

3. (a) Explain why the age of the Universe can be approximated as $\frac{1}{H_0}$, where H_0 is the Hubble constant. [2]

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- (b) The megaparsec (Mpc) is a unit of astronomical distance equal to 3.09×10^{22} m. Use Hubble's law to show that the expected redshift for a supernova at a distance of 1 Mpc for a wavelength of 486.1 nm is approximately 0.11 nm. [3]

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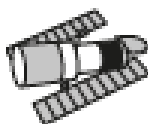
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- (c) The spiral galaxy shown is rotating anticlockwise and is viewed by the Hubble Space Telescope.

Hubble Space
Telescope



Images not to scale



Rotating spiral
galaxy

The measured blue shift at point A of the galaxy is 0.22 nm (for the 486.1 nm wavelength) and the measured redshift at point B of the galaxy is 0.66 nm (for the same wavelength). Calculate the recessional velocity of the galaxy and the rotational speed of the galaxy at A (assume that A and B have the same rotational speeds). [5]

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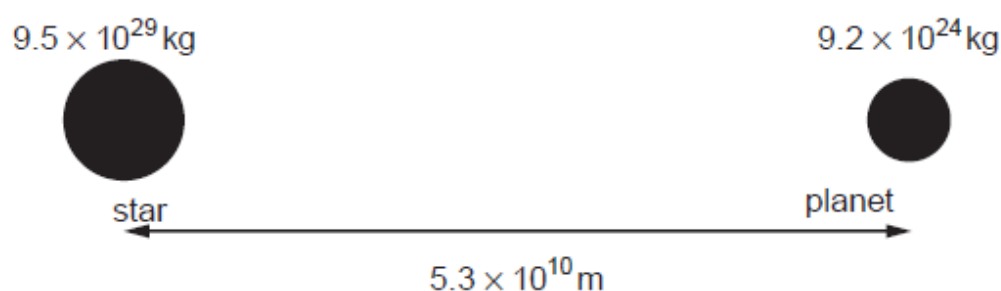
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The planet (Kepler-186f) orbits the star (Kepler-186) and is considered to be the most Earth-like planet yet discovered. Approximate details of the system are shown in the diagram below.



- (a) (i) Show that the radius of orbit of the star around the centre of mass of the system is 513 km. [2]

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- (ii) The period of orbit of the planet is 130 days. Calculate the orbital velocity of the star. [3]

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- (iii) The velocity calculated in part (a)(ii) is small. Explain why small velocities are difficult to measure using red shift. [2]

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- (b) (i) Calculate the gravitational force between the star and the planet. [2]

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- (ii) An historic alternative to Newton's Law of Gravitation involved invisible steel rods connecting stars and planets and providing the force for keeping planets in orbit. A hypothetical cylindrical steel rod of radius $7.0 \times 10^6 \text{ m}$ and length $5.3 \times 10^{10} \text{ m}$ joins the planet to the star. Determine whether or not this steel rod is strong enough to keep the planet in orbit and explain why this theory is not accepted. (Breaking stress of steel = $4.5 \times 10^8 \text{ Pa}$.) [4]

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- (c) (i) Calculate the potential energy of the star-planet system (see diagram in part (a)). [2]

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- (ii) In fact the planet has a slightly elliptical orbit. Explain how conservation of energy applies to this elliptical orbit. [2]

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Einstein's theory of stimulated emission, published in 1916, laid the foundation for lasers but it took humans another 44 years before the first successful ruby laser was produced. However, seven years beforehand a maser had been produced. This is similar to a laser but involves microwaves instead of light – **microwave amplification by stimulated emission of radiation**.

It turns out that masers are easier to build than lasers because the lifetime of metastable energy levels tends to be proportional to $\frac{1}{\text{frequency of radiation}}$.

The ingenuity of humankind is often marvelled upon when such complicated devices as lasers and masers are used in devices such as DVD players and atomic clocks. However, nature itself is somewhat more modest – natural masers have been produced for billions of years in the atmospheres of stars, comets, star-forming regions, supernova remnants and even super massive black holes. And all this time the Universe has kept quiet about its technological achievements.

