Question 1

This question relates to changes occurring when a material is deformed.

a. A long, thin bar of metal is stetched elastically. Its initial dimensions are 1.0000 mm x 1.0000 mm x 1000.0 mm. After the stretching, these dimensions became 0.9997 mm x 0.9997 mm x 1001.0 mm. You will notice that the bar became thinner as well as elongated.

i) Calculate the change in volume of the bar. Has it increased or decreased?

ii) In what way is your conclusion consistent with there being the same amount of material in the bar both before and after the stretching?

iii) The *Poisson Ratio* of a material is the ratio *lateral strain: longitudinal strain.* (strain is change in length/original length).
Calculate the Poisson Ratio for this sample of metal.

b. Rubber has the property of deforming at virtually constant volume.

i) A long sample of rubber with uniform square cross-section is stretched to a strain of 2%. By what proportion should the cross-sectional area reduce to maintain a constant volume?

ii) Hence determine the percentage reduction in each side of the square cross-section.

iii) From this calculate the Poisson Ratio for rubber.

6 marks

Question 2

This question investigates the effect of scaling in living organisms and engineering models.

a)

(i) A human femur (thigh bone) has a diameter of approximately 30 mm. Assuming that it carries half the weight of a person of mass 70 kg, calculate the stress in the femur when the person is standing normally.

(ii) Without calculation, suggest what will happen to this stress when the person stands on one leg, then begins to run.

b) In the story *Gulliver's Travels*, the land of *Brobdingnag* is inhabited by giants. Gulliver estimates that the giants are approximately 100 times as big as humans in all dimensions.

(i) By what factor does the mass of an average Brobdingnagian exceed the mass of a human?

(ii) Therefore, by what factor does the stress in a Brodingnagian's femur exceed that of a human femur?

(iii) The breaking stress of bone may be taken as 100 MPa. What does this tell us about the way a Brobdingnagian might walk and run?

c) Engineers sometimes make use of scale models to examine aspects of new designs. Use the outcomes of the illustration above to comment on the usefulness of scale models in predicting the strength of a lightweight bridge structure.

12 marks

Question 3

There are several factors which determine the maximum height of a mountain. Everest at only 8 km is not very high in terms of the maximum height that can be attained given the strength of the rock. All mountains on Earth suffer from erosion which reduces their height significantly. The maximum height is limited by the rock flowing under the enormous weight above it, which is related to the Young's Modulus value for the rock, E. We can suggest that an equation for the maximum height of a mountain would depend upon the density of the rock, ρ, Young's Modulus, E, and the strength of Earth's gravity, g. An insight into the solution of a problem can often be made by looking at the dimensions of the relevant physical quantities.

a) E is a measure of how much the rock deforms when a load is applied to it. E has units of N m^{-2}. Write down the units of E, ρ and g in terms of meters, kilograms and seconds (m, kg, s).

b) If the height of the mountain is given by the formula h = constant x E x ρ^{α} x g^{β}, by comparing the units on the left and right hand sides of the equation (the constant has no units), determine the values of α and β and write down the equation for h.

c) If the value of E for rock is 10^{10} Pa, the density of rock is 3 x 10^{3} kg m^{-3} and the value of the constant is 1, estimate to one significant figure the height of a mountain that will be given by the formula. (Such a mountain is to be found on Mars).

(9 marks)

Question 4

A man has a typical mass of 70 kg and the minimum cross sectional area of the bones in each leg is approximately 5.0×10^{-4} m^2. The compressive breaking stress of bone is approximately 1.0×10^7 Nm^{-2}. If the man stands with his weight equally supported by each leg, calculate the following:

(a) The maximum stress in his leg bones

(2 marks)

(b) The ratio of the maximum stress to the breaking stress

(1 marks)

If a giant grew to such a size that each of the linear dimensions of his body increased by a factor of nine calculate:

(c) The mass of the typical giant

(2 marks)

(d) The new ratio of the maximum stress to the breaking stress

(2 marks)

(e) Is the giant able to stand on one leg?

