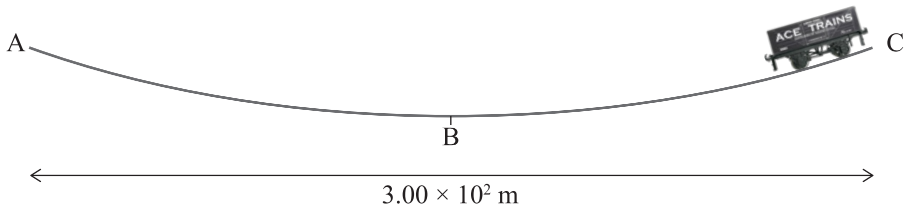
QUESTION ONE: THE RAILWAY WAGON

Acceleration due to gravity = 9.81 m s^{-2}

A railway wagon full of sand is released from rest at point C. The wagon oscillates on a vertically curved track between A and C with simple harmonic motion of period 60.0 s. The effects of friction can be ignored.



(a) (i) State the conditions that must apply for the motion to be simple harmonic motion.

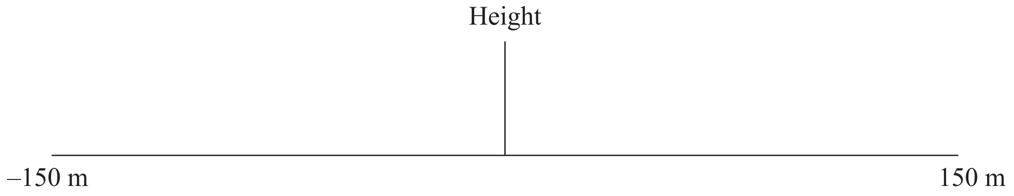
(ii) Show that the maximum speed attained is 15.7 m s^{-1}.

(b) When the wagon is halfway between B and C, calculate its approximate height above B.

(c) (i) The wagon has a small hole from which sand leaks onto the rail track at a steady rate.

Sketch a height profile of the sand on the graph below and explain the shape of the profile.

State any assumptions made.



(ii) When the wagon arrives back at C, the remaining sand is suddenly dumped from the wagon.

Explain what effect this removal of mass will have on the physical parameters of the wagon's motion.

(d) The track A to C is an arc of a circle.

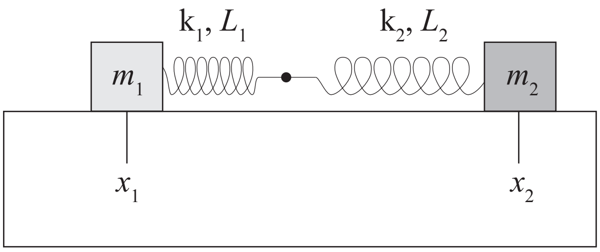
By first calculating the radius of the circle, discuss whether the original assumption that this motion is simple harmonic motion is valid.

(8)

QUESTION TWO: SPRINGS

(a) A spring of length L and spring constant k is cut into two parts of lengths L_1 and L_2 with spring constants k_1 and k_2, respectively.

Show that $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$.



Consider the situation shown above with two springs of spring constants k_1 and k_2 connected together and linking masses m_1 and m_2, which sit on a frictionless surface. The equilibrium lengths of the springs are L_1 and L_2, respectively, and the centre of mass of m_1 lies at x_1 and the centre of mass of m_2 lies at x_2.

(b) Mass m_1 is held fixed while a force F_0 is applied to mass m_2 in the direction of m_1.

Using the result of part (a), show that the change of position for mass m_2 is $\Delta x_2 = F_0 \dfrac{k_1 + k_2}{k_1 k_2}$

Both masses are now released simultaneously. The values for the masses and spring constants are $m_1 = 2.0$ kg, $k_1 = 5.0$ N m^{-1}, $m_2 = 3.0$ kg, $k_2 = 10$ N m^{-1}, and the force $F_0 = 2.0$ N. Assume the mass of the springs is negligible compared to m_1 and m_2.

(c) Describe in detail the resulting motions of masses m_1 and m_2.

