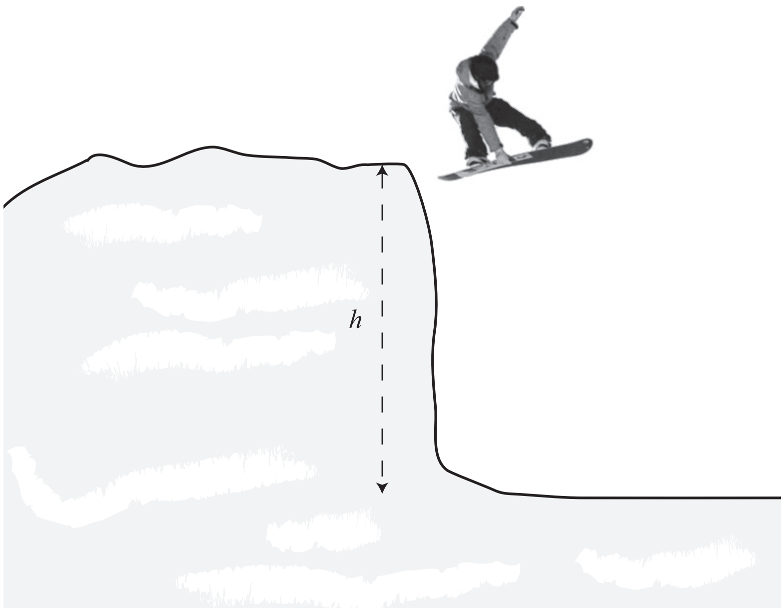
QUESTION ONE: SNOWBOARDING (8 marks)

A snowboarder of mass m rides over an icy ledge onto a horizontal surface below.
The snowboard leaves the ledge at 0 m s^{-1} in the vertical direction and at only a very small horizontal velocity.



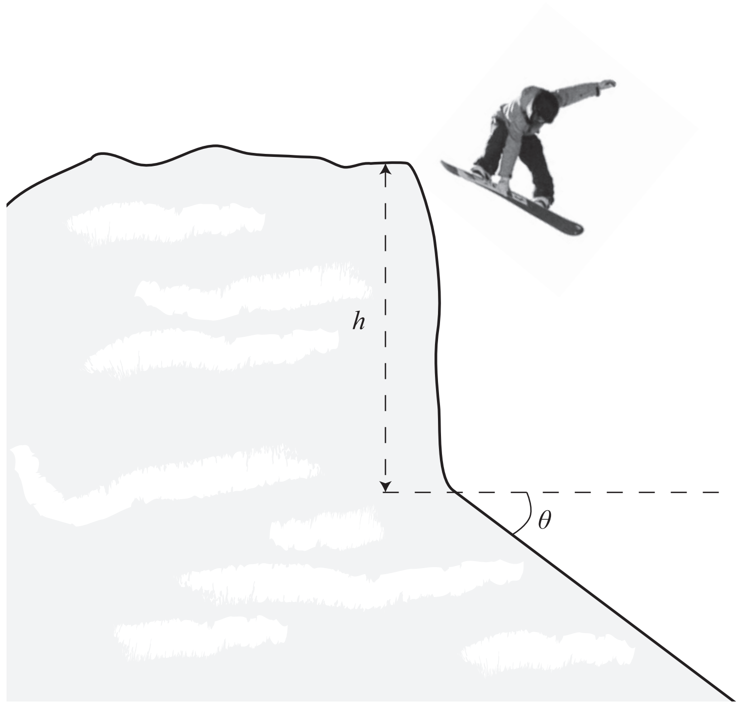
(a) Assuming that the centre of mass drops by a distance b (through the bending of the knees) on impact, show that the average reaction force acting on the snowboarder is

$$F_R = mg\left(1 + \frac{h}{b}\right)$$

(b) By using a height, h, of 3 m and a reasonable estimate of b, calculate the size of the average reaction force experienced by the snowboarder. Comment on how the actual force might differ from the average force.

(c) Show that the time to come to a stop is given by $t = b\sqrt{\frac{2}{gh}}$ and discuss the effect of landing on soft snow (a sample calculation is required).

(d) Having survived his first fall, the intrepid snowboarder makes sure his next fall is onto a surface sloping at angle θ to the horizontal, as shown.



It can be shown that $F_R = mg\left(1+\frac{h}{b}\right)\cos\theta$.