(1 marks)

[8]

(I) A smooth ball of radius 10.0 cm, mass 0.600 kg, hangs by a weightless string from a support. What is the speed of a horizontal wind necessary to keep the string inclined at 39° to the vertical? Make the assumption that the wind speed drops to zero on collision with the ball. The density of the air is 1.293 kg m^{-3}.

[4]

Question 5

a) **Figure 6** shows a simplified model of a crystal, with atoms represented by spheres and the bonds between them represented by springs, of spring constant k. Each bond is of length a, and a stress σ is applied to the front and rear faces of the model such that a force F is applied to each bond going from front to rear. The stress causes each bond in the front to rear direction to extend by an amount x. (It should be noted that this is a highly simplified model: the primitive cubic arrangement, although common for compounds, is very unusual for elements.)

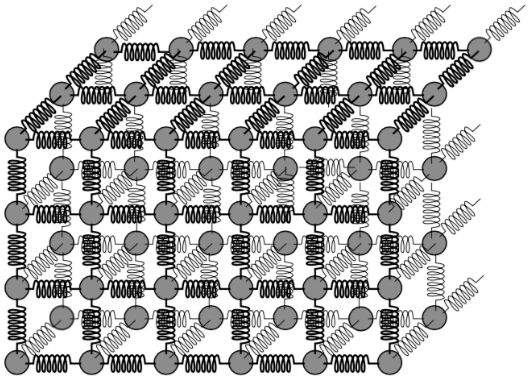


Figure 6

The coordination number of the atoms in a solid is equal to the number of bonds acting on each atom within the body of the solid (i.e. not those on the surface).

(i) What is the coordination number for the arrangement shown in **Figure 6**?
(ii) What area on the front face of the model is associated with each atom?
(iii) Derive an expression for the stress, σ, in terms of F and a.
(iv) Find the strain in the front-to-rear bonds.
(v) Hence derive an expression for the Young Modulus E of the material in terms of k and a.

[5 marks]

b) A more realistic model, applicable to many elements, is to consider what is known as a close-packed layer, as shown in **Figure 7**.

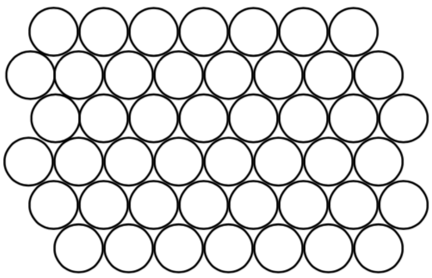


Figure 7

(i) What is the co-ordination number for atoms in the middle of this single layer?
(ii) Now consider placing more identical atoms in the dimples on the top of this layer to form a new close-packed layer. How many atoms in the new layer are in contact with any given atom in the layer beneath?
(iii) Hence, what is the co-ordination number of atoms in the middle of a structure consisting of many close-packed layers stacked on top of each other?
(iv) When this structure is heated it is often erroneously stated that it melts because the bonds are weakened. If all the bonds remained intact but were of lower spring constant, what change would actually be observed in the macroscopic behaviour of the material? Give a brief reason for your answer.

(v) In fact melting occurs when a small proportion or the bonds rupture and are therefore able to re-form with different partners, thus enabling relative motion of the atoms. Use the data below to estimate the proportion of bonds which typically break when simple solids melt.

Specific latent heats for some simple elements:

Element	SLH(fusion)/$\mathrm{kJ\,kg^{-1}}$	SLH(vaporisation)/$\mathrm{kJ\,kg^{-1}}$
argon	29.5	161
helium	3.45	20.7
hydrogen (H_2)	59.5	445
krypton	16.3	108
neon	16.8	84.8

[8 marks]

Question 6

The ball-release mechanism of a pinball machine is shown in Fig. 17.1.

