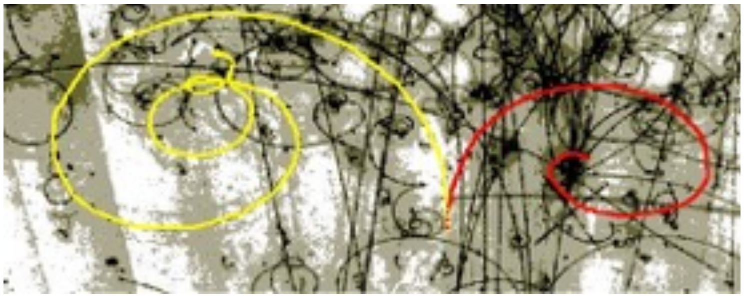
4 Show that the value of the ratio $\Delta E/\Delta m$ is approximately consistent with the relationship $\Delta E = \Delta mc^2$.

The large value of c^2 (9×10^{16} J kg^{-1}: use this value from now on in calculations) means that a small change in mass represents a vast change in rest energy. This relationship between mass and energy is why particle physicists measure masses in MeV / c^2; any unit of energy divided by c^2 is a unit of mass.

Creating massive particles

Energy is 'materialised' in matter–antimatter production. A photon of electromagnetic radiation can produce an electron and a positron. In this case, the energy of the photon vanishes and the rest energy of the particles appears. (This reaction needs to take place near to the nucleus of a heavy atom to conserve momentum but this is not going to affect your calculations here.)

In this bubble chamber photograph a photon enters from the bottom. It is uncharged and so produces no observable track. After some distance the photon disappears and produces the electron–positron pair. These two charged particles ionise the liquid in the chamber and bubbles form near the ions and are photographed.



In this case the chamber is filled with liquid hydrogen mixed with liquid neon. It is held under pressure which is released just as the particles enter the chamber to encourage bubbles to form and enlarge near the ions.

5 The bubble chamber is in a magnetic field, so charged particles bend due to the force Bqv on a moving charge. How does the photograph show that the two particles have opposite charges?

6 The mass of the electron is 5.5×10^{-4} u. What is the minimum energy photon that will produce an electron–positron pair? From what part of the electromagnetic spectrum is this? (Planck constant $h = 6.63 \times 10^{-34}$ J Hz^{-1}.)

Nuclear binding energy

If protons and neutrons (together known as nucleons) are bound together in a nucleus, the bound nucleus must have less energy than the nucleons of which it is made. That is, the rest energy of the nucleus must be less than the sum of the rest energies of its nucleons. In turn, this means that the mass of the nucleus must be less than the sum of the masses of its nucleons.

The simplest compound nucleus is the deuteron, the nucleus of hydrogen-2. It consists of a proton and a neutron bound together by the strong nuclear force. The masses of these particles are:

- proton: 1.0073 u
- neutron: 1.0087 u
- deuteron: 2.0136 u.

7 Calculate the difference in mass between a deuteron and one proton and one neutron.

8 Calculate the binding energy of the deuteron in J and in MeV.

9 Calculate the binding energy per nucleon of the deuteron.

10 Express the difference in mass as a percentage of the sum of the masses of the proton and neutron.

Mass change in nuclear fission

A possible reaction for the nuclear fission of uranium-235 is:

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{133}_{51}\mathrm{Sb} + {}^{99}_{41}\mathrm{Nb} + 4\,{}^{1}_{0}\mathrm{n}.$$

The masses of the particles are

- U-235 = 235.0439 u
- Sb-133 = 132.9152 u

- Nb-99 = 98.9116 u
- neutron (n) = 1.0087 u.

11 Show that the energy change per atom of uranium is about 200 MeV and calculate Δm/m.

Summary

Einstein's famous equation $E_{rest} = mc^2$ reveals a Universe that is not as simple as it seems at first sight. The mass of a particle is generally a very large part of its total energy. The existence of rest energy was not suspected until after Einstein had predicted it, because the change in mass is usually so small, because changes in energy are usually a small fraction of the rest energy. Only in nuclear reactions where Δm/m ~ 0.1% or more are you able to see the change in mass, accompanied by what appears to be a huge change in energy.

Hints

1 Compare masses of H plus Li with mass of two He nuclei.

2 Two 8.5 MeV alpha particles come out, but one 800 keV proton goes in.

3 Compare the answers to questions 1 and 2.

4 Don't expect to get exactly the speed of light. Remember to take the square root of c^2!

5 What is the difference between forces F and - F?

6 Start with the mass of an electron in atomic mass units. Convert to kilograms. Write down the mass of an electron–positron pair. Use $E_{rest} = mc^2$ to get the rest energy of the pair in joules. Then use E = hf.

