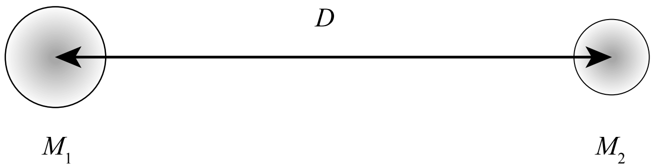
QUESTION ONE

A binary star consists of two stars of masses M_1 and M_2 separated by a distance D and both rotate about their stationary centre of mass.



(i) Express the distance, r, of the star of mass M_1 from the centre of mass in terms of M_1, M_2 and D.

(ii) By equating the gravitational force between the stars with the magnitude of the centripetal force on one of them, show that the orbital period of the binary system, T, is related to the distance between the stars, D, by the relationship:

$$\frac{T^2}{D^3} = \frac{4\pi^2}{G(M_1 + M_2)}$$

QUESTION TWO (6 marks)

Universal Gravitational Constant = 6.67×10^{-11} N m^2 kg^{-2}

(a) The fastest rate of rotation of a planet is that for which the gravitational force on matter at the equator just provides the centripetal force necessary for that matter to move with circular motion.

Show that the period of rotation in this case is given by

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

where the planet is assumed to be a uniform sphere of density ρ (which has units of kg m^{-3}).

The volume of a sphere is $\frac{4}{3}\pi r^3$.

(b) The mean density of the Earth is 5500 kg m^{-3}. Is Earth close to disintegration?

(c) The reading on a bathroom scale is less at the equator than at the poles. Suggest TWO reasons for this.

(6)

QUESTION THREE: ARMAGEDDON (8 marks)

Universal gravitational constant = 6.67×10^{-11} N m^2 kg^{-2}

The volume of a sphere = $\frac{4}{3}\pi r^3$

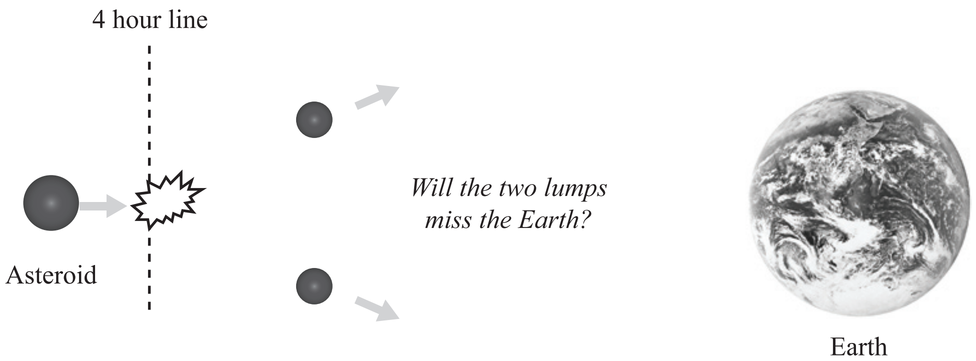
The movie "Armageddon" is based on an asteroid "the size of Texas", which is about to collide with the Earth. To protect the Earth, NASA proposes to land a drilling team on the asteroid, who will drill a hole to the centre and detonate a nuclear warhead there. This blast is supposed to split the asteroid into two equal pieces that will each move far enough sideways to safely miss the Earth. However, there is not much time and the explosion will take place only four hours before impact.

(a) NASA scientists expect that the blast will not affect the motion of the centre of mass.

Explain.

(b) Using the information and diagram below, work out if the Earth will be saved. Ignore any gravitational attraction between the two lumps, and list any other assumptions you make during your calculation.

Width of Texas = 1.45×10^6 m
Average density of the asteroid = 3 000 kg m^{-3}
Energy released by the nuclear warhead = 5×10^{18} J
(this is equivalent to 10^5 Hiroshima bombs)