Essentially what happens in star masers is this – light and explosions from stars excite nearby gas regions. In these nearby gases, light and collisions lead to high energy levels becoming populated by electrons. Some of these higher energy levels will be metastable, setting up a maser amplifying region. As in any laser, the process has to start with a spontaneous emission accidentally shooting off in the correct direction, but afterwards, stimulated emission takes over and the maser beam starts towards infinity; infinity is stretching the truth a bit but it does sound good. In reality the intensity of the maser beam increases rapidly and the beam propagates at the speed of light as em radiation tends to do.

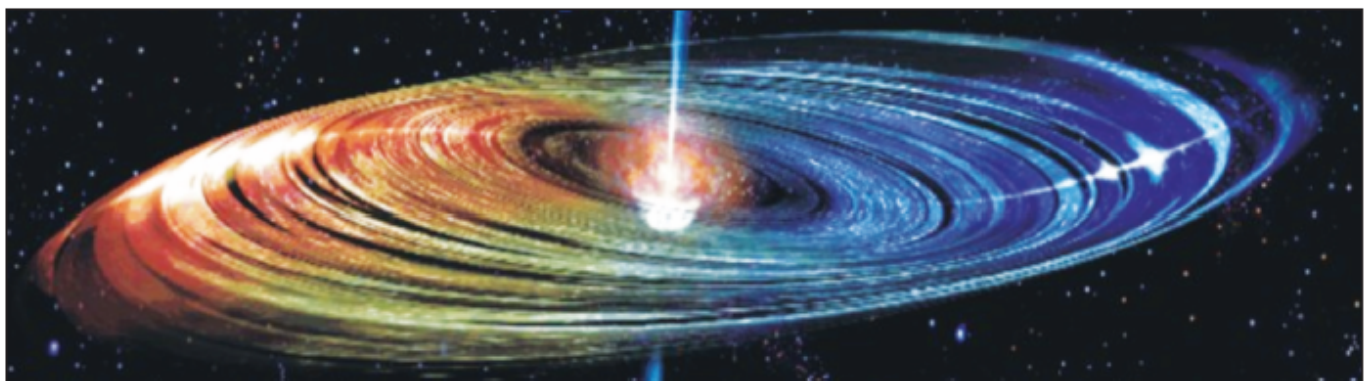


Diagram 1

In all fairness to humans, there is one aspect of laser design that nature has not succeeded in producing by itself and that is the resonant cavity of a laser. Multiple passes through a laser amplifying medium improves the quality of laser operation immensely. This is achieved among humans using two mirrors but as yet this technological advancement seems to be missing in gas regions around stars.

Although a resonant cavity seems to be missing in natural masers there does seem to be a high degree of beaming. Small differences across the irregularly shaped maser cloud disk lead to vast differences in intensity due to exponential gain. The directions in the gas disk that have a longer length of population inversion will appear much brighter (as the increased maser amplification leads to an exponential increase). The majority of the radiation will emerge along this line of greatest length in a "beam"; this is termed *beaming*. Not quite as good as a laboratory laser cavity but pretty impressive for a seemingly random gas cloud.

Megamasers is the term used for water masers in the gas cloud around black holes.

Most large galaxies with a nucleus and a bulge have, at their centres, a supermassive black hole. When the black hole is actively accumulating matter, it releases a tremendous amount of energy, and the galaxy is said to have an active nucleus. Buffered by the dust in the flow of matter into the black hole, molecules can survive there and get energised by collisions with other molecules and dust particles. Water molecules are common in this environment, and they can emit maser radiation. A water maser can shine a beam of microwaves at a very specific

can emit maser radiation. A water maser can shine a beam of microwaves at a very specific frequency, 22.235 GHz, which corresponds to microwaves about 1 cm in wavelength. The primary reason megamasers have been studied in detail is that they make excellent tools to help us understand the environment around black holes. Small changes in the detected frequency of the masers, owing to the Doppler effect, mean that we can determine the line-of-sight velocity of maser clouds very precisely. All this means that the mass of black holes can be determined 20 times more accurately.

Another primary science goal that we can address with studies of water megamasers is measuring distances to galaxies. Measuring distances is a notoriously difficult problem in astronomy, and one of the most important. By analysing the internal dynamics of water maser systems, we can measure the rotation velocity, v , of maser clouds as they orbit the black hole from the Doppler shift of the maser lines. We can also measure the centripetal acceleration, a , of maser clouds by observing how the Doppler shift velocity changes over time. Using the simple relation for centripetal acceleration:

$$a = \frac{v^2}{r}$$

we can then calculate the radius of the gas 'disk' (see below) orbiting the black hole.