Explain, using physical principles, the effect of the slope on the force experienced by the snowboarder.

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(e) For snowboarders approaching the ledge with a non-zero velocity, the slope can be made so that the reaction force on landing is zero.

Discuss what shape of the slope would be required and why.

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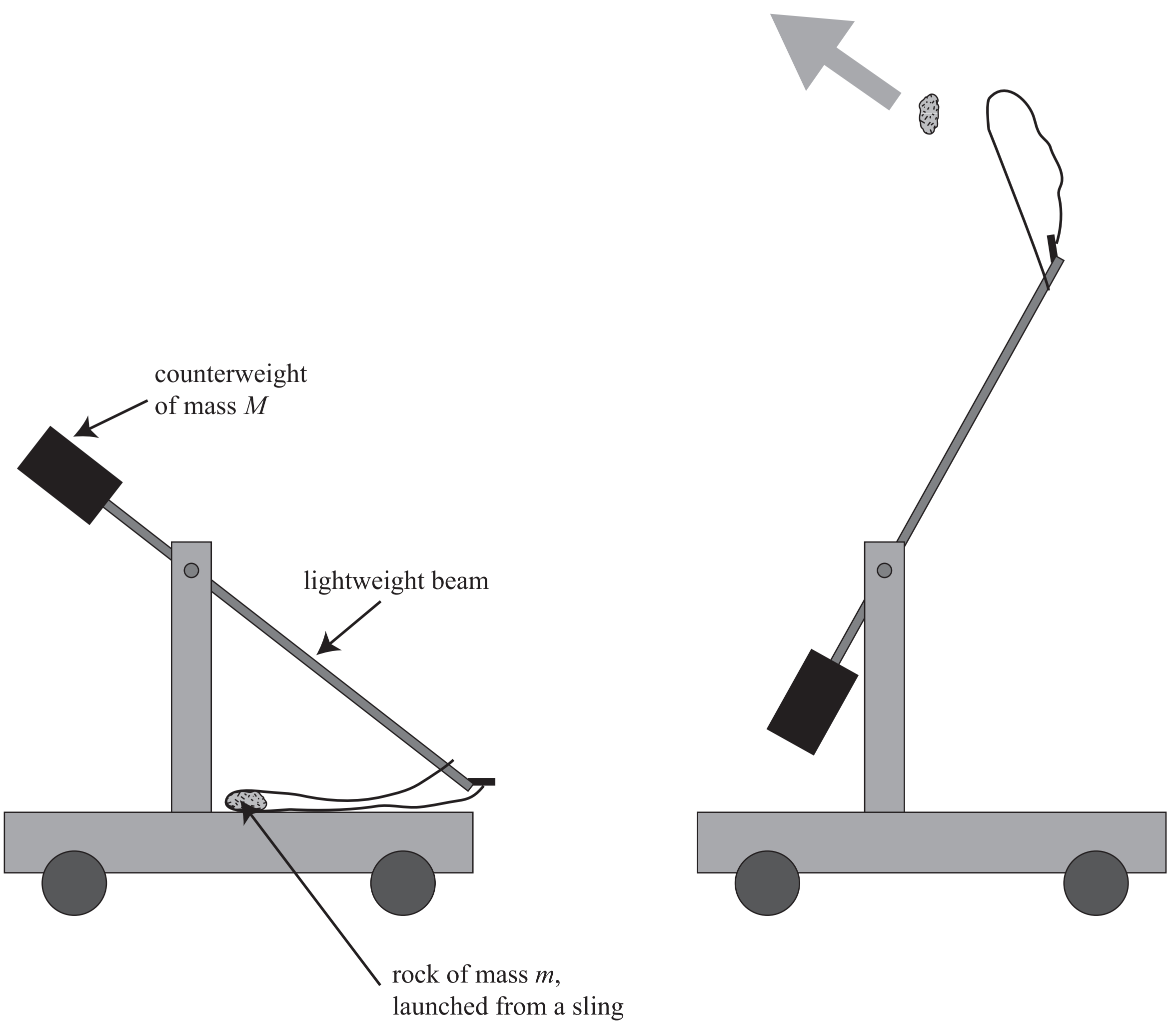
QUESTION TWO: THE TREBUCHET (8 marks)

Acceleration due to gravity = 9.81 m s^{-2}

The trebuchet is a medieval weapon for hurling rocks at fortifications.



(a) State the energy changes that take place when the machine fires the rock.