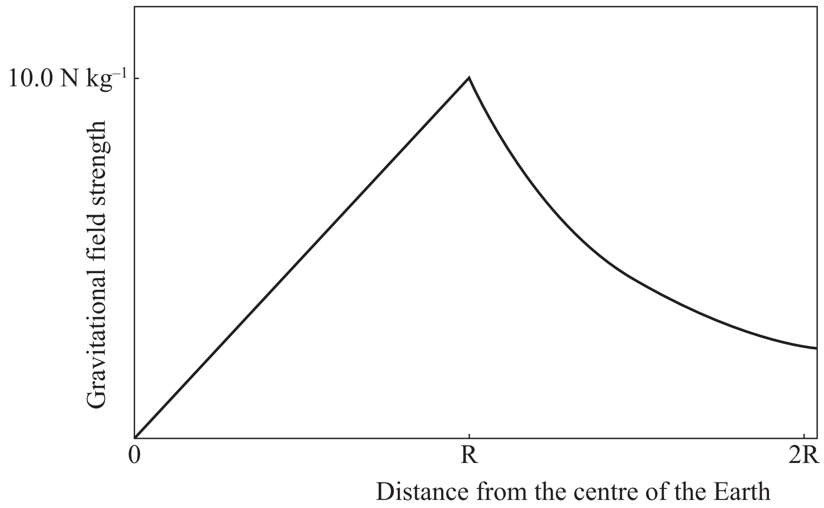
(c) In reality the two pieces will gravitationally attract each other.

Assuming the asteroid splits into two equal-sized spherical lumps, calculate the acceleration caused by the gravitational force of each sphere on the other. State the significance of the result.

(8)

QUESTION FOUR: GRAVITY (8 marks)

Some text books show the change in gravitational field strength (g) as one moves outwards from the centre of the Earth, by using a graph, such as the one following, where R is the radius of the Earth.



(a) Explain, using physical principles, why the gravitational field strength (g) is zero at the centre of the Earth.

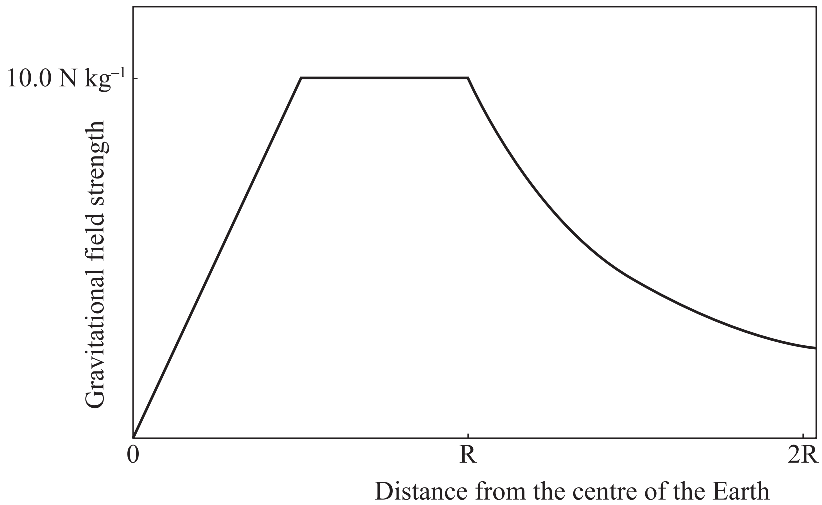
(b) An object within a hollow sphere experiences no gravitational force from the mass of that sphere.

Use this result, and Newton's law of gravitation, to show that g increases linearly as one moves from the centre of the Earth outwards to the surface.

Note: volume of sphere $= \frac{4}{3}\pi r^3$ and density $= \frac{\text{mass}}{\text{volume}}$.

(c) Explain why g decreases as shown, when one moves outwards from the Earth's surface.

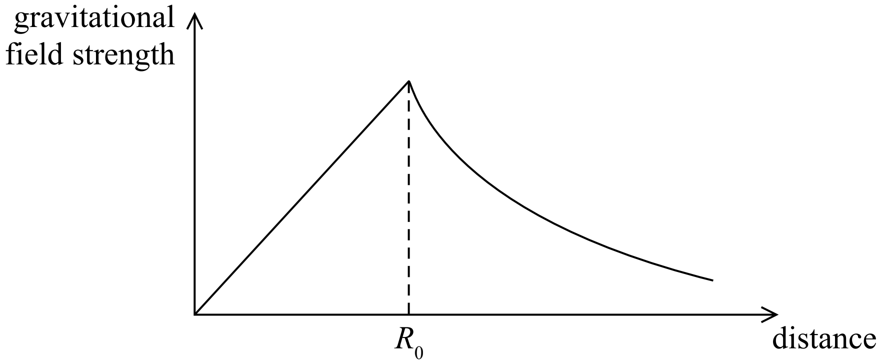
(d) In fact, the variation in g is more accurately shown in this graph.