7 Do this one in the same way as question 1.

8 $E_{rest} = mc^2$ again. But now use the electron charge to get to electron volts and MeV.

9 How many nucleons in a deuteron?

10 Best to take the difference as a fraction of the mass before.

11 Add up before and after masses in atomic mass units first. Don't forget there's one extra neutron to start with and four extra neutrons afterwards. Then convert mass changes first to joules and then to MeV.

Answers and worked solutions

1.

$$\text{Mass of H plus Li} = 1.0073\ \text{u} + 7.0160\ \text{u} = 8.0233\ \text{u}$$

$$\text{Mass of two He} = 2 \times 4.0015\ \text{u} = 8.0030\ \text{u}$$

$$\text{Difference } \Delta m = 8.0030\ \text{u} - 8.0233\ \text{u} = -0.0203\ \text{u}$$

So we can find the mass difference in kg:

$$\Delta m = -0.0203\ \text{u} \times 1.6605 \times 10^{-27}\ \text{kg} = -3.3708 \times 10^{-29}\ \text{kg}$$

2.

Increase in energy:

$$\Delta E = 2 \times 8.5\ \text{MeV} - 0.8\ \text{MeV} = 16.2\ \text{MeV}$$

In joules:

$$\Delta E = (16.2 \times 10^{6}\ \text{eV}) \times (1.6 \times 10^{-19}\ \text{J eV}^{-1}) = 2.6 \times 10^{-12}\ \text{J}$$

3.

$$\frac{\Delta E}{\Delta m} = \frac{2.60 \times 10^{-12}\ \text{J}}{3.37 \times 10^{-29}\ \text{kg}} = 7.7 \times 10^{16}\ \text{J kg}^{-1}.$$

4. If $\Delta E = \Delta mc^2$, then $c^2 = 7.7 \times 10^{16}$ J kg^{-1}, so $c = 2.8 \times 10^{8}$ m s^{-1}.

5. The force on a moving charged particle is Bqv. If the charge q changes sign, the direction of the force is reversed, so the curvature is opposite.

6. The mass of an electron or positron is equal to:

$$(5.5 \times 10^{-4}\ \text{u}) \times (1.66 \times 10^{-27}\ \text{kg}) = 9.1 \times 10^{-31}\ \text{kg}.$$

From $E_{rest} = mc^2$, the rest energy of an electron–positron pair is:

$$E_{rest} = 2 \times 9.1 \times 10^{-31}\ \text{kg} \times (3 \times 10^{8}\ \text{m s}^{-1})^2 = 1.6 \times 10^{-13}\ \text{J}.$$

If this energy is supplied by a photon of energy *E* = hf, then:

$$f = \frac{1.6 \times 10^{-13}\ \text{J}}{6.63 \times 10^{-34}\ \text{J Hz}^{-1}} = 2.5 \times 10^{20}\ \text{Hz}.$$

This is the frequency of a gamma ray.

7. The mass difference is:

$$2.0136\ \text{u} - (1.0073\ \text{u} + 1.0087\ \text{u}) = -0.0024\ \text{u}.$$

In kg the mass difference is:

$$-0.0024\ \text{u} \times (1.66 \times 10^{-27}\ \text{kg}) = -3.98 \times 10^{-30}\ \text{kg}.$$

8.

$$\begin{aligned} \text{Binding energy} &= -3.98 \times 10^{-30}\ \text{kg} \times (3 \times 10^{8}\ \text{m s}^{-1})^2 \\ &= -3.58 \times 10^{-13}\ \text{J} \\ &= -\frac{3.58 \times 10^{-13}\ \text{J}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} = -2.2 \times 10^{6}\ \text{eV} \\ &= -2.2\ \text{MeV}. \end{aligned}$$

9. The deuteron has two nucleons so the binding energy per nucleon is
–2.2 MeV / 2 = –1.1 MeV.

10. As a percentage the mass difference is equal to:

$$\frac{0.0024\ \text{u}}{1.0073\ \text{u} + 1.0087\ \text{u}} = 1.2 \times 10^{-3} \times 100 = 0.1\%\ \text{(approximately)}$$

11.

$$\text{Mass after} = 132.9152\ \text{u} + 98.9116\ \text{u} + (4 \times 1.0087\ \text{u}) = 235.8616\ \text{u}$$

$$\text{Mass difference} = 236.0526\ \text{u} - 235.8616\ \text{u} = 0.191\ \text{u}.$$

$$\text{Change in rest energy} = \frac{0.191\ \text{u} \times (1.66 \times 10^{-27}\ \text{kg}) \times (3 \times 10^{8}\ \text{m s}^{-1})^2}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} = 1.78 \times 10^{8}\ \text{eV} = 178\ \text{MeV}.$$