(d) Show that the maximum velocity reached by mass m_2 is = 0.40 m s^{-1}.

(e) Explain how the motion of the system would be altered if the mass of the springs was not negligible compared to m_1 and m_2.

(8)

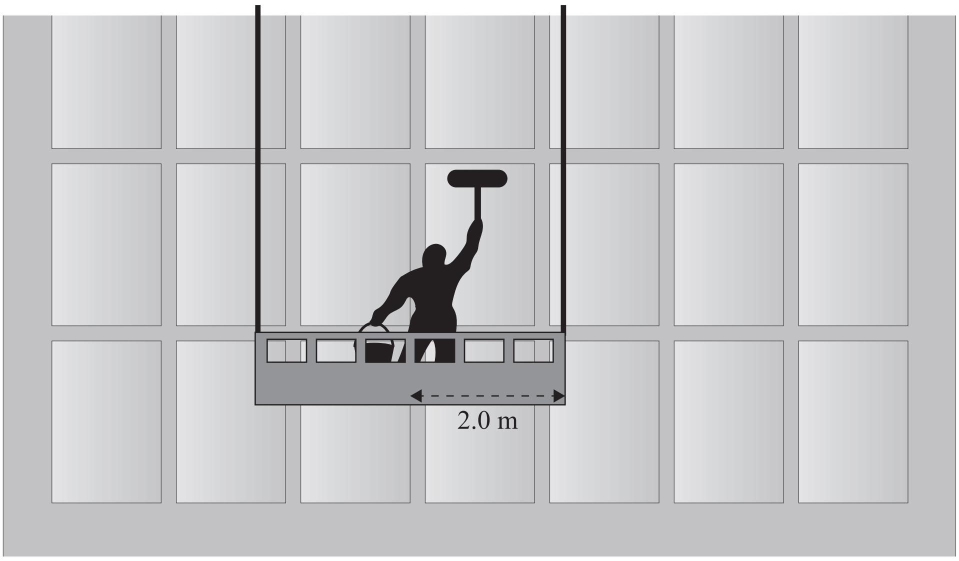
QUESTION THREE: VIBRATING WIRES

Acceleration due to gravity = 9.81 m s^{-2}

Long wires, stretched between two points, can vibrate when a steady wind blows past them. Engineers, in dealing with the problems caused by these vibrations, have found it useful to define the Strouhal number, St, as $\text{St} = f\dfrac{r}{v_w}$, where f is the frequency of vibration, r is the radius of the wire, and v_w is the wind speed.

(a) Show that the Strouhal number is dimensionless.

A window-washing cradle of width 4.0 m is suspended from two cables of equal length.



A steady wind of 10 m s^{-1} causes the cables to vibrate with a frequency of 200 Hz. In this situation, a Strouhal number of 0.20 is typical. The wave speed in a wire is given as $v = \sqrt{\dfrac{T}{\mu}}$, where T is the tension and μ is the mass / unit length.

(b) Explain, using physical principles, why the wave speed in a wire depends on the tension and the mass / unit length.

(c) The cradle has a mass of 100 kg and the window washer, who is standing in the middle of the cradle, has a mass of 75 kg (including his mop and bucket).

Given that the density of the cable material is 8.0×10^3 kg m^{-3}, show that the wave speed in the cables is 18 m s^{-1}.

(d) The window washer moves from the centre of the cradle to a position 1.0 m from the centre.

(i) Compare the wavelengths of the vibrations in the two cables.

(ii) In this scenario, the window washer does not hear beats.

Explain the physical conditions required for him to hear beats from vibrations in the wires when he is standing in the window-washing cradle.