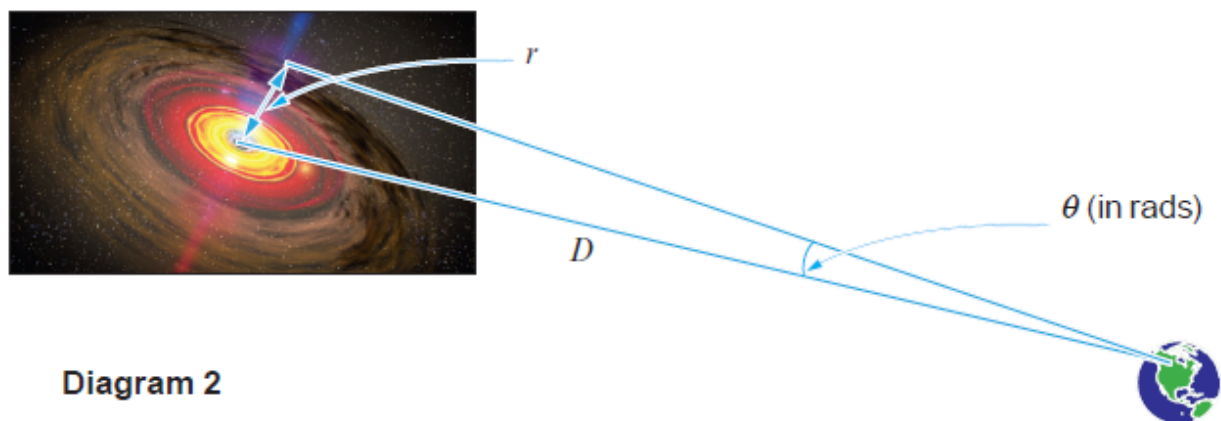


Diagram 2

We can also measure the angular size (θ) of the gas disk (in radians). Given the angular size (θ) and the radius, r , of the gas disk, we can obtain the distance, D by assuming that the angle (θ) is small. The simplicity of this method is remarkable and gives reliable results that are not dependent on controversial or unproven theories. In fact, the results obtained thus far from a couple of suitable galaxies with megamasers leads to a Hubble constant of $(68.9 \pm 7.1) \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Answer the following questions in your own words. Direct quotes from the original article will not be awarded marks.

- (a) In your own words explain what a megamaser is and how it works. (See paragraphs 1, 4, 7 and 8.) [5]

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- (b) If the lifetime of a metastable level leading to visible light is $2\mu\text{s}$, estimate the lifetime of a 22.235 GHz maser transition. (See paragraph 2.) [3]

- (c) The recessional velocity of the galaxy used in paragraph 10 is $3\,300\text{ km s}^{-1}$. Use Hubble's law to calculate the distance of the galaxy in Mpc. (N.B. use the value of H_0 in paragraph 10. There is no need to change the units.) [2]

- (d) What does the author mean by "We can also measure the centripetal acceleration, a , of maser clouds by observing how the Doppler shift velocity changes over time"? (See paragraph 9.) [3]

- (e) (i) Doppler shift measurements are obtained for a gas disk orbiting a black hole. The orbital velocity of gas around the black hole is measured as 410 km s^{-1} and the acceleration of the gas disk is measured as 6 km s^{-1} **per year**. Calculate the distance of this region of the gas disk from the black hole. (See paragraph 9.) [3]

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- (ii) The angular size (θ) in radians of the gas disk is measured as 5.1×10^{-9} rad $\pm 10\%$ and its distance from Earth is measured as 1.53×10^{23} m $\pm 10\%$. Evaluate whether or not these values are consistent with your calculation in part (e)(i). (See Diagram 2.) [4]
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- (b) The radius of the geostationary orbit above the Earth's equator is 42000 km and the Earth-Moon distance is 380000 km. Use Kepler's 3rd law to calculate the period of the Moon's orbit. [3]

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- (c) Estimate the minimum possible orbital period for a satellite orbiting the Earth, stating any assumption that you make (the radius of the Earth is 6370 km). [3]

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