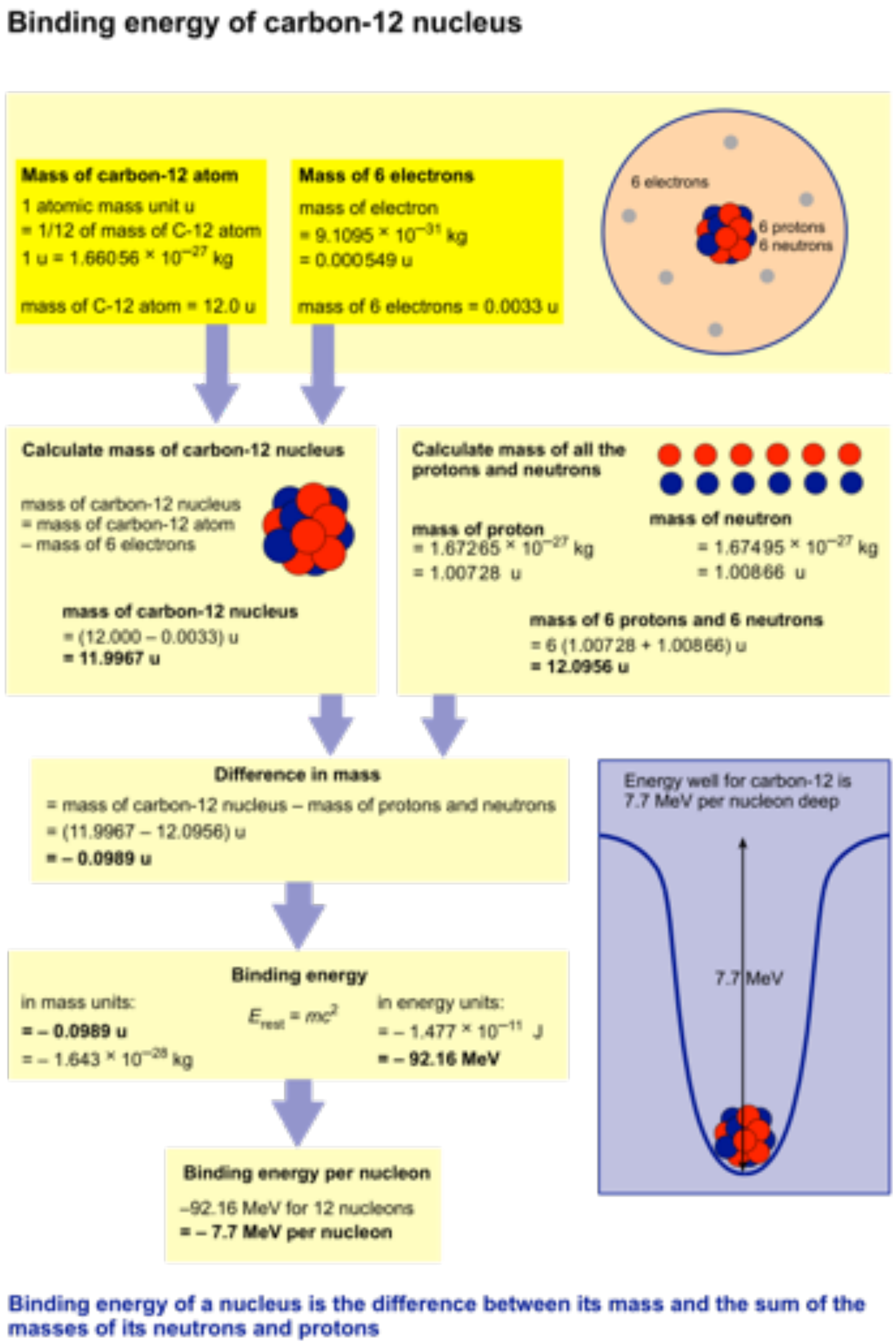
The ratio is given by:

$$\Delta m / m = 0.191\ \text{u} / 236\ \text{u} = 8.1 \times 10^{-4} \sim 0.1\%.$$

External reference

This activity is taken from Advancing Physics chapter 18, 200S

TAP 525-2: Finding binding energy



Binding energy of carbon-12 nucleus

Mass of carbon-12 atom

1 atomic mass unit u

= 1/12 of mass of C-12 atom

1 u = 1.66056×10^{-27} kg

mass of C-12 atom = 12.0 u

Mass of 6 electrons

mass of electron

= 9.1095×10^{-31} kg

= 0.000549 u

mass of 6 electrons = 0.0033 u

Calculate mass of carbon-12 nucleus

mass of carbon-12 nucleus
= mass of carbon-12 atom
– mass of 6 electrons

mass of carbon-12 nucleus
= (12.000 – 0.0033) u
= 11.9967 u

Calculate mass of all the protons and neutrons

mass of proton
= 1.67265×10^{-27} kg
= 1.00728 u

mass of neutron
= 1.67495×10^{-27} kg
= 1.00866 u

mass of 6 protons and 6 neutrons
= 6 (1.00728 + 1.00866) u
= 12.0956 u

Difference in mass

= mass of carbon-12 nucleus – mass of protons and neutrons
= (11.9967 – 12.0956) u
= – 0.0989 u

Binding energy

in mass units:
= – 0.0989 u
= -1.643×10^{-28} kg

$E_{rest} = mc^2$

in energy units:
= -1.477×10^{-11} J
= – 92.16 MeV

Binding energy per nucleon

–92.16 MeV for 12 nucleons
= – 7.7 MeV per nucleon

Binding energy of a nucleus is the difference between its mass and the sum of the masses of its neutrons and protons

TAP 525-3: Fusion in a kettle?

A change of scale

When you are confident with basic calculations of fission and fusion energy changes, you should work through these questions that try to put the energies of these changes into a more human scale for you. You will also need to understand the conversion of atomic mass units to energy and the meaning of the term ‘electron volt’.

Try these

One of the reactions that fuel the stars is the fusion of two protons to give deuterium. In turn the deuterium goes through a series of reactions, the end product being helium. This is also a process that releases energy. In this question you are asked to consider the energy that would be released if all the deuterium in the water contained in an electric kettle were to be converted by fusion into helium.

The kettle contains 1 litre of water. The data you need are listed below.

1 atomic mass unit (u) = 931 MeV

1 eV = 1.6×10^{-19} J

$N_A = 6.02 \times 10^{23}$ mol^{-1}

Particle	Mass / u
$^{1}_{1}H$	1.007 825
$^{2}_{1}H$	2.014 102
$^{3}_{2}He$	3.016 030
$^{1}_{0}n$	1.008 665

1 Two deuterium nuclei $^{2}_{1}H$ can fuse to give one nucleus of helium $^{3}_{2}He$ with the ejection of one other particle. Write down the balanced equation that represents this reaction.

2 Calculate the mass change that occurs in this reaction.

3 Convert this energy into joules.

This gives you the energy released when two deuterium nuclei fuse. The next steps take you through the calculation of the total energy released if all the deuterium in the kettle water were to fuse to make helium-3. The ratio of deuterium atoms to hydrogen in water is roughly 1 to 7000.

4 What is the mass of 1 mole of water (H = 1 u; O = 16 u roughly)?

5 How many moles of water are contained in the litre?

6 How many molecules of water (H_2O) are in the kettle?

7 How many molecules of deuterium oxide (D_2O) are in the kettle?

8 Each heavy water molecule has two atoms of deuterium; what total energy is released if all the deuterium in the kettle is converted to helium-3?

Now to put this number in a new perspective. It requires 4200 J to increase the temperature of 1kg of water by 1K.

9 How many litres of water could be heated through 100 K by the fusion energy you calculated in question 8?

Hints

1 It is important to consider the atomic electrons in this equation. You begin with two, one for each hydrogen. How many electrons does an un-ionised atom of deuterium have? So what must one of the emitted particles be? This should lead you to the other particle.

2 The conversions you need are near the data table in the question.

4 The formula of water shows that there are two hydrogen atoms and one oxygen for each water molecule.

5 1 litre of water has a mass of 1 kg.

6 1 mole contains 6×10^{23} molecules of water.

Practical advice

These questions can be modified in many ways, not least by changing the homely example of a kettle to perhaps a bath full of water or even to Lake Windermere or the local reservoir.

Social and human context

The 6000 litres of heated water may not seem so significant until you realise that this has come from the fusion of deuterium which had an original volume of 0.15 cm^3.

Answers and worked solutions

1.

$${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$

2. Δm = (3.016 030 u + 1.008 665 u) – 2 × 2.014 102 u = – 0.0035 u

3. 0.003509 u × 931 × 10^{6} eV u^{-1} × 1.6 × 10^{-19} J eV^{-1} = 5.23 × 10^{-13} J

4. 18 g

5. 1 litre of water has a mass of 1 kg.

   number of moles = 1000 g / 18 gmol^{-1} = 56 mol

6. 56 mol × 6.02 × 10^{23} mol^{-1} = 3 × 10^{25}

7. (3.4 ×10^{25})/7000= 4.9 × 10^{21}

8. energy released = 4.9 ×10^{21} × (5.23 ×10^{-13} J) = 2.49 × 10^{9} J

9. (2.49 ×10^{9} J)/ (4200 J kg^{-1} K^{-1} × 100K) = 6000 kg = 6000 litres

External reference

This activity is taken from Advancing Physics chapter 18, 260S