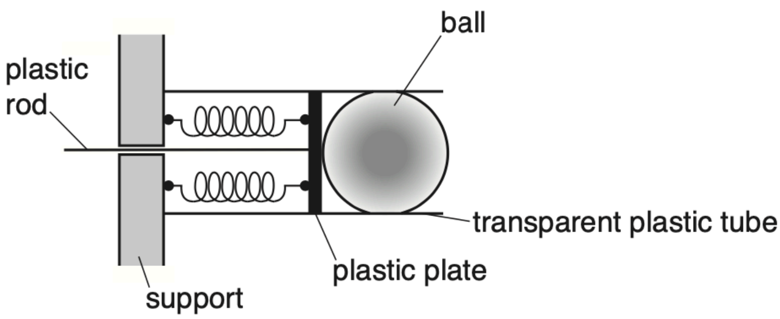


Fig. 17.1

A pair of identical compressible springs are fixed between a plastic plate and a support. The springs are in parallel. A plastic rod attached to the plate is pulled to the left to compress the springs. A ball, initially at rest, is fired when the plate is released.

(a) The force constant of each spring is *k*.
Explain why the force constant of the two springs in parallel in Fig. 17.1 is equal to 2*k*.

....

....

.... **[1]**

(b) A group of students are conducting an experiment to investigate the ball-release mechanism shown in Fig. 17.1. The students apply a force F and measure the compression x of the springs.

The table below shows the results.

***F*/N**	***x*/cm**
1.1 ± 0.2	2.0
2.0 ± 0.2	4.0
2.9 ± 0.2	6.0
4.0 ± 0.2	8.0
5.1 ± 0.2	10.0

Fig. 17.2 shows four data points from the table plotted on a F against x graph.