What does this graph tell us about the Earth's interior structure?

(8)

5 The graph shows the variation of the gravitational field strength with distance from the centre of the Earth. R_0 is the radius of the Earth.



(a) Describe how gravitational field strength varies with distance from the centre of the Earth

- for distances between 0 and R_0
- for distances greater than R_0

(3)

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(b) A scientist suggests the following:

> "If a tunnel were made through the centre of the Earth, an object dropped at one end would accelerate downwards until it reached the centre. It would then decrease in speed until it reached the other end of the tunnel with a speed of zero. The object would then return the other way, undergoing simple harmonic motion."

Using the graph between 0 and R_0, determine whether simple harmonic motion would occur.

(4)

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(c) The scientist also suggests that the period of oscillation for a body dropped through the tunnel would be the same as the orbital period for a body orbiting just above the surface of the Earth. Its radius of orbit is assumed to be R_0.

(i) Derive an expression for the period of oscillation of the body dropped through the tunnel.

(4)

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(ii) Derive an expression for the orbital period for a body that is orbiting the Earth with radius R_0.

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(Total for Question 5 = 14 marks)

Qno.6

The value of the acceleration due to gravity, g decreases as $\frac{1}{r^2}$ where r is the radial distance measured from the centre of the Earth (this follows from Newton's Law of Gravitation).

a) By what percentage is g less than the value of 9.81 m s^{-2} measured at the surface of the Earth, when measured at the height of a satellite orbit of 300 km above the Earth's surface?

b) What would be the value of g at the distance of the Moon (the Moon is 400,000 km away from the Earth)?

Radius of Earth = 6,400 km

(5 marks)

Qno.7

Johannes Kepler used Tycho Brahe's detailed observations on planetary motion, made without the use of telescopes, to determine the elliptical orbits of the planets. He also ascertained that the square of the period of orbit T is proportional to the cube of R (Kepler's Third Law). The radial distance R for a planet is the simple arithmetic average of the closest distance of approach to the Sun, R_{min} and the furthest distance from the Sun, R_{max}.

(a) Sketch a diagram of a planetary orbit, marking on it R_{min} and R_{max}.

[1]

(b) From the statement above, write down two equations, the first one relating T and R with a constant of proportionality k, and a second equation relating R, R_{min} and R_{max}.

[2]

(c) The average distance of the Earth from the Sun is defined as 1 Astronomical Unit (1 AU). Determine the value of k for part (a), including units.
(The period T can be measured in years).

[1]

(d) Halley's Comet also orbits the Sun and so the value of k is the same as in (b). Its period of orbit is 75.3 years. Determine the value of R for its orbit about the Sun.

[1]

(e) The closest distance of approach to the Sun for the comet is 0.585 AU, when it is visible to the naked eye. Calculate the furthest distance of the comet from the Sun.

[1]

(f) The comet's speed is 70.6 km s^{-1} at closest approach to the Sun. Is the speed greater or smaller than this at the comet's furthest distance from the Sun? Give a reason for your answer.

[3]

(g) As a man-made satellite orbits the Earth, there is always a point on the Earth directly below it. This point follows the path of a satellite's orbit and is plotted on a map of the Earth, as shown below in **Figure 4**. Describe or sketch the satellite's orbit i.e. how it is oriented about the Earth, and its shape.

[3]

(h) This orbit is known as a Molnya orbit and is used for some spy satellites. Apart from the obvious feature that it covers Russia and the USA, what is its advantage?

[1]