(b) Assuming that the rock is released from ground level, show that the theoretical maximum range is:

$R = 2\frac{M}{m}h$, where M = mass of counterweight
m = mass of rock
h = height counterweight falls

(c) The maximum range can be increased by mounting the trebuchet on wheels (rather than fixing it to the ground).

Explain.

(d) If a trebuchet with a maximum range of 100 m on Earth were taken to the Moon (where the gravitational field strength is one sixth of that on the surface of the Earth), what would be its range?

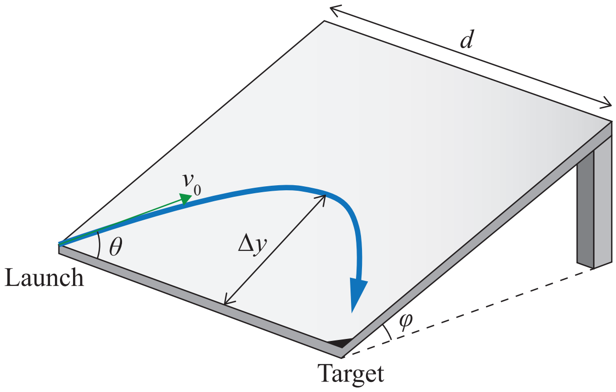
Using physical principles, explain your answer.

(8)

QUESTION THREE: THE RAMP PROJECTILE

Acceleration due to gravity = 9.81 m s^{-2}

$\sin^2\theta + \cos^2\theta = 1$
$\sin 2\theta = 2\sin\theta\cos\theta$



A spring-loaded plunger launches a ball at a speed v_0 from one corner of a smooth flat board that is tilted at an angle φ in order to make the ball hit a small target at the adjacent corner, a distance d away, as shown in the diagram. The ball can be considered to be sliding without friction.

(a) (i) Assuming the angle θ required to hit the target is known,

show that the maximum distance up the board is given by $\Delta y = \dfrac{1}{2}\dfrac{(v_0 \sin\theta)^2}{g \sin\varphi}$.

(ii) By considering the horizontal motion, show that the time to reach the target, Δt, is given by $\Delta t = \dfrac{d}{v_0 \cos\theta}$.

(iii) Derive a relationship for the time to reach the maximum height Δy.

Express your answer in terms of v_0, θ and φ.

(b) Show that the angle θ at which the ball should be launched so that the target is reached is

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{gd\sin\varphi}{v_0^{\ 2}}\right).$$

(c) We now consider the case where the ball rolls without slipping.

If the ball, with mass m and radius r, has a rotational inertia of $\frac{2}{5}mr^2$, and assuming again that θ is known, show, by considering conservation of energy, that $\Delta y = \frac{7}{10}\frac{(v_0\sin\theta)^2}{g\sin\varphi}$.

Explain all reasoning.

(d) (i) Consider the situation when the angle $\varphi = 0°$.

Explain the result produced.

(ii) Comparing the answers for (a)(i) and (c), explain why the answer for (c) is larger.

(8)

QUESTION FOUR: CRICKET – THROW IN FROM THE BOUNDARY

Acceleration due to gravity = 9.81 m s^{-2}

(a) Show that the range, R, of a projectile thrown from ground level at angle, ϕ, to the horizontal with starting velocity, v, is $\dfrac{v^2 \sin 2\phi}{g}$.

(Note that $2\sin\phi\cos\phi = \sin 2\phi$.)

(b) A cricket ball is thrown from ground level with a velocity 28.0 m s^{-1}, and hits a target on the ground 80.0 m away.

Show that the time of flight of the ball is 4.04 s.

The effects of air resistance can be ignored.

(c) The ball is now thrown at the same target, with the same initial speed, but at a lower angle. This time, it is aimed to bounce in front of the target, so that it hits the target on the second bounce. When the ball bounces the first time, it rebounds with the same angle as it came in, but it loses half its speed.

(i) Calculate the time taken for the ball to reach the target.

(ii) Discuss, with physical reasons, the difference in times between parts (b) and (c)(i).

(d) Any real throw of a ball would be from approximately head height, rather than from ground level.