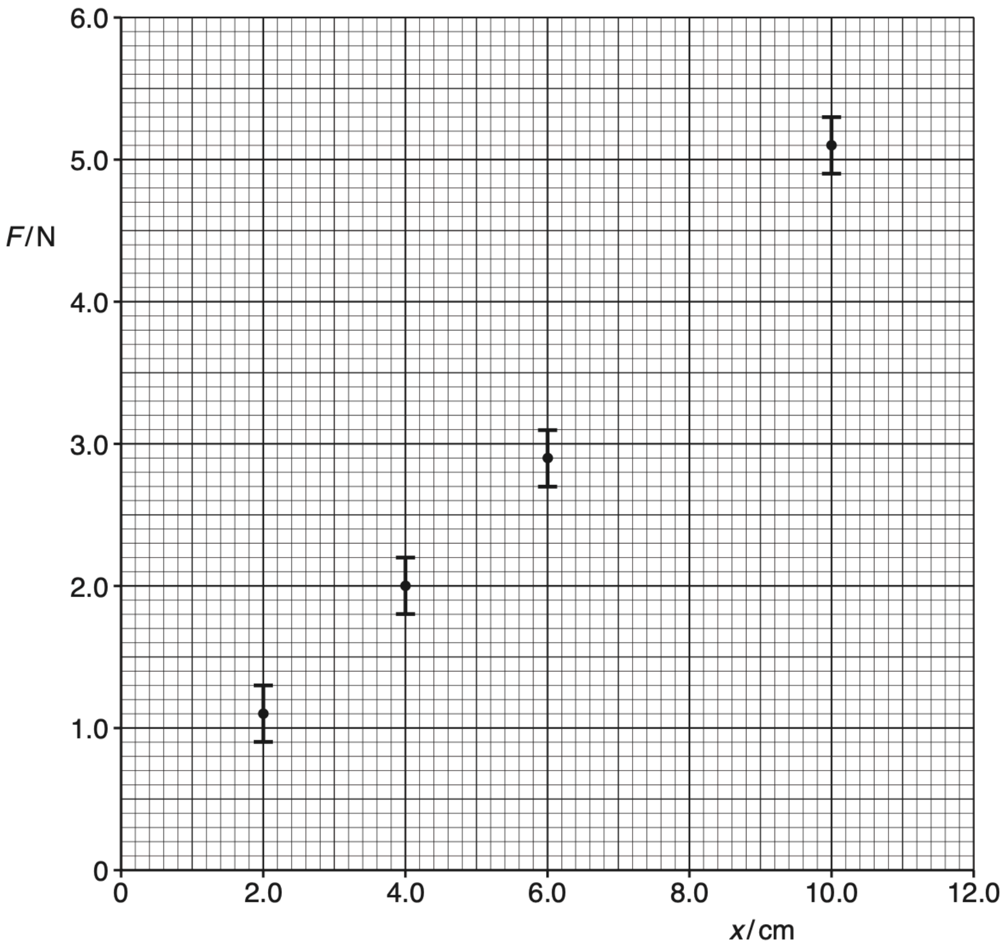


Fig. 17.2

(i) Plot the missing data point and the error bar on Fig. 17.2. **[1]**

(ii) Describe how the data shown in the table may have been obtained in the laboratory.

....

....

.... **[2]**

(iii) Draw the best fit and the worst fit straight lines on Fig. 17.2.
Use the graph to determine the force constant k for a **single** spring and the absolute uncertainty in this value.

k = ... ± .... $N\,m^{-1}$ **[4]**

(iv) State the feature of the graph that shows Hooke's law is obeyed by the springs.

....

.... **[1]**

(v) The mass of the ball is 0.39 kg.

Use your answer from **(iii)** to calculate the launch speed v of the ball when the plastic plate shown in Fig. 17.1 is pulled back 12.0 cm.

v = ... $m\,s^{-1}$ **[3]**

(c) A new arrangement for the ball-release mechanism using three identical springs is shown in Fig. 17.3.

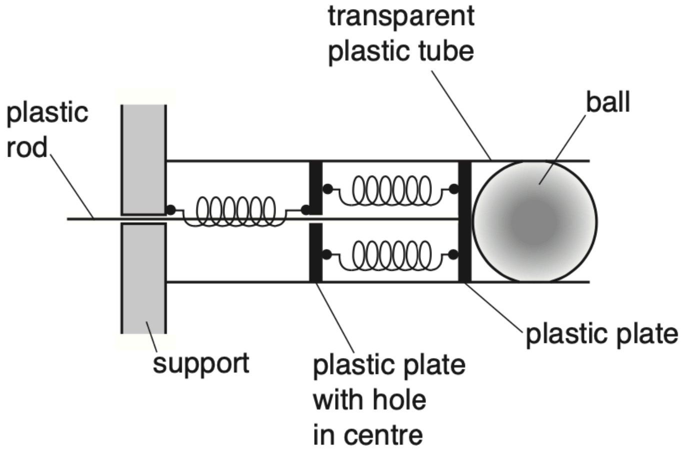


Fig. 17.3

The force constant of each spring is k.
The same ball of mass 0.39 kg is used. The plastic rod is pulled to the left by a distance of x.

Show that initial acceleration a of this ball is given by the equation

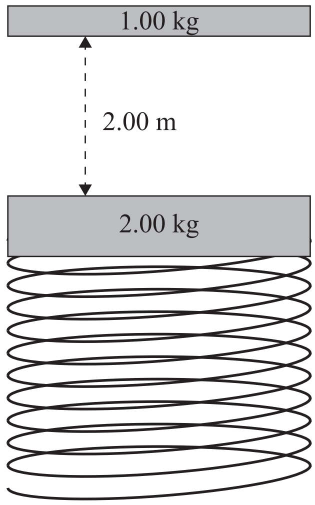
$$a = 1.7\,kx.$$

QUESTION SEVEN: THE SPRING

Acceleration due to gravity = 9.81 m s^{-2}

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

A mass of 1.00 kg is dropped 2.00 m onto a sticky platform of mass 2.00 kg. The platform sits on a spring of spring constant = 1.00×10^2 N m^{-1}.



(a) By using appropriate conservation laws, show that, during the collision between the mass and the platform, ⅔ of the gravitational potential energy lost by the mass is converted into heat.

State any assumptions made.

(b) Show that the maximum amount of additional compression that occurs in the spring is 0.473 m.

(c) Calculate the amplitude of the resulting vibrations.

(d) Show that the total energy at the bottom of the motion is equal to the total energy at the top of the motion.

(8)