(8)

QUESTION FOUR: SIMPLE HARMONIC GRAVITY (8 marks)

Volume of a sphere = $\frac{4}{3}\pi r^3$

Radius of the Earth = 6.37×10^6 m

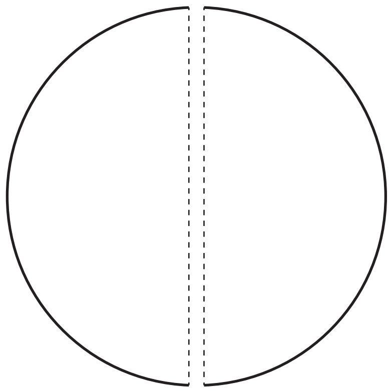
Mean density of the Earth = 5.5×10^3 kg m^{-3}

Universal Gravitational Constant = 6.67×10^{-11} N m^2 kg^{-2}

(a) Imagine that a hole is drilled from the South Pole, through the centre of the Earth to the North Pole.

By using Newton's Law of Gravitation, show that, if Earth has uniform internal density ρ, and ignoring air resistance, an object dropped into the hole will perform simple harmonic motion.

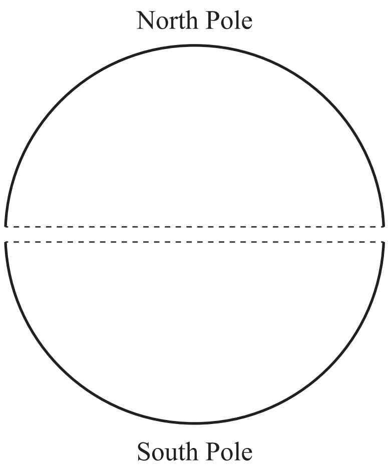
Hint: For an object at distance r from the centre of the Earth, the net gravitational force on the object, due to those parts of the Earth's mass that lie at distances greater than r, is zero.



(b) Calculate the period of this simple harmonic motion.

(c) Explain in what way this period is related to the orbital period of satellites in low Earth orbits.

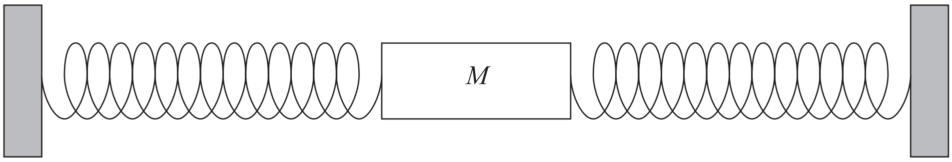
(d) Discuss the differences in the motion of the falling object that would be observed if the hole were drilled from an equatorial position rather than from the South Pole.



(8)

QUESTION FIVE: SIMPLE HARMONIC MOTION (8 marks)

An object of mass M is connected to a pair of identical springs as shown.

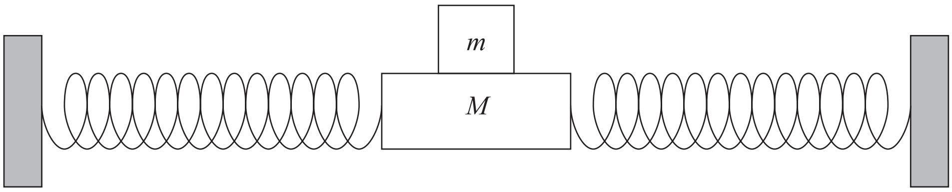


(a) Show that M will oscillate with SHM when given a small horizontal displacement, x.

(b) This oscillating mass system starts with zero momentum when released from its initial displacement. At the mid point of the motion the system seems to have gained quite a lot of momentum. This appears to contradict the law of conservation of momentum.

Explain why there is no contradiction.

(c) A second object of mass m, initially at rest, is placed gently onto mass M and sticks to it.



(i) By what factor does the energy of the oscillation change if mass m is added when the mass M is moving through its position of maximum velocity?

(ii) By what factor does the energy of the oscillation change if mass m is added when mass M is at its position of maximum acceleration?

(d) The maximum frictional force between masses m and M is μN, where μ is the coefficient of static friction between the pair of masses and N is the normal reaction force on mass m.

Show that the maximum amplitude of oscillation, A, for which the mass m will not slip is given by

$\frac{\mu g(M+m)}{2k}$, where k is the spring constant of each spring, and g is the acceleration due to gravity.

(8)