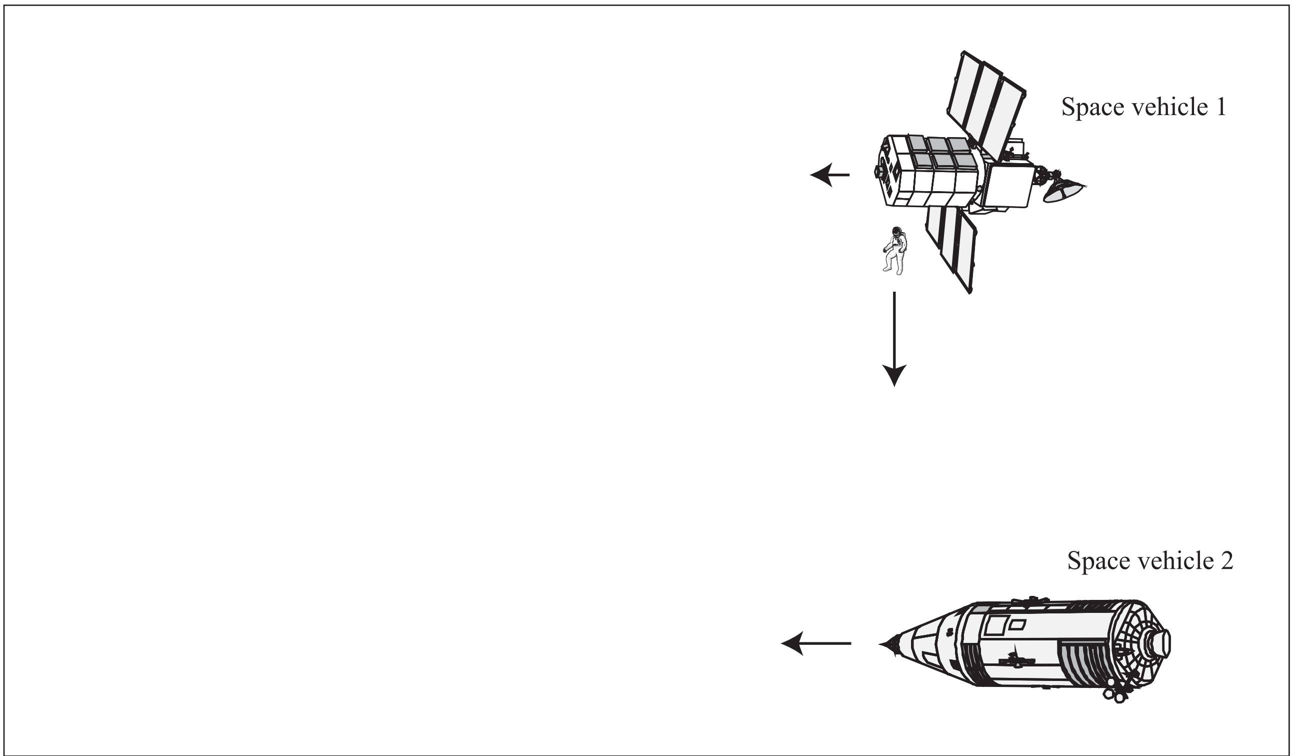
Show that the range achieved by a throw from a height of 2 m above the ground would be

$$v\cos\phi\left(\frac{v\sin\phi+\sqrt{v^2\sin^2\phi+4g}}{g}\right)$$

(8)

QUESTION FIVE: THE ASTRONAUT (8 marks)

Nicole, an astronaut, is assigned to make external repairs to a second space vehicle which is travelling in a direction parallel to her own, but with a greater relative velocity of 2.6 m s^{-1}. The second space vehicle is 85 m in length. Nicole launches herself directly towards the nose of the vehicle with a speed of 6.0 m s^{-1} in a direction perpendicular to the motion of the second space vehicle. At this point, the vehicles are separated by a distance of 200 m. As a safety measure, Nicole is equipped with a launcher that fires a magnetic puck. The magnetic puck is attached to a 4 m length of rope, the other end of which is tied to Nicole. The magnetic puck can only be fired in the direction in which Nicole is travelling. The magnetic puck has a mass equal to 1% of Nicole's mass, but only 0.004% of the mass of the second space vehicle that she is heading towards.



(a) Show that the magnetic puck allows Nicole to attach herself to the second space vehicle.

(b) Nicole can control the speed at which she fires the puck, from 10 m s^{-1} to 750 m s^{-1} relative to herself.

If Nicole fires the magnetic puck when she is 4 m away from the second space vehicle:

(i) Discuss the effect of the puck velocity on the success of the mission.

(ii) Discuss the effect of the puck velocity on the second space vehicle, for the full range of puck velocities.

(8)