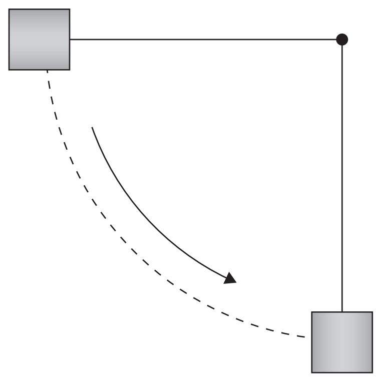
QUESTION ONE: THE SWINGING MASS

A mass *m*, connected to an inextensible light rope of length *L*, is allowed to swing down from a horizontal position to the vertical as shown in the diagram.



(a) (i) Show that the maximum tension in the rope is $3mg$.

(ii) Explain why the length of the rope does not affect the maximum tension.

(b) (i) The rope is replaced by a piece of extensible rubber (spring constant k) of the same length, L.

Show that the maximum tension reached in the rubber when the mass is allowed to swing down is given by:

$$T_{\text{max}} = \frac{3mg(L + x)}{L + 2x}$$

where L is the unstretched length of the rubber, and x is the maximum extension of the rubber.

(ii) Explain why, despite the mass falling a greater distance, the maximum tension in the rubber is less than the maximum tension in the rope.

(8)

QUESTION TWO: BUNGY JUMPING

Acceleration due to gravity = 9.81 m s^{-2}

Standing on a platform that is 25.0 m above a river, Emma, of height 2.00 m and mass m, is tied to one end of an elastic rope (the bungy) by her ankles, while the other end of the bungy is fixed to a platform. The length of the bungy is adjusted so that Emma's downward motion stops at the instant her head reaches the water surface. When Emma is at rest, in equilibrium, at the end of the bungy, her head is 8.00 m above the water. The unstretched length of the bungy is L, and it has a spring constant of k. Assume Emma's centre of mass is halfway up her body.

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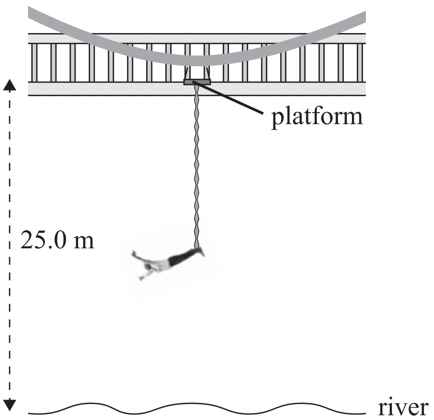
The bungy jump site

http://thebostonjam.files.wordpress.com/2011/11/heidi-jump1.jpg?w=474



(a) By considering energy conservation, show that at the lowest point in the jump, $mgh = \frac{1}{2}\text{k}(23 - L)^2$, where h is the change in height of Emma's centre of mass.

Explain all reasoning.

(b) Show that, at the equilibrium position, $mg = \text{k}(15 - L)$.

(c) Show that the value of L is 13.0 m.

(d) (i) Calculate Emma's maximum speed.

(ii) Calculate Emma's maximum acceleration.

(e) Explain what will happen to the spring constant of the bungy when its length is reduced by 50%.

(8)

QUESTION THREE: COLLISIONS (8 marks)

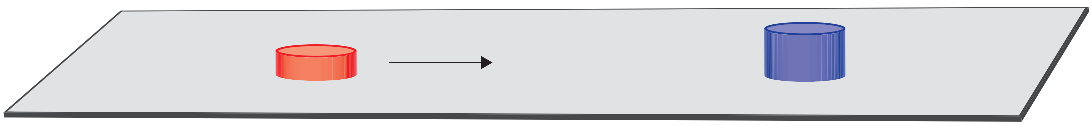
A red puck of mass 2.0×10^{-3} kg is set moving along a long, frictionless track towards a blue puck of mass 4.0×10^{-3} kg. The red puck has a velocity of 21 m s^{-1}, while the blue puck is at rest, but free to move. Both pucks carry a positive charge of 1.5×10^{-6} C. The pucks meet along the line of their centre of mass in an elastic interaction.

The charges cause an electrostatic force between the two pucks.

When they are separated by a distance r, the repulsive force is given by $F = \dfrac{kQ_1Q_2}{r^2}$.

The electric potential energy is given by $E_p = \dfrac{kQ_1Q_2}{r}$.

Q_1 and Q_2 are the two charges and k is a constant ($= 9.0 \times 10^9$ N m^2 C^{-1}).



- 2.0×10^{-3} kg
- 21 m s^{-1}
- +1.5 μC

- 4.0×10^{-3} kg
- +1.5 μC

(a) Describe (without calculations) the motion of each puck as it interacts with the other.

(b) At the instant of closest approach, both pucks have the same velocity.

Explain why this is so, and show that the velocity is 7.0 m s^{-1}.

(c) If the pucks are initially 10 m apart, show that the electrostatic potential energy at this separation is negligible compared with the kinetic energy of the moving puck.

(d) By considering the kinetic and electrostatic energies, calculate the distance of closest approach.

(e) Calculate the final velocities of the pucks (when they are a long distance apart).

(8)

QUESTION FOUR: ELEMENTS OF MECHANICS (8 marks)

(a) If a light object (small mass) and a heavy object (large mass) have the same kinetic energy, explain which of the two has the larger momentum.

(b) (i) A television safety advertisement features a car taking corners at dangerously high speeds. The danger is symbolised by land-mines appearing scattered around the corners. As the vehicle approaches a corner, the voice-over says "There is more force taking you off the road and less force keeping you on it." The car skids across the road and rolls over an embankment.

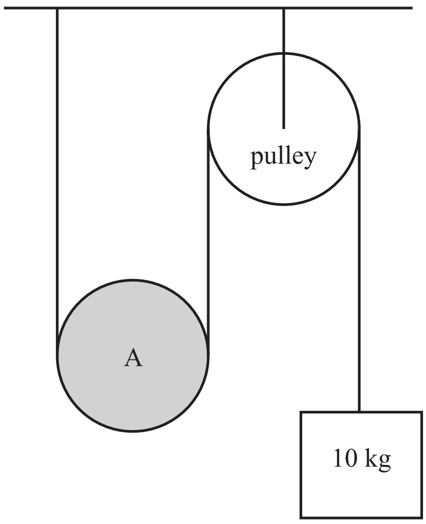
Discuss the accuracy of the voice-over statement, with reference to centripetal force and friction.

(ii) In another safety advertisement, a car is shown going into a skid. We are told that if the driver had started the skid while doing 50 km h^{-1}, he would have stopped at point A. However he started the skid while doing 60 km h^{-1}.

Calculate his speed when he reached point A.

(c) A light, frictionless belt carries a smooth cylinder (A), as shown in the diagram. The 10 kg mass is falling at constant velocity.

Assume the pulley is frictionless.



Calculate the mass of cylinder A.