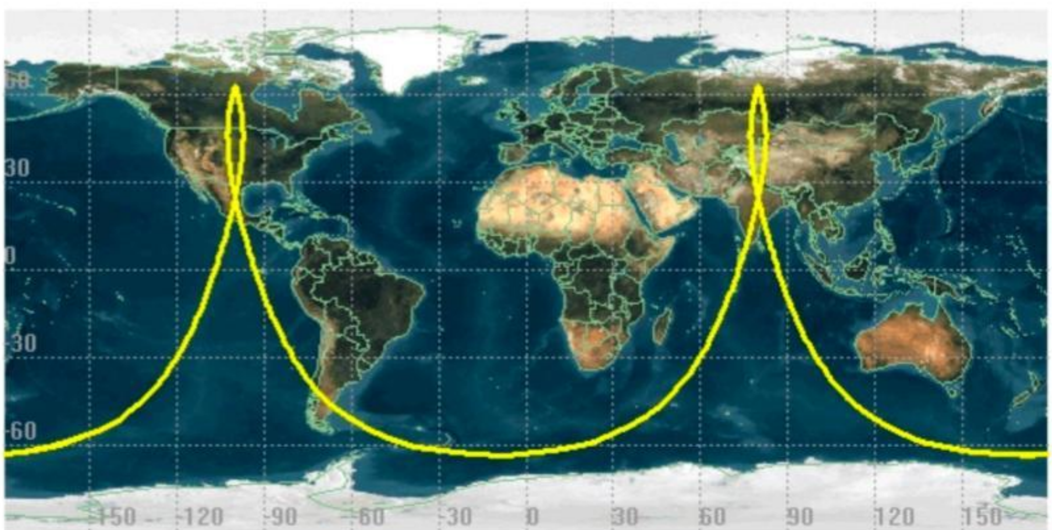

Figure 4. *Path of a satellite in a Molnya orbit around the Earth.*

(13 marks)

Qno.8

A star with a diameter larger than that of the Sun can collapse to form a neutron star which has a diameter of only a few kilometres. As the core collapses to form a neutron star, its electrical conductivity becomes very high. This results in the magnetic field lines being trapped in the collapsing matter so that the field lines become denser and the field strength increases.

If the magnetic field pattern is similar to that of a bar magnet, as shown in Fig. 7 below, then as the star radius r decreases, its cross sectional area decreases, and the density of the field lines at the equator changes as $1/r^2$.

If the radius decreases from 1.4×10^6 km to 10 km, and the initial magnetic field strength of the star at the equator is 10^{-2} T, calculate the magnetic field strength of the neutron star at the equator.

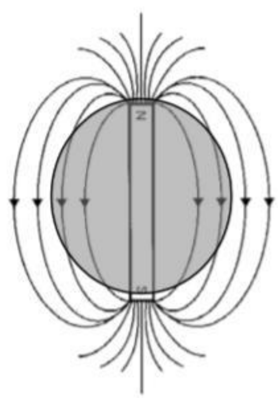

Fig. 7

(4 marks)

Qno.9

This question contains several sections that are independent of each other.

A star more massive than the sun can collapse under its own gravity to form a neutron star. Here the electrons and protons combine to form neutrons. A star which is initially rotating and collapses will rotate with a shorter period, as its rotational momentum (called angular momentum) is **conserved**. The angular momentum of the star is given by, $J=\Omega R^2$ (ignoring some constant), where R is the radius of the star and Ω is the angular rate of rotation in radians per second (2π radians is one full rotation).

a) If the volume of the star decreases by 15 orders of magnitude, and the shape of the star remains the same, then what is the ratio of the final to initial radii, $\frac{R_{final}}{R_{initial}}$?

b) If J is conserved, and $\Omega_{initial} = \frac{2\pi}{20 days}$, then calculate Ω_{final} in radians/second and also the new period of rotation in seconds.

c) As the core of the star collapses to form the neutron star, the electrical conductivity becomes very high. In this case the star's magnetic field lines become frozen into the material of the star and collapse down with the star, increasing the flux density. The neutron star will thus have a very strong magnetic field. If we take the flux $\Phi = BR^2$, with B being the magnetic field strength whose initial value is 10^{-2} T, then determine the final magnetic field strength after the collapse.

d) If a neutron star spins too fast, it will start losing material from its equatorial region. Show that this implies a minimum period, T_{min} given by

$$T_{min} = const \cdot M^{-\frac{1}{2}} R^{\frac{2}{3}}$$

$$\text{where } const = \frac{2\pi}{\sqrt{G}}$$

Taking M=1.4 solar masses, R=10 km, then calculate T_{min}.

e) The binding energy of a neutron star is the gravitational potential energy lost when it is formed from a cloud of atoms all separated a great distance apart. The binding energy of a star of mass, M, and radius, R, is given by $BE = k_1 \frac{GM^2}{R}$, where k_1 is a numerical constant. Neutron stars do not behave in the same way as ordinary matter and they have a mass-radius relationship given by $RM^{\frac{1}{3}} = k_2$ where k_2 is a constant. Two neutron stars of identical mass collide and form a more massive star. Assuming the mass-radius constraint holds, what is the ratio of the final binding energy of the star to the total initial binding energy?

Mass of the sun = 2.0×10^{30} kg
G = 6.67×10^{-11} Nm^{2}kg^{-2}

(16 marks)

Qno.10

The Sun emits light (and other parts of the electromagnetic spectrum). The light can be described in terms of particles called photons. The energy of a single photon E_{ph} is given by $E_{ph} = hf$, where h is Planck's constant and f is the frequency of the light. The photon also has a momentum, somewhat like the particles in a gas, and will produce a force F on a reflecting surface, given by $F = 2E_{ph}/c$ (the factor "2" is because the photons arrive and get reflected back).

A spacecraft can be driven by a 'solar sail', of unknown dimensions, which consists of a large sheet of reflective material kept facing the Sun. When a photon from the Sun hits the sail it is reflected off and, as a result, the sail experiences a force. **Consider one square metre of area**. If n photons (per square metre) are reflected each second, then the average force will be given by

$F_{av} = \frac{2nE_{ph}}{c}$ per square metre.

As photons spread out radially from the Sun, the intensity of photons (the number crossing each square metre of a sphere surrounding the Sun every second) follows an inverse square law, *i.e.* n is proportional to $\frac{1}{r^2}$, where r is the distance measured from the centre of the Sun.

A solar sail fixed to a spacecraft, with the full area of the sail facing the Sun, causes an acceleration of 1.2 mms^{-2} when the spacecraft is far from the Sun, crossing the orbit of Jupiter. With this low mass satellite you can ignore any other effects.

Assume that the photons correspond to a wavelength of 549×10^{-9} m. The total number of photons reaching the Earth from the Sun, $n_E = 3.6 \times 10^{21}$ m^{-2} s^{-1}.

Data: $\lambda_{light} = 549 \times 10^{-9}$ m
Number of photons per square metre reaching the Earth each second = 3.6×10^{21} m^{-2} s^{-1}.
Distance Sun - Earth = 150×10^6 km
Distance Sun - Jupiter = 780×10^6 km
Mass of spacecraft = 150 kg

a) Calculate the area of the sail, *A*.

[Here is a set of quantities that might prove useful in obtaining the result: E_{ph}, n_J (at Jupiter), ma, total force on the sail].

<u>Work out any quantities that you can</u> and make it clear what each result is. There is no particular order.

b) If the mass of the spacecraft was about five times greater, the acceleration drops to almost zero. This is true at Jupiter's orbital distance from the Sun or the Earth's orbital distance, or in fact at any distance from the Sun. Why is this?

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____ [2]

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Qno.11

An early measurement of the speed of light was made by a Danish scientist Ole Rømer in the 1670s. He observed the period of orbit of Io, the closest known moon of Jupiter at that time. The mean time interval between successive eclipses of Io by Jupiter is equal to its period of orbit, 42 h 28 min 42 s. However, for some months during the Earth year (region **A** in the Earth's orbit), the period of Io's orbit increased by a few seconds, whilst during other times (region **B** in the Earth's orbit) the period decreased.
It can be assumed that radius of Jupiter's orbit about the Sun is much larger than the Earth's.

a) Sketch a diagram of the orbits of the Earth (showing its direction of motion) and Jupiter to indicate where (**A** and **B**) these variations from the mean period would be greatest.

Diagram:

[2]

b) The radius of the Earth's circular orbit is 1.5×10^{11} m and its period can be taken as 365 days. If the variation of the period of the orbit of Io is up to 15 s longer or shorter than the mean, what value does this give for the speed of light?

[3]

c) Sketch a graph on the axes showing the variation in the period of Io's orbit for one Earth year. Give values on the axes and mark on **A** and **B**.

[1]

/6

Qno.12

The neutral point between the Earth and the Moon is the point where the gravitational pull of the Moon is equal to the gravitational pull of the Earth. If the energy a 1000 kg spacecraft needs in order to reach the neutral point from the Earth is 6.0×10^{10} J and to reach the neutral point from the Moon is 0.25×10^{10} J, what is the minimum energy needed to send a 1 kg rock from the Moon to the Earth?

A. 0.25×10^7 J B. 5.75×10^7 J C. 6.0×10^7 J D. 6.25×10^7 J

Qno.13

The intensity of radiation received at the Earth from the Sun is about 1.4 kW m^{-2}. The faintest stars in the Milky Way Galaxy have intensity at the earth of about 10^{-20} of the solar intensity. At what distance away should a 100 W bulb be placed in order to produce the same intensity as the faintest stars?

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